Transceiver Design for Cognitive Full-Duplex Two-Way MIMO Relaying System

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Transceiver Design for Cognitive Full-Duplex Two-Way MIMO Relaying System

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***Abstract*—In this paper, we present the design of optimal precoders and receive filters for a full-duplex (FD) two-way amplify-and-forward (AF) multiple-input multiple-output (MIMO) relaying system in a cognitive radio network. The secondary user (SU) or cognitive nodes share the spectrum with the licensed primary user (PU) nodes while keeping the interference to PU nodes below a threshold. The cognitive transceivers communicate with each other with the assistance of a cognitive AF relay. The precoders for the cognitive transceivers are designed so as to maximize the signal to noise ratio (SNR) at the relay while limiting the interference to the PU node below a specified threshold. Due to FD mode of operation, all the nodes suffer from self-interference (SI). We assume the available channel state information (CSI) of loopback self-interference channels at relay and transceivers to be imperfect. We design the optimal relay precoder and transceiver receive filter, for suppressing the residual SI, by minimizing the sum of mean square error (SMSE) at the cognitive transceivers with constraint on transmit power at relay. The precoders and receive filters are updated at each time slot, taking into account the cumulative interference effect from all previous time slots, which enhances the performance of the system over time. The simulation results illustrate the effectiveness of the proposed design in terms of the achievable sum-rate.**

I. Introduction

In-band full-duplex wireless communication has the potential to double spectral efficiency by simultaneous transmission and reception on the same frequency and time resource. Recent improvements in self-interference cancellation (SIC) techniques [1],[2] have made MIMO FD systems viable. Hence, FD is regarded as one of the promising technologies for the next generation wireless networks. Relaying techniques have been investigated as a means to facilitate better quality of service, improved link capacity, and enhanced network coverage. Cooperative relaying, where one or more relay nodes are used for communication between end users, is envisioned to be a key enabling technology for 5G.

Two-way in-band full-duplex relaying [3] incorporates the merits of both these technologies. In a two-hop two-way FD AF relaying system, two transceivers transmit and receive simultaneously on the same frequency in each time slot, with the aid of a FD relay. The two-way FD AF MIMO relaying system consisting of a MIMO relay, but single antenna transceivers is studied in [4]. A similar system in [5] comprises of two MIMO transceivers and a MIMO relay, wherein the beamformers at relay and transceivers were jointly designed for minimizing the SMSE at the two transceivers. The effect of path loss on the system design is neglected in these and most of such previous work on two-way FD AF MIMO relaying.

Cognitive radio technology enhances the spectrum utilization in wireless communications. The secondary users in a cognitive radio network operate under a severe power constraint in order to limit the interference to the primary user. In such a scenario, FD two-way relaying, which has better efficiency than one-way relaying, will be beneficial to enhance the range as well as rate of communication A robust MSE-based transceiver design problem for a one-way FD MIMO cognitive cellular system was studied in [6], where the relay base station was FD, but the transceivers were operating in half-duplex (HD) mode. The problem of sum-rate maximization for a HD two-way relaying cognitive radio network was studied in [7] for a MIMO relay and single antenna secondary transceivers and primary user.

In this paper, we present the design of a cognitive two-way in-band FD AF relaying MIMO system with two secondary transceivers, one secondary relay, and a primary user. First we design the precoders at the secondary transceivers by maximizing the SNR at the secondary relay, while keeping the total interference to the primary user node, due to the operation of cognitive nodes, under a threshold. Then, we jointly design the precoder at the relay and the receive filters at the secondary transceivers by minimizing the SMSE at the secondary transceivers. This problem is non-convex in nature. Finally, we analyze the performance of the proposed designs in a LTE cellular cognitive network scenario, in terms of the achievable transmit power efficiency and sum-rate. The main contributions of this paper are twofold: (*i*) primary user interference constrained precoder design and power optimization at the transceivers, and (*ii*) recursive computation of relay precoder, receive filters with minimal memory requirements.

The rest of this paper is organized as follows. Section II describes the system model. The design of optimal precoders and receive filters is discussed in Section III. Simulation results are presented in Section IV and the conclusions in Section V.

$Notations$: We denote a scalar, vector, and matrix by italic lowercase, boldface lowercase, and boldface uppercase, respectively. For a matrix $\mathbf{Y}$, its transpose, conjugate, conjugate transpose, inverse, determinant, vectoriza-

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tion operation, trace, and Frobenius norm are denoted by $\mathbf{Y}^T, \mathbf{Y}^C, \mathbf{Y}^H, \mathbf{Y}^{-1}, |\mathbf{Y}|, \mathrm{vec}(\mathbf{Y}), \mathrm{tr}(\mathbf{Y}), \|\mathbf{Y}\|_{\mathbb{F}}$, respectively. $\mathbf{X}\otimes\mathbf{Y}$ represents the kronecker product of matrices $\mathbf{X}$ and $\mathbf{Y}$. $\mathbb{E}\{\cdot\}$ is the expectation operator, $\|\mathbf{y}\|$ represents 2-norm of $\mathbf{y}$, $\mathrm{mat}(\cdot)$ performs the inverse operation of $\mathrm{vec}(\cdot)$. If a variable $m = 1$, then $\underline{m} = 2$ and vice versa. Time-slot dependent variables $b, \mathbf{b}, \mathbf{B}$ denote a scalar, vector, matrix, respectively, in the current time slot t, while $b^{(j)}, \mathbf{b}^{(j)}, \mathbf{B}^{(j)}$ denote a scalar, vector, matrix,respectively, in any time slot j.

