

PHASOR MEASUREMENTS IN DYNAMIC STATE ESTIMATION OF POWER SYSTEMS

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Abstract—As the expansion of power systems continue, the challenge of real monitoring and control gets bigger. Important network parameters are measured at various points on the system and transferred to the control center, where the data is used for various Energy Management System (EMS) functions. State Estimation forms one of the primary EMS functions at the control center. Accuracy of the state estimation is directly dependent on the accuracy of the measurements used. Hence, it is highly essential to use accurate measuring devices. Phasor Measurement Units (PMU), with their inherent high accuracy and the unique ability to measure the voltage angles, offers a great advantage in improving the overall accuracy of the state estimation. Few techniques have been proposed in the literature for using PMUs in static state estimation. It is also important to use PMU data in to Dynamic State Estimation (DSE), which predict the state of the system one step ahead. Hence, an attempt has been made in this paper to present a technique, which can incorporate the PMU measurements in to dynamic state estimation.

Index Terms—dynamic state estimation, energy management systems, phasor measurement units, static state estimation

I. INTRODUCTION

Energy Management Systems (EMS) forms the basis for efficient operation and control of the modern day power systems. State Estimation, forms one of the most important EMS functions whose output will be used in other EMS functions such as, security analysis, economic load dispatch etc. State estimation is the method of estimating the voltage magnitude and angles at all the buses of a power system, from the available measurements. These measurements are obtained through a suitable communication medium and processed through state estimation algorithms, to obtain the voltage magnitude and angle at all the buses of the network under consideration. This estimation technique is called static state estimation, as it calculates the state of the system at a given instant of time by using the measurements at the same instant of time.

State estimation is a computationally intense process. Hence, it is run either at fixed intervals of time or once it is found that the power system has sufficiently changed from its

pervious state. But the power system is a quasi-static system, hence changes continuously, albeit slowly. Hence, a state estimation run at fixed intervals of time, gives a snap shot of the system at that point of time, but fails to capture this dynamic nature of the power system. This lead to the development of the Dynamic State Estimation (DSE) algorithms, which calculate the state of the system by modeling the time varying behavior of the power system. One of the first papers on DSE was published by Debs and Larson [1]. DSE algorithms use mathematical modeling of the time behavior of the system to cater to the dynamic changes of the power system and predict the state of the system one step ahead. This ability of prediction has tremendous advantages in performing security analysis and allows more time for the operator to take control decisions in case of an emergency.

The measuring devices and their accuracy, plays a major role on the accuracy of state estimation. Higher the accuracy of measurements, higher is the accuracy of estimation. Normally the measurements processed in the state estimation algorithms include, power flows, injections and voltage measurements. But in the mid 1980s, with growing applications of microprocessors, a new device called the Phasor Measurement Unit (PMU) was developed. Although PMUs were initially developed for protection applications, its importance as highly accurate measuring device was quickly realized and incorporated in to state estimation techniques [2]. Its advantage lies in the fact, that it can measure both voltage and current phasors at the installed bus. More over, PMU measurements are much more accurate than the normal SCADA measurements. The importance of PMUs can be gauged by the fact that for the first time both voltage magnitude and voltage angular measurements could be obtained directly from PMU. In other words, if PMUs were to be installed at all buses of a power system then we could directly measure the state of a power system, instead of estimating it. The process of state estimation would then reduce to solving a set of linear equations. But due to the high costs involved, PMUs have not been installed at all buses of the power system. As a result, the measurements obtained at the control center are a mixture of both PMU and the normal SCADA data. Since PMU measurements, are highly accurate, the weightage given to them in the state estimation process is much higher than the normal measurements. But the weights cannot be arbitrarily high, as this may drive the Information Matrix of the estimation process towards singularity. More over, PMUs also introduce a new measurement parameter, in the form of voltage angles, which the traditional instruments could not measure. This marked a permanent change in the way state estimation was being performed. Hence, many researchers have developed algorithms, to incorporate both

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PMU and normal SCADA measurements in to the state estimation algorithms.

Though various techniques have been developed to incorporate the phasor measurements in to the static state estimation, so far there have not been many attempts towards incorporating them in to dynamic state estimation. Hui Xue *et al.* [3] have proposed a PMU based DSE technique, which predicts the load flow data at the next instant of time by a historical data base and combines the PMU data at the next instant of time for dynamic state estimation of power systems.