II. System Model

We consider a cognitive two-way FD relaying system as shown in Fig. 1. The system comprises of a primary user P, two secondary transceivers S_1, S_2; and a secondary AF relay R. Nodes P, S_1 and S_2 are each equipped with N_s antennas. Relay R has N_r antennas, of which N_s antennas are used for signal reception while all N_r antennas are used for signal transmission, such that $N_\mathrm{r} \geq N_\mathrm{s}$. All the nodes operate in FD mode, on the same frequency which was intended for use by P only. We assume that there is no direct communication link between the secondary nodes S_1 and S_2, and so all their communication happens via the relay R.

All the channel links are modeled as independent and frequency-flat Rayleigh fading channels and are assumed to be static for the duration of each time slot. The matrix $\mathbf{G}_{ij}$ represents the MIMO channel matrix between transmitter i and receiver j. Thus, the matrices $\mathbf{G}_{m\mathrm{r}} \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{s}}$, $\mathbf{G}_{\mathrm{r}m} \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{r}}$, $\mathbf{G}_{n\mathrm{p}} \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{s}}$ and $\mathbf{G}_{\mathrm{p}n} \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{s}}$, $m \in \{1, 2, \mathrm{p}\}$, $n \in \{1, 2\}$, represent the MIMO channels as shown in Fig. 1. These channel matrices are assumed to be perfectly known. The matrices $\mathbf{G}_{\mathrm{rr}} \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{r}}$ and $\mathbf{G}_{mm} \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{s}}$, $m \in \{1, 2, \mathrm{p}\}$, represent the loopback self-interference MIMO channels. We assume that the available CSI of the loopback channels is imperfect , such that

$$\mathbf{G}_{jj} = \widehat{\mathbf{G}}_{jj} + \mathbf{\Phi}_j, \quad j = 1, 2, \mathrm{p}, \mathrm{r}, \tag{1}$$

where $\widehat{\mathbf{G}}_{jj}$ is the available channel estimate and $\mathbf{\Phi}_j$ is the CSI error with zero mean and covariance $\mathbb{E}\{\mathbf{\Phi}_j\mathbf{\Phi}_j^H\} = N_\mathrm{s}\sigma_{\mathrm{e}j}^2\mathbf{I}_{N_\mathrm{s}}$ [8]. The symbol α_{ij} represents the path loss between nodes i and j; $i, j \in \{1, 2, \mathrm{p}, \mathrm{r}\}$, such that $\alpha_{ij} = \alpha_{ji}$. We assume $\alpha_{ii} = 1$ and it will be not be explicitly mentioned henceforth.

During each time slot t the following events occur :

(*i*) The primary user P transmits signal $\mathbf{x}_\mathrm{p} \in \mathbb{C}^{N_s\times 1}$ which acts as interference for all the secondary nodes.

(*ii*) Each secondary transceiver precodes data vector $\mathbf{d}_m \in \mathbb{C}^{N_s\times 1}$ having covariance $\mathbb{E}\{\mathbf{d}_m\mathbf{d}_m^H\} = \mathbf{I}_{N_\mathrm{s}}$ with precoding matrix $\mathbf{T}_m \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{s}}$ to generate $\mathbf{x}_m$, $m \in \{1, 2\}$, and transmits it to R, where the signal received is given by

$$\mathbf{y}_\mathrm{r} = \sqrt{\alpha_{1\mathrm{r}}}\mathbf{G}_{1\mathrm{r}}\mathbf{x}_1 + \sqrt{\alpha_{2\mathrm{r}}}\mathbf{G}_{2\mathrm{r}}\mathbf{x}_2 + \mathbf{G}_{\mathrm{rr}}\mathbf{x}_\mathrm{r} + \sqrt{\alpha_{\mathrm{pr}}}\mathbf{G}_{\mathrm{pr}}\mathbf{x}_\mathrm{p} + \mathbf{n}_\mathrm{r}, \tag{2}$$

where the noise $\mathbf{n}_\mathrm{r}$ is a circularly symmetric complex Gaussian random vector with zero mean and covariance $\mathbb{E}\{\mathbf{n}_\mathrm{r}\mathbf{n}_\mathrm{r}^H\} = \sigma_{nr}^2\mathbf{I}_{N_\mathrm{s}}$. The signal at the relay after SIC is given by

$$\begin{aligned}\widetilde{\mathbf{y}}_\mathrm{r} =& \sqrt{\alpha_{1\mathrm{r}}}\mathbf{G}_{1\mathrm{r}}\mathbf{T}_1\mathbf{d}_1 + \sqrt{\alpha_{2\mathrm{r}}}\mathbf{G}_{2\mathrm{r}}\mathbf{T}_2\mathbf{d}_2 + \mathbf{\Phi}_\mathrm{r}\mathbf{x}_\mathrm{r} + \\ & \sqrt{\alpha_{\mathrm{pr}}}\mathbf{G}_{\mathrm{pr}}\mathbf{x}_\mathrm{p} + \mathbf{n}_\mathrm{r}.\end{aligned} \tag{3}$$

The third term in (3) refers to the residual SI due to error in the available loopback CSI as modeled in (1).