This paper presents a novel technique to include PMU measurements in to a dynamic state estimation technique and study its effects on the estimation process. In the technique presented here, incorporation of PMU based voltage magnitude and voltage angular measurements in to the DSE has been studied. With the power system being expanded, especially in the developing countries, more accurate monitoring and control of the system becomes essential, necessitating more PMUs to be installed. Hence, PMU based dynamic state estimation becomes very important in the modern day energy management systems.

II. PHASOR MEASUREMENT UNITS

In the 1980s, a device called the Synchronous Component Distance Relay (SCDR) was developed for transmission line protection. This device had the ability to calculate the synchronous components of the voltage and current on the transmission lines. Since the state vector of a power system consists of positive sequence voltage vectors at all the buses of a power system, this device clearly had the ability to directly measure the state vector [2]. Though the device was developed primarily for protection applications, its ability to directly measure the voltage phasors paved the way for its usage in state estimation and this led to the eventual development of Phasor Measurement Units (PMU). The PMUs main advantage comes from the fact that, probably for the first time we can have synchronized measurements of parameters across the power system. The PMUs use the GPS system to synchronize the measurements. The accuracy of the synchronisation can be as low as $1\mu\text{s}$. The PMUs receive the one pulse per second GPS signal, along with the time tag, which is used as an initiation for the measuring the positive sequence voltages and currents. This data is in turn transmitted to the control center, through suitable communication channels for monitoring and control applications [4], [5]. As PMUs can measure the phasors with extremely high accuracy which are also synchronized, their addition in to the state estimators, even if in a few locations, should improve the performance of the state estimation process considerably [2].

III. PHASOR MEASUREMENTS IN STATIC STATE ESTIMATION

Before venturing in to dealing PMUs in dynamic state estimation, it's important to take a look at some of the conclusions obtained from the experience of including PMUs in static state estimation. These conclusions drawn from incorporating PMUs in static state estimation have been taken in to account while developing algorithms for incorporating PMU measurements in DSE. As we know, PMUs may not be

installed at all the location of a power system, the estimation algorithms have to process a mixture of both the PMU measurements and the normal SCADA measurements [6], [7].

There are two different schools of thought regarding the way of incorporating the PMU measurements in to state estimation. The first method, called the Direct Substitution technique, suggests that the PMU data are far superior to the normal SCADA data and hence have to be collected and used separately from the SCADA data. This method considers that the PMU based voltage magnitude and angular measurements at a particular bus, to be equal to the state at that particular bus and the estimation process is carried out only for other buses where PMUs are not installed. The other school of thought suggests that, though the PMU data are different from the SCADA data, they can nevertheless be used along with SCADA data in the estimation process, albeit with higher weights [6], [7]. The first method uses the voltage magnitude and angle measuring ability of the PMUs fully. But the disadvantage of this method is that the high accuracy of the PMU measurements is not contributing to the estimation process. But in the second method, the high accuracy of the voltage and angular measurements of PMU participate in the estimation process and hence help in reducing the errors in estimation due to other less accurate measurements. In the second method, the accuracy can further be improved by replacing the voltage magnitude and angular estimates corresponding to the PMU measurements by PMU measurements themselves, once the state estimation is complete [6]. This method is found to produce lesser error in the final state estimates

Some of the conclusions obtained from the PMU based static state estimation techniques are [2], [6], [8] and [9]:

1. The angular measurements play the same role in active measurement residuals as the voltage measurement plays in the reactive measurement residuals.
2. The introduction of highly accurate angular measurements increases the convergence speed especially in larger systems.
3. State estimation results are more sensitive to angular measurement errors than to errors in power flow measurements.
4. Of the two estimation techniques presented, the second method of incorporating PMU measurements with larger weights is preferable as it has better accuracy.

These conclusions form the basis of our model for incorporating the phasor measurements in dynamic state estimation.