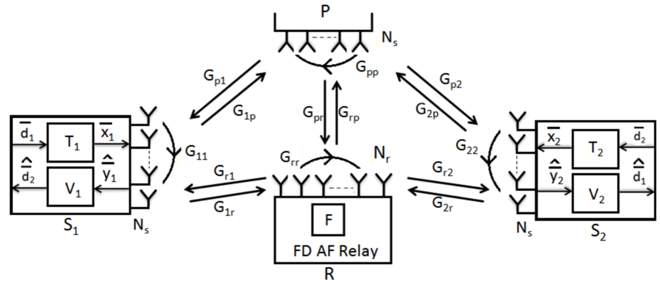

Fig. 1: System diagram of a FD two-way cognitive radio network.

(*iii*) The secondary AF relay precodes the signal received in the previous time slot, $\mathbf{y}_\mathrm{r}^{(t-1)}$, using the precoding matrix $\mathbf{F} \in \mathbb{C}^{N_\mathrm{r}\times N_\mathrm{s}}$ and transmits the resulting signal

$$\mathbf{x}_\mathrm{r} = \mathbf{F}\widetilde{\mathbf{y}}_\mathrm{r}^{(t-1)}, \quad t \geq 2. \tag{4}$$

(*iv*) Each secondary transceiver receives the relay signal as

$$\mathbf{y}_m = \sqrt{\alpha_{\mathrm{r}m}}\mathbf{G}_{\mathrm{r}m}\mathbf{x}_\mathrm{r} + \mathbf{G}_{mm}\mathbf{x}_m + \sqrt{\alpha_{\mathrm{p}m}}\mathbf{G}_{\mathrm{p}m}\mathbf{x}_\mathrm{p} + \mathbf{n}_m, m = 1, 2, \tag{5}$$

where noise $\mathbf{n}_m$ is a circularly symmetric complex Gaussian random vector with zero mean and covariance $\mathbb{E}\{\mathbf{n}_m\mathbf{n}_m^H\} = \sigma_{nm}^2\mathbf{I}_{N_\mathrm{s}}$. Since, each secondary transceiver has knowledge of its own transmitted signals, $\mathbf{d}_m^{(t-1)}$ and $\mathbf{x}_m$, as well as of CSI $\sqrt{\alpha_{\mathrm{r}m}}\mathbf{G}_{\mathrm{r}m}, \widehat{\mathbf{G}}_{mm}$, and $\mathbf{F}$, it cancels out the SI resulting in

$$\begin{aligned}\widetilde{\mathbf{y}}_m =& \sqrt{\alpha_{\mathrm{r}m}}\mathbf{G}_{\mathrm{r}m}\mathbf{F}\{\sqrt{\alpha_{\underline{m}\mathrm{r}}}\mathbf{G}_{\underline{m}\mathrm{r}}^{(t-1)}\mathbf{T}_{\underline{m}}^{(t-1)}\mathbf{d}_{\underline{m}}^{(t-1)} + \mathbf{\Phi}_\mathrm{r}^{(t-1)}\mathbf{x}_\mathrm{r}^{(t-1)} \\ & + \mathbf{n}_\mathrm{r}^{(t-1)}\} + \mathbf{\Phi}_m\mathbf{T}_m\mathbf{d}_m + \sqrt{\alpha_{\mathrm{p}m}}\mathbf{G}_{\mathrm{p}m}\mathbf{x}_\mathrm{p} + \mathbf{n}_m.\end{aligned} \tag{6}$$

Here again, the residual SI is represented by the term containing $\mathbf{\Phi}_m$. Each secondary transceiver then applies a receive filter $\mathbf{V}_m \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{s}}$ to $\widetilde{\mathbf{y}}_m$ to obtain an estimate of the data, $\widehat{\mathbf{d}}_{\underline{m}}$, transmitted by the other secondary transceiver.

(*v*) The signals transmitted from the secondary nodes cause interference at the primary transceiver. The total power of interfering signals at P is

$$I_\mathrm{p} = I_1 + I_2 + I_\mathrm{r}, \tag{7}$$

where,

$$I_m = \alpha_{m\mathrm{p}}tr(\mathbf{G}_{m\mathrm{p}}\mathbf{x}_m\mathbf{x}_m^H\mathbf{G}_{m\mathrm{p}}^H), \quad m = 1, 2, \mathrm{r}. \tag{8}$$

III. Design of optimal Precoders and Receive Filters

In this section, we present the proposed design of the relay precoder $\mathbf{F}$, the transceiver precoders $\mathbf{T}_1$ and $\mathbf{T}_2$, and the transceiver receive filters $\mathbf{V}_1$ and $\mathbf{V}_2$.

A. Transceiver Precoder Design

We start with the QR-decomposition of the hermitian of the MIMO channel matrix $\mathbf{G}_{m\mathrm{r}}$ as: $\mathbf{G}_{m\mathrm{r}}^H = \mathbf{Q}_m\mathbf{R}_m$. Then, the precoder matrices are given by

$$\mathbf{T}_m = \beta_m\mathbf{Q}_m\mathbf{W}_m, \quad m = 1, 2. \tag{9}$$

and the resultant transmit power is given by

$$P_m = \mathbb{E}\{\|\mathbf{T}_m\mathbf{d}_m\|^2\} = \beta_m^2 tr(\mathbf{W}_m\mathbf{W}_m^H), \; m = 1, 2, \tag{10}$$

where, the scaling factor β_m will be derived in the sequel and $\mathbf{W}_m$ is derived from $\mathbf{R}_m$ such that:

$$\mathbf{G}_{m\mathrm{r}}\mathbf{T}_m = \beta_m\mathbf{I}_{N_\mathrm{s}}. \tag{11}$$

We design $\mathbf{T}_1$, $\mathbf{T}_2$ by maximizing the received SNR at R under a constraint on total interference power, I_{p}, at P and bounds on transmit power of $\mathrm{S}_1, \mathrm{S}_2$. The constraint on I_{p} is

$$I_{\mathrm{p}} \leq \theta, \tag{12}$$

where, θ is the maximum permissible interference to P from all the secondary nodes in a cognitive scenario. From (7), (8), (9), (10) and (11), (12) can be written as

$$\begin{aligned} \theta &\geq \alpha_{1\mathrm{p}}\|\mathbf{G}_{1\mathrm{p}}\mathbf{x}_1\|^2 + \alpha_{2\mathrm{p}}\|\mathbf{G}_{2\mathrm{p}}\mathbf{x}_2\|^2 + \alpha_{\mathrm{rp}}\|\mathbf{G}_{\mathrm{rp}}\mathbf{x}_{\mathrm{r}}\|^2 \\ &= \alpha_{1\mathrm{p}}\beta_1^2 b_1 + \alpha_{2\mathrm{p}}\beta_2^2 b_2 + \alpha_{\mathrm{rp}} p_r \|\mathbf{G}_{\mathrm{rp}}\|_{\mathbb{F}}^2, \end{aligned} \tag{13}$$

where, $b_m = tr(\mathbf{G}_{m\mathrm{p}}\mathbf{Q}_m\mathbf{W}_m\mathbf{W}_m^H\mathbf{Q}_m^H\mathbf{G}_{m\mathrm{p}}^H), m \in \{1,2\}$ and $\mathbb{E}\{\mathbf{x}_{\mathrm{r}}\mathbf{x}_{\mathrm{r}}^H\} = p_{\mathrm{r}}\mathbf{I}_{N_{\mathrm{s}}}$, where, p_{r} is the per-antenna power of R. As will be seen in the next section, R always transmits full power, such that

$$p_{\mathrm{r}} = P_{\mathrm{r}}^{max}/N_{\mathrm{r}} = p_{\mathrm{r}}^{max}. \tag{14}$$

As a result, (13) can be written as

$$\alpha_{1\mathrm{p}}\beta_1^2 b_1 + \alpha_{2\mathrm{p}}\beta_2^2 b_2 \leq \theta - \alpha_{\mathrm{rp}} p_r^{max}\|\mathbf{G}_{\mathrm{rp}}\|_{\mathbb{F}}^2 = \theta'. \tag{15}$$

Equation (14) is also the reason for choosing $\mathrm{SNR}_{\mathrm{R}}$ as the function to be maximized. Since R is always transmitting at full power, we optimize P_1, P_2 to limit the interference at P.

From (2) and (11), the SNR at R at time t is given by:

$$\mathrm{SNR}_{\mathrm{R}} = \frac{\alpha_{1\mathrm{r}}\beta_1^2 N_{\mathrm{s}} + \alpha_{2\mathrm{r}}\beta_2^2 N_{\mathrm{s}}}{\sigma_{\mathrm{nr}}^2 N_{\mathrm{s}}}. \tag{16}$$

We observe that maximizing the SNR at the relay is same as maximizing the signal power in (16), for fixed noise power. So, the optimization problem can instead be formulated as:

$$\begin{aligned} \max_{\beta_1^2,\beta_2^2} \quad & \alpha_{1\mathrm{r}}\beta_1^2 N_{\mathrm{s}} + \alpha_{2\mathrm{r}}\beta_2^2 N_{\mathrm{s}} \\ \text{subject to} \quad & \alpha_{1\mathrm{p}}\beta_1^2 b_1 + \alpha_{2\mathrm{p}}\beta_2^2 b_2 \leq \theta' \\ & P_m^{min} \leq P_m \leq P_m^{max} \quad, m = 1,2. \end{aligned} \tag{17}$$

The problem in (17) is a linear optimization problem in β_1^2 and β_2^2 and can be easily solved by a optimization tool such as $linprog$ in MATLAB. The optimal values thus obtained are $\beta_1^{2^*}$ and $\beta_2^{2^*}$. The corresponding optimal precoder matrices and optimal transmit power are given by (9) and (10), respectively.