IV. DYNAMIC STATE ESTIMATION

Power system is a dynamic system, and hence changes continuously but slowly. The static state estimators, failed to model this quasi-static behavior of the power system. The DSE techniques have a mathematical model for the time behavior of the power system and hence deliver more realistic estimates. DSE uses the present (and some times previous) state of the power system along with the knowledge of the system's physical model, to predict the state vector for the next time instant. Once the new measurements at the next instant of time arrive, the predicted values are filtered to obtain a more

accurate estimate of the states. This prediction feature of the DSE provides vital advantages in system operation, control, and decision-making. It allows the operator more time to act in cases of emergency, helps in detection of anomalies, bad data etc. [9], [10]. The following are the main steps followed in the DSE procedure:

- **Mathematical modeling:** The first step in DSE is to identify a suitable mathematical model for the time behavior of the power system.
- **Parameter Identification:** This step involves the calculation of the parameters that make up the mathematical model describing the behavior of the power system
- **State Prediction:** This step, predicts the state vector at the instant of time (say 'k+1'), from the knowledge of state vector at the present instant (say 'k') and the mathematical model.
- **State Filtering:** Once the measurements are obtained for the instant 'k+1', the predicted state vector is simply updated to obtain a more accurate estimate of the state of the system.

At the end of these three steps, the DSE gives an updated estimated state vector of the power system, which in turn will be used by other EMS functions.

Leite Da Silva et al [11] has presented a technique, which uses Holt's double exponential smoothing technique for prediction and Extended Kalman Filter (EKF), for the filtering process. This paper uses the above DSE technique for incorporating phasor measurements. The mathematical model for the time behavior of the system is given by:

$$x_{k+1} = F_k x_k + G_k + w_k \quad (1)$$

Where x_{k+1} and x_k represent the state vector at instants 'k+1' and 'k' respectively, F_k is the function representing the state transition between two instants of time, G_k is associated with the trend behavior of the state trajectory and w_k is the white Gaussian noise with zero mean and covariance Q . The measurement model is given by:

$$Z = Hx + v \quad (2)$$

Where Z is the measurement vector, x is the state vector, H is the Jacobian of the nonlinear function relating the state vector and the measurements and v is the noise in measurements with a standard deviation of R . Here R is of the form:

$$R = \begin{pmatrix} \sigma^{-2} & 0 \\ 0 & \sigma_{PMU}^{-2} \end{pmatrix} \quad \text{Where } \sigma^{-2} \ll \sigma_{PMU}^{-2} \quad (3)$$

Where ' σ ' is the standard deviation of the normal SCADA measurements and ' σ_{PMU} ' is the standard deviation of the PMU measurements.

The parameters, F_k and G_k are calculated using the Holt's double exponential smoothing method and the predicted state vector is obtained by the following equation:

$$\bar{x}_{k+1} = F_k \hat{x}_k + G_k \quad (4)$$

Where, \bar{x} is the predicted value at instant 'k+1'. The covariance of predicted values at instant 'k+1' is given by:

$$N_{k+1} = F_k P_k F_k^T + Q \quad (5)$$

Where, ' P_k ' is the covariance of the estimate at instant 'k'.

This predicted estimates, will then be used to predict the measurements at the next instant of time. When the measurements at the k+1 instant arrive, the predicted state vector \bar{x} has to be updated, to obtain the filtered estimates.

The optimization function here would be, to minimize the difference between predicted measurements and the actual measurements at the 'k+1'th instant. The second factor to minimize is the difference between the predicted and the actual state vector. Hence the optimization function is given by:

$$J(x) = [Z - h(\bar{x})]^T R^{-1} [Z - h(\bar{x})] + [x - \bar{x}]^T N^{-1} [x - \bar{x}] \quad (6)$$

The Extended Kalman Filter (EKF) technique is used for optimizing the above equation and the final equation for the filtering step can be written as:

$$\hat{x}_{k+1} = \bar{x}_{k+1} + K_{k+1} [Z_{k+1} - h(\bar{x}_{k+1})] \quad (7)$$

Where $K_{k+1} = \Sigma H^T R^{-1} = [H^T R^{-1} H + N^{-1}]^{-1} H^T R^{-1}$, is called the Gain matrix [9], [10], [11], [12].

At the end of these steps, the DSE gives an updated estimated state vector of the power system, which can in turn be used by other EMS functions for their operation.

V. PMUS IN DYNAMIC STATE ESTIMATION

From the experience of incorporating PMUs in static state estimation, it can be concluded that, PMUs by and large increase the performance of the state estimation. Keeping in mind the conclusion drawn from the PMU based static state estimators we propose a PMU based DSE technique. With PMUs we can have much more accurate filtered states, resulting in more accurate predicted states for the next instant of time. Hence, the operator will have much more reliable data for performing security analysis and taking control decisions. In these days of expanding power grids and the system being pushed towards its limits for maximum utilization, any improvement in accuracy of the prediction and accuracy of estimated state variables can be extremely useful.