B. Relay Precoder and Transceiver Receive Filter Design

We design $\mathbf{F}$, $\mathbf{V}_1$, $\mathbf{V}_2$ by minimizing the SMSE of the transceivers $\mathrm{S}_1, \mathrm{S}_2$, with a constraint on the transmit power of relay R. Matrix $\mathbf{F}$ is decomposed as $\mathbf{F} = \gamma\bar{\mathbf{F}}$, where γ is a positive scaling factor and $||\bar{\mathbf{F}}||_{\mathbb{F}} = 1$. This decomposition simplifies the derivation of the relay precoder in closed form. For the same reason, the term γ^{-1} is introduced in (19). Thus, the SMSE = $f(\gamma, \bar{\mathbf{F}}, \mathbf{V}_1, \mathbf{V}_2)$. The optimal design is obtained by solving the following optimization problem:

$$\begin{aligned} \min_{\gamma,\bar{\mathbf{F}},\mathbf{V}_1,\mathbf{V}_2} \quad & f(\gamma, \bar{\mathbf{F}}, \mathbf{V}_1, \mathbf{V}_2) \\ \text{subject to} \quad & \mathbb{E}\{\|\mathbf{x}_{\mathrm{r}}\|^2\} \leq P_{\mathrm{r}}^{max}. \end{aligned} \tag{18}$$

Due to space constraints, throughout this paper we'll denote $(\alpha_{1\mathrm{r}}\beta_1^{(j)^2} + \alpha_{2\mathrm{r}}\beta_2^{(j)^2} + \sigma_{\mathrm{nr}}^2)\mathbf{I}_{N_{\mathrm{s}}} + \alpha_{\mathrm{pr}} p_{\mathrm{p}}^{(j)}\mathbf{G}_{\mathrm{pr}}^{(j)}\mathbf{G}_{\mathrm{pr}}^{(j)^H}$ by $\Psi^{(j)}$. The SMSE of the two transceivers is given by

$$\begin{aligned} & f(\gamma, \bar{\mathbf{F}}, \mathbf{V}_1, \mathbf{V}_2) \\ &= \sum_{i=1}^{2}\mathbb{E}\{\|\mathbf{d}_i^{(t-1)} - \gamma^{-1}\widehat{\mathbf{d}}_i\|^2\} \\ &= \sum_{i=1}^{2}\mathbb{E}\{\|\mathbf{d}_i^{(t-1)}\|^2\} + \gamma^{-2}\mathbb{E}\{\|\mathbf{V}_{\underline{i}}^H\widetilde{\mathbf{y}}_{\underline{i}}\|^2\} - \\ &\quad \gamma^{-1}[tr(\mathbb{E}\{\mathbf{V}_{\underline{i}}^H\widetilde{\mathbf{y}}_{\underline{i}}\mathbf{d}_i^{(t-1)^H}\}) + tr(\mathbb{E}\{\mathbf{d}_i^{(t-1)}\widetilde{\mathbf{y}}_{\underline{i}}^H\mathbf{V}_{\underline{i}}\})]. \end{aligned} \tag{19}$$

To proceed further, we express each term of SMSE equation in terms of the relevant optimization variables. Thus,

$$\begin{aligned} &\mathbb{E}\{||\mathbf{d}_i^{(t-1)}||^2\} = N_{\mathrm{s}}, \\ &\mathbb{E}\{\mathbf{d}_i^{(t-1)}\widetilde{\mathbf{y}}_{\underline{i}}^H\mathbf{V}_{\underline{i}}\} = \beta_i^{(t-1)}\sqrt{\alpha_{i\mathrm{r}}\alpha_{\underline{i}\mathrm{r}}}tr(\mathbf{F}^H\mathbf{G}_{\mathrm{r}\underline{i}}^H\mathbf{V}_{\underline{i}}). \end{aligned} \tag{20}$$

Further, using (6) and (11), we have

$$\begin{aligned} &\mathbb{E}\{||\mathbf{V}_i^H\widetilde{\mathbf{y}}_i||^2\} \\ &= \alpha_{i\mathrm{r}}tr[\mathbf{V}_i^H\mathbf{G}_{\mathrm{r}i}\mathbf{F}(\mathbb{E}\{\mathbf{J}_0\} + \alpha_{\underline{i}\mathrm{r}}\beta_{\underline{i}}^{(t-1)^2}\mathbf{I}_{N_{\mathrm{s}}} + \sigma_{n\mathrm{r}}^2\mathbf{I}_{N_{\mathrm{s}}} + \\ &\quad \alpha_{\mathrm{pr}}p_{\mathrm{p}}^{(t-1)}\mathbf{G}_{\mathrm{pr}}^{(t-1)}\mathbf{G}_{\mathrm{pr}}^{(t-1)^H})\mathbf{F}^H\mathbf{G}_{\mathrm{r}i}^H\mathbf{V}_i] + \sigma_{ni}^2 tr(\mathbf{V}_i\mathbf{V}_i^H) \\ &\quad + \mathbb{E}\{tr(\mathbf{V}_i^H\mathbf{\Phi}_i\mathbf{T}_i\mathbf{d}_i\mathbf{d}_i^H\mathbf{T}_i^H\mathbf{\Phi}_i^H\mathbf{V}_i)\} + \\ &\quad \alpha_{\mathrm{p}i}p_{\mathrm{p}}tr(\mathbf{V}_i^H\mathbf{G}_{\mathrm{p}i}\mathbf{G}_{\mathrm{p}i}^H\mathbf{V}_i), \end{aligned} \tag{21}$$

where,

$$\mathbf{J}_0 = \sum_{k=0}^{t-2}\prod_{l=1}^{t-k-1}\mathbf{\Phi}_{\mathrm{r}}^{(t-l)}\mathbf{F}^{(t-l)}\Psi^{(k)}\prod_{l=1}^{t-k-1}\mathbf{F}^{(k+l)^H}\mathbf{\Phi}_{\mathrm{r}}^{(k+l)^H}. \tag{22}$$

for $t \geq 3$ and $\mathbf{0}_{N_{\mathrm{s}}}$ for $t = 1, 2$. In this paper, it's assumed that $\prod_{i=a}^{b}(\cdot) = 1$, if $b < a$. Using Lemma 1 from [9] and proceeding as in Theorem 1 from [5], we obtain $\mathbb{E}\{\mathbf{J}_0\}$ as:

$$\begin{aligned} \mathbf{J}_q = \mathbb{E}\{\mathbf{J}_0\} = &\sum_{k=1}^{t-1}(\sigma_{\mathrm{er}}^2)^k tr(\mathbf{F}^{(t-k)}\Psi^{(t-k-1)}\mathbf{F}^{(t-k)^H})\mathbf{I}_{N_{\mathrm{s}}} \\ &\prod_{l=1}^{k-1}tr(\mathbf{F}^{(t-l)}\mathbf{F}^{(t-l)^H}), \end{aligned} \tag{23}$$

The term $\mathbf{J}_q$ represents the contribution of all previous $\mathbf{F}^{(k)}$, $t = 2$ onwards, which is required to suppress the SI caused by FD operation. Since, the first $\mathbf{F}$ is computed at $t = 2$, the value of $\mathbf{J}_q$ is $\mathbf{0}_{N_{\mathrm{s}}}$ for the first two time slots. After a few algebraic manipulations on (23), $\mathbf{J}_q$ can be recursively computed as

$$\begin{aligned} \mathbf{J}_q = &\sigma_{\mathrm{er}}^2[tr(\mathbf{F}^{(t-1)}\Psi^{(t-2)}\mathbf{F}^{(t-1)^H})\mathbf{I}_{N_{\mathrm{s}}} \\ &+ \mathbf{J}_q^{(t-1)}tr(\mathbf{F}^{(t-1)}\mathbf{F}^{(t-1)^H})]. \end{aligned} \tag{24}$$

The recursive structure of $\mathbf{J}_q$ plays a significant role in the design of $\mathbf{F}, \mathbf{V}$. As a consequence, the nodes need not store all the previous relay precoders and transceiver receive filters. This not only greatly reduces the memory requirement at the relay but also results in reduced computation time for $\mathbf{J}_q$. Moreover, the precoder designed using (24) will lead to better performance than that of the system proposed in [5] where only the n latest time slots are used for computing $\mathbf{J}_q$.

Using Lemma 1 from [9] and (10), we can rewrite the second to last term of (21) as

$$\mathbb{E}\{tr(\mathbf{V}_i^H\mathbf{\Phi}_i\mathbf{T}_i\mathbf{d}_i\mathbf{d}_i^H\mathbf{T}_i^H\mathbf{\Phi}_i^H\mathbf{V}_i)\} = \sigma_{ei}^2 P_i tr(\mathbf{V}_i\mathbf{V}_i^H). \tag{25}$$

Using (3), (4), (11), we express the relay transmit power as

$$\mathbb{E}\{\|\mathbf{x}_{\mathrm{r}}\|^2\} = tr[\mathbf{F}(\mathbf{J}_q + \mathbf{\Psi}^{(t-2)})\mathbf{F}^H] = tr(\mathbf{F}\mathbf{J}_{\mathrm{r}}\mathbf{F}^H). \tag{26}$$

Having expressed the terms of SMSE and relay transmit power in terms of the optimization variables, we now turn to

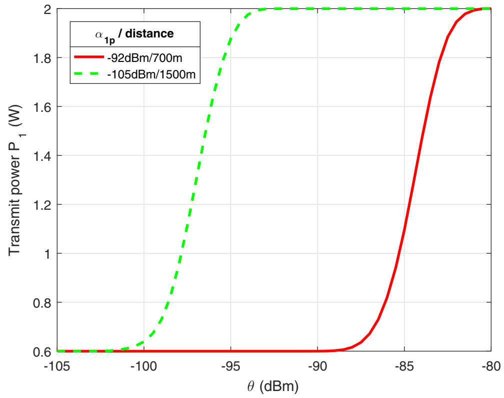

Fig. 2: Variation of transmit power versus θ, for different values of path loss.