The DSE model used here is as described in the previous section. In this paper incorporation of PMU based voltage magnitude and voltage angular measurements in to the DSE will be studied. Some of the observations from the PMU based static state estimation techniques, which can be useful in modeling the PMU based dynamic state estimation are:

1. The system will consist of a mixture of both PMU measurements and normal SCADA measurements.
2. The PMU measurements will be used in the state estimation along with the SCADA measurements, albeit with higher weights.
3. Once the filtered estimates are obtained, voltage magnitude and angular estimates corresponding to the PMU measurements are replaced by the PMU based voltage and angular measurements themselves, to increase the accuracy.
4. The behavior of the DSE will be studied by observing its response for varying weights of the PMU measurements.

The PMU based DSE technique presented in this paper, has been implemented and tested on a 5-bus system. The system dynamics is simulated through change in the injections at various buses. The PMU will be introduced at each bus, one at a time, and the DSE program is run for each such placement

of PMU. At each location, the weight values given to the PMU measurements are varied and the behavior of the estimation process studied. Once the placement of PMU at all the buses has been completed, a combination of PMUs at 2 buses of the system will be introduced. These combinations are then repeated for various weight values of the PMU based measurements. The accuracy of the estimates for various combinations of the above experiments will give a fair idea about the performance of the PMU based DSE technique that has been implemented. The simulation procedure and other details of the implementation are given in the next section.

VI. IMPLEMENTATION DETAILS

The steps of implementation of the algorithm follow the same order as described for the DSE model in the previous section. The value of Q is fixed at 0.0001. The values of the constants used in Holt's method α and β , are fixed at 0.4 and 0.75. The measurement error covariance for the normal SCADA based measurements and the PMU measurements are fixed through out the simulation procedure, as they are assumed to vary little during the time interval of the simulation.

For the basic steps of DSE simulation, the model envisaged by Leite da Silva et al [11] has been used, with a few changes as per the current problem's requirement. The details of the 5-bus system, used for implementing the PMU based DSE is as shown below:

TABLE I
STAGG 5-BUS SYSTEM BASE CASE DETAILS

Bus	V	Theta (°)	PL (MW)	QL (MW)	PG (MW)	QG (MW)
1	1.06	0	0	0	-	-
2	1.0711	-2.27	20	10	40	30
3	1.0877	-3.76	45	15	40	-
4	1.0734	-4.17	40	5	0	0
5	1.049	-5.24	60	10	0	0

Where PG is the Value of active generation at the bus, QG is the value of the reactive generation at the bus, PL is the active load at the bus and QL is the reactive load at the bus.

It should be noted that the above system details hold good only for the base case. The above values will be changed as per the requirements for simulating a dynamic system environment. The details of the network that has been used for simulation can be found in reference [13]:

The dynamics of the system is simulated by incrementing the injections at all the buses of the system apart from bus1, which serves as the slack bus. The increments are carried out on ten successive sample time intervals. At every sample time interval, the injections at all buses, apart from the slack bus and the reactive injections at PV buses, are varied at a fixed rate of about 5% of the value at the previous time sample and a small additional jitter. The initial and final values of the injections after the 10 sample time intervals are as shown below.

TABLE II
INITIAL AND FINAL INJECTIONS AT ALL THE BUSES

Bus No.	Initial values		Final Values	
	Active Injection (MW)	Reactive Injection (MW)	Active Injection (MW)	Reactive Injection (MW)
1	88.0543	-55.6247	92.68	-73.7255
2	20	20	45.1558	45.156
3	-5	43.27	4.4334	24.8115
4	-40	-5	-52.5779	4.4333
5	-60	-10	-85.1558	-3.711

The measurement set consists of 11 normal SCADA based measurements, which includes active and reactive flow measurements on three lines, active and reactive injections at two buses and a voltage magnitude measurement at one bus. As mentioned before, the PMU based measurements, considered presently are voltage magnitude and voltage angular measurements. These measurements are introduced in addition to the existing eleven normal measurements. The simulation results and its impact are discussed in the next section.