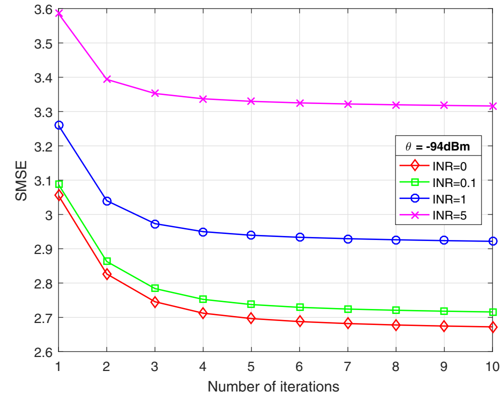

Fig. 3: Number of iterations required to obtain optimal **F**,**V**, for different values of INR, for θ = -94dBm and SNR = 5dB.

the solution of the problem in (18). The Lagrangian corresponding to this problem is given by

$$\begin{aligned}&\mathcal{L}(\gamma,\bar{\mathbf{F}},\mathbf{V}_1,\mathbf{V}_2,\lambda)\\&=f(\gamma,\bar{\mathbf{F}},\mathbf{V}_1,\mathbf{V}_2)+\lambda(\mathbb{E}\{\|\mathbf{x}_{\mathrm{r}}\|^2\}-P_{\mathrm{r}}^{max}+z^2),\end{aligned}\tag{27}$$

where λ is the lagrangian variable and z is the slack variable. On substituting the results of (19), (20), (21), (26), (23),(25) in (27) and putting $\mathbf{F}=\gamma\bar{\mathbf{F}}$, we get

$$\begin{aligned}\mathcal{L}=&\sum_{i=1}^{2}\{N_{\mathrm{s}}-\beta i^{(t-1)}\sqrt{\alpha_{i\mathrm{r}}\alpha_{\underline{i}\mathrm{r}}}tr(\mathbf{V}_{\underline{i}}^H\mathbf{G}_{\mathrm{r}\underline{i}}\bar{\mathbf{F}}+\bar{\mathbf{F}}^H\mathbf{G}_{\mathrm{r}\underline{i}}^H\mathbf{V}_{\underline{i}})+\\&\gamma^{-2}(\sigma_{n\underline{i}}^2+\sigma_{e\underline{i}}^2P_{\underline{i}})tr(\mathbf{V}_{\underline{i}}\mathbf{V}_{\underline{i}}^H)+\alpha_{\mathrm{p}i}p_{\mathrm{p}}tr(\mathbf{V}_i^H\mathbf{G}_{\mathrm{p}i}\mathbf{G}_{\mathrm{p}i}^H\mathbf{V}_i)\\&+\alpha_{\underline{i}\mathrm{r}}tr(\mathbf{V}_{\underline{i}}^H\mathbf{G}_{\mathrm{r}\underline{i}}\bar{\mathbf{F}}\mathbf{J}_{\underline{i}}\bar{\mathbf{F}}^H\mathbf{G}_{\mathrm{r}\underline{i}}^H\mathbf{V}_{\underline{i}}\}+\lambda[\gamma^2tr(\bar{\mathbf{F}}\mathbf{J}_{\mathrm{r}}\bar{\mathbf{F}}^H)-\\&P_{\mathrm{r}}^{max}+z^2],\end{aligned}\tag{28}$$

where

$$\mathbf{J}_{\underline{i}}=\mathbf{J}_q+(\alpha_{\underline{i}\mathrm{r}}\beta_{\underline{i}}^{(t-1)^2}+\sigma_{nr}^2)\mathbf{I}_{N_{\mathrm{s}}}+\alpha_{\mathrm{pr}}p_{\mathrm{p}}^{(t-1)}\mathbf{G}_{\mathrm{pr}}^{(t-1)}\mathbf{G}_{\mathrm{pr}}^{(t-1)^H}.$$

The optimization problem in (28) can be solved using the Karush-Kuhn-Tucker (KKT) conditions [10]. Since the SMSE function is not jointly convex in the optimization variables, we use the coordinate descent method. Thus, the optimal values of $\mathbf{F},\mathbf{V}_i$, $i\in\{1,2\}$, are obtained iteratively.

First, keeping $\mathbf{V}_i$ fixed, we apply the KKT conditions to get:

$$\frac{\partial\mathcal{L}}{\partial\gamma}=0\Rightarrow\ \lambda\gamma tr(\bar{\mathbf{F}}\mathbf{J}_{\mathrm{r}}\bar{\mathbf{F}}^H)=\gamma^{-3}(v_1+v_2),\tag{29}$$