VII. SIMULATION RESULTS

From the PMU based static state estimation, we know that PMUs can be used along with the normal measurements in the state estimation procedure, so that the high quality measurements of the PMUs can drive the estimates of other low quality measurements towards their true values. Hence, the normal understanding goes that, more the number of PMU measurements, higher is the performance of the estimation algorithm. The performance of the DSE is judged by calculating the amount of variation of the estimates from their actual values. The actual values are obtained for simulation purposes, by performing load flow, at every time sample.

To understand the impact of the PMUs on the DSE, we need to understand how the location and the weightage associated with the PMU affects the estimation procedure. At least 2 PMUs are necessary in order to calculate the voltage angular vector, as the voltage angle at a particular bus is the angular deviation of the voltage vector at that particular bus, with reference to the voltage vector at the reference bus. Hence, one PMU is always assumed to be present on the reference bus.

In our study, PMUs are introduced one at a time (apart from the one at reference bus) and increased from there on. The test cases considered for the present discussion and their results are as shown below:

A. Single PMU scenario

As the title suggests a single PMU is introduced, apart from the PMU at the reference bus (which is implicitly assumed), in to the DSE process along with the normal measurements. The PMU can be placed at any of the four available buses. The performance of the DSE when PMU is placed at any of these locations with varying weight vectors (obtained by varying the standard deviations of PMU measurements) are as shown below.

The standard deviation of the normal measurements is held constant through out the simulation. The weight values of

the PMUs are varied from 1 to about 1000 times the weight of the normal measurements. That is, the simulation is run with weights of the PMU measurements being varied, around 1/10/100/200/400/1000 times the normal measurements. The error is calculated by the amount of variation of the filtered states with reference to the actual state variables at each time sample. The error is then averaged over the ten sample time intervals. The average error after the filtering step, for the ten time samples, when PMU is placed at bus 2 is as shown below.

TABLE III

VARIATION OF AVERAGE VOLTAGE MAGNITUDE AND ANGULAR ESTIMATES ERROR WHEN PMU IS PLACED AT BUS AT 2, FOR DIFFERENT WEIGHTS OF PMU

Weightage given to PMU w.r.t normal measurements	Average Voltage magnitude estimate error (%)	Average Voltage angular estimate error (%)
No PMU	0.0580	1.2537
1	0.0195	0.8579
10	0.0157	0.8541
100	0.0150	0.7253
200	0.0146	0.4792
400	0.0143	0.4200
1000	0.0142	0.7066

It can clearly be seen that higher the weightage of the PMU, lesser is the error in the overall filtered values. For the present case it can be seen that, when the weightage given to PMU measurements is about 400 times the normal measurements, we achieve the least average error for angular estimates, while the voltage magnitude estimates are better of, when the weightage for PMU measurements are 1000. But since the voltage magnitude estimates in general have a high accuracy for all weights (%error always less than 0.1), we can give preference to the case where we achieve least error in angular estimates. In this case a weightage of 400 is ideal for achieving good estimates, when the PMU is placed at bus 2.

Similar studies were carried out, by placing the PMU at other locations. The combination of weight value which gives the least error for various locations of the PMU is as shown below:

TABLE IV

MINIMUM ERROR IN VOLTAGE MAGNITUDE AND VOLTAGE ANGULAR ESTIMATES, FOR VARYING LOCATIONS OF PMU

Location of PMU	Weightage of PMU measurements, for least error	Average Voltage magnitude estimate error (%)	Average Voltage angular estimate error (%)
No PMU	-	0.058	1.2537
2	400	0.0143	0.42
3	100	0.0131	0.3825
4	10	0.0135	0.7995
5	1000	0.0233	0.7269

For the above result the combination, which gave least error in angular estimates, was preferred to the combination, which gave least error in voltage magnitude estimate, as it is the angular estimates, which are generally found to have higher percentage error than the voltage magnitude estimates.

In order to visualize the impact of PMUs in DSE more clearly, a study of the variation of error in the estimated states for different PMU locations, for a particular weight value of the PMU, is carried out. The results of one such case, where the PMU weightage was fixed at 100, is given as below:

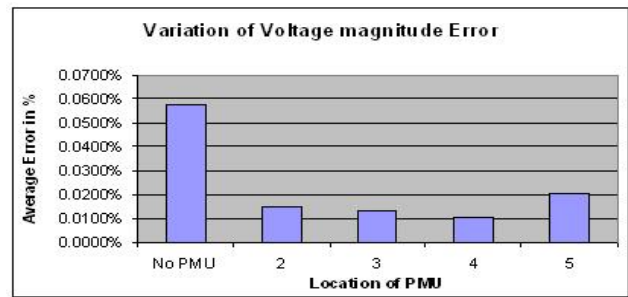


Figure 1: Variation of Voltage Magnitude Estimates with different locations of PMU and the PMU weights fixed at 100.

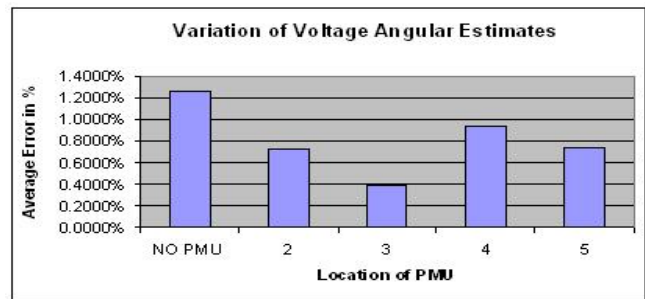


Figure 2: Variation of Voltage Angular Estimates with different locations of PMU and the PMU weights fixed at 100.

It can clearly be seen that, introduction of PMU, has a great impact on reducing the amount of error in the final filtered states of PMU. It can also be observed that, there is a higher reduction in error in voltage magnitude estimates than in the voltage angular estimates, for the given case. In the above case, placement of PMU at bus 3 gives the highest reduction in error for voltage angular estimates. During the course of the study, it was observed that the least error in the estimates (especially angular estimates) was obtained, when the PMU weights were of the order of 100 or 200. The study was extended for all the other combinations of the weight values and PMU locations.

B. Multiple PMUs:

A similar study as mentioned in the previous section was conducted for the case of multiple PMUs. The combinations of PMU locations that were studied include, PMUs at bus (2, 3), (2, 4), (2, 5), (3, 4), (3, 5) and (4, 5). The system being a 5-bus system, with a PMU being assumed at the reference bus, combinations with higher number of PMUs other than the ones mentioned would have made the estimation process quite linear. Moreover the percentage of PMUs with respect to the number of buses in the system becomes quite high (80 to 100%). Therefore, the cases of three or more PMUs (apart from reference bus PMU) were not considered.

Due to space constraints, only the results for the combination of PMUs at buses (2, 3), are presented, as shown in Figure 3 and 4. The graphs in Fig. 3 and Fig. 4 show that the percentage error has clearly dropped with the usage of PMUs at buses 2 and 3. A minimum error of 0.2953% for angular estimates is obtained, when the weightage of the PMUs at buses 2 and 3, is of the order of 200. By comparing the results for single and double PMUs, it can be observed that, higher accuracy of the estimates were obtained with two PMUs at bus

2 and 3, than having individual PMUs at either bus2 or at bus3.

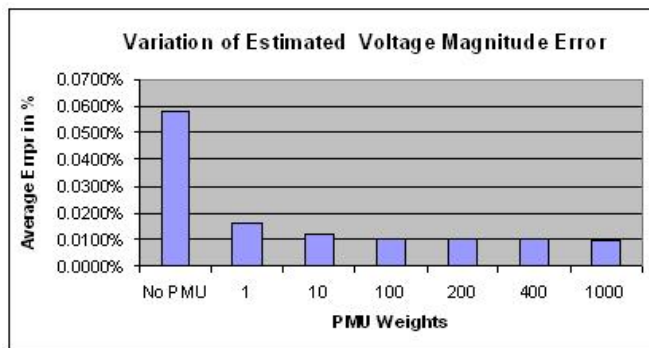


Figure 3: Variation of Voltage Magnitude error when PMUs are at bus 2 and 3, for various Weights of PMUs

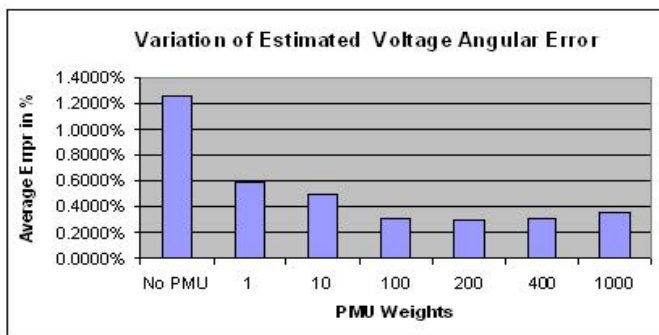


Figure 4: Variation of Voltage Angular error when PMUs are at bus 2 and 3, for various Weights of PMUs

The simulations were performed for all the other combinations of two PMUs and the results have been encouraging. The algorithm is also being tested for bigger systems and the results of them will be published in due course, in a separate paper.

VIII. CONCLUSIONS

PMUs are extremely accurate measuring devices, with the ability to directly measure the voltage and current phasors. These high accuracy measurements of the PMUs may not be warranted, as many state estimation techniques are not equipped for handling them. Many researchers have proposed techniques to handle PMU data in static state estimation. An attempt has been made in this paper to present a PMU based dynamic state estimation technique. Some of the conclusions obtained by other researchers from PMU based static state estimation techniques have also been considered to improve the efficiency of the PMU based DSE. Accordingly, the technique uses the phasor measurements in the estimation process along with normal SCADA data, but with higher weights. The case of Single PMU and Multiple PMUs in a 5-bus system has been studied and the results are presented. It is quite clear from the results obtained, that the PMUs have a tremendous impact in reducing the error in the estimates of the DSE. The authors plan to further this work by performing the required simulation studies for larger standard and practical systems. Further analysis and results will be presented in future works.

With the power sector being deregulated and the utilities

trying to monitor the system as accurately as possible, for more efficient operation, the usage of PMUs is bound to increase. From this point of view, PMU based dynamic state estimators are of extreme importance in the modern day EMS.

IX. REFERENCES

- [1] Atif. S. Debs and Robert. E. Larson, "A dynamic estimator for tracking the state of a power system", *IEEE Transactions on Power apparatus and Systems*, Vol. PAS 89, NO.7, pp. 1670-1678, September-October, 1970
- [2] J. S. Thorp, A. G. Phadke, K. J. Karimi, "Real time voltage-phasor measurements for static state estimation", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-104, No. 11, pp. 3098-3106, November 1985.
- [3] Hui Xua, Qing-quan Jia, Ning Wang, Zhi-qian Bo, Hai-tang Wang, and Hong-xia Ma, "A Dynamic State Estimation Method with PMU and SCADA Measurement for Power Systems," *Proceedings of The 8th International Power Engineering Conference (IPEC 2007)*, Singapore, 2007, pp. 848-853.
- [4] A. G. Phadke, "Synchronized phasor measurements in power systems", *IEEE Computer Applications in Power*, pp 10-15, 1993.
- [5] David G. Hart, David Uy, Vasudev Gharpure, Damir Novosel, Daniel Karlsson, Mehmet Kaba, "PMUs – A new approach to power network monitoring", *ABB Review* 1/2001, pp 58-61.
- [6] Mark Rice, Gerald T. Heydt, "Phasor measurement unit data in power system state estimation", Intermediate Project Report for the PSERC Project "Enhanced State Estimators", Power Systems Engineering Research Center, January 2005.
- [7] R. Zivanovic, C. Cairns, "Implementation of PMU technology in state estimation: an overview," *IEEE AFRICON 4th*, Vol. 2, September 1996, pp. 1006-1011.
- [8] Hongga Zhao, "A new estimation model for utilizing PMU measurements", *International Conference on Power System Technology*, 2006.
- [9] Shivakumar. N. R and Amit Jain, "Including Phasor Measurements in Dynamic State Estimation of Power Systems", *Proceedings of International Conference on Power System Analysis Control and Optimization*, pp 958-963, March 13-15, 2008, Visakhapatnam.
- [10] Sunita Chohan, "Static and tracking state estimation in power systems with bad data analysis", PhD Dissertation, Centre for Energy Studies, IIT-Delhi, July 1993.
- [11] A. M. Leite da Silva, M. B. Do Coutto Filho, J. F. de Queiroz, "State forecasting in electric power systems", *IEE Proceedings*, Vol. 130, Pt. C. No. 5, pp 237-244, September 1983.
- [12] Jegatheesan, R. Leena, I, "Dynamic State Estimation", *TENCON '91.1991 IEEE Region 10 International Conference on EC3-Energy, Computer, Communication and Control Systems*, On page(s): 492-496, 28-30 Aug 1991
- [13] Glenn W. Stagg and Ahmed H. El Abiad, "Computer Methods in Power System Analysis", chapter 8, McGraw Hill Book Company, 1968.