where $\quad v_i=(\sigma_{ni}^2+\sigma_{ei}^2P_i)tr(\mathbf{V}_i\mathbf{V}_i^H)$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\bar{\mathbf{F}}^c}=\mathbf{0}_{N_{\mathrm{r}}\times N_{\mathrm{s}}}\Rightarrow&\sqrt{\alpha_{\mathrm{r}1}\alpha_{\mathrm{r}2}}(\beta_1^{(t-1)}\mathbf{G}_{\mathrm{r}2}^H\mathbf{V}_2+\beta_2^{(t-1)}\mathbf{G}_{\mathrm{r}1}^H\mathbf{V}_1)\\&=\lambda\gamma^2\bar{\mathbf{F}}\mathbf{J}_{\mathrm{r}}+\alpha_{\mathrm{r}1}\mathbf{G}_{\mathrm{r}1}^H\mathbf{V}_1\mathbf{V}_1^H\mathbf{G}_{\mathrm{r}1}\bar{\mathbf{F}}\mathbf{J}_1\\&+\alpha_{\mathrm{r}2}\mathbf{G}_{\mathrm{r}2}^H\mathbf{V}_2\mathbf{V}_2^H\mathbf{G}_{\mathrm{r}2}\bar{\mathbf{F}}\mathbf{J}_2.\end{aligned}\tag{30}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda}=0\Rightarrow tr(\bar{\mathbf{F}}\mathbf{J}_{\mathrm{r}}\bar{\mathbf{F}}^H)=\gamma^{-2}(P_{\mathrm{r}}^{max}-z^2).\tag{31}$$

$$\frac{\partial\mathcal{L}}{\partial z}=0\Rightarrow\lambda[\gamma^2tr(\bar{\mathbf{F}}\mathbf{J}_{\mathrm{r}}\bar{\mathbf{F}}^H)-P_{\mathrm{r}}^{max}]=0.\tag{32}$$

Now, from (29) and (32), we observe that at optimal point, $\lambda\neq0$ and so the constraint in (18) becomes equality constraint, i.e., the relay R transmits at full power P_{r}^{max}. So, (31) becomes:

$$tr(\bar{\mathbf{F}}\mathbf{J}_{\mathrm{r}}\bar{\mathbf{F}}^H)=\gamma^{-2}(P_{\mathrm{r}}^{max}).\tag{33}$$

Now, from (29) and (33), we get: $\lambda\gamma^2=\frac{(v_1+v_2)}{P_{\mathrm{r}}^{max}}$. Substituting this value in (30) and following Theorem 2 of [5], we get

$$\bar{\mathbf{F}}^*=\mathrm{mat}(\mathbf{M}^{-1}\sqrt{\alpha_{\mathrm{r}1}\alpha_{\mathrm{r}2}}\mathrm{vec}[\beta_1^{(t-1)}\mathbf{G}_{\mathrm{r}2}^H\mathbf{V}_2+\beta_2^{(t-1)}\mathbf{G}_{\mathrm{r}1}^H\mathbf{V}_1]),\tag{34}$$

where

$$\begin{aligned}\mathbf{M}=&(\mathbf{J}_1^T\otimes\alpha_{\mathrm{r}1}\mathbf{G}_{\mathrm{r}1}^H\mathbf{V}_1\mathbf{V}_1^H\mathbf{G}_{\mathrm{r}1})+(\mathbf{J}_2^T\otimes\alpha_{\mathrm{r}2}\mathbf{G}_{\mathrm{r}2}^H\mathbf{V}_2\mathbf{V}_2^H\mathbf{G}_{\mathrm{r}2})\\&+(\mathbf{J}_{\mathrm{r}}^T\otimes\frac{(v_1+v_2)\mathbf{I}_{N_{\mathrm{r}}}}{P_{\mathrm{r}}^{max}}),\\\gamma^*=&\sqrt{\frac{P_{\mathrm{r}}^{max}}{tr(\bar{\mathbf{F}}\mathbf{J}_{\mathrm{r}}\bar{\mathbf{F}}^H)}}.\end{aligned}$$

Therefore, optimal relay precoder is given by $\mathbf{F}^*=\gamma^*\bar{\mathbf{F}}^*$.

Now, using the optimal $\mathbf{F}^*$, we compute optimal receive filters by applying the KKT condition: $\frac{\partial\mathcal{L}}{\partial\mathbf{V}_i^c}=0_{N_{\mathrm{s}}},i\in\{1,2\}$, which gives the optimal receive filter at the transceiver as:

$$\begin{aligned}\mathbf{V}_i^*=&[(\sigma_{ni}^2+\sigma_{ei}^2P_i)\mathbf{I}_{N_{\mathrm{s}}}+\alpha_{\mathrm{r}i}\mathbf{G}_{\mathrm{r}i}\mathbf{F}^*\mathbf{J}_i\mathbf{F}^{*^H}\mathbf{G}_{\mathrm{r}i}^H]^{-1}\times\\&\gamma^*\sqrt{\alpha_{\mathrm{r}1}\alpha_{\mathrm{r}2}}\beta_{\underline{i}}^{(t-1)}\mathbf{G}_{\mathrm{r}i}\mathbf{F}^*.\end{aligned}\tag{35}$$

It must be noted that the optimal matrices $\mathbf{F}^*,\mathbf{V}_i^*$ are obtained by iteratively computing each other's filter until a stable value of SMSE is reached. The initial value of $\mathbf{V}_i$ for computing $\mathbf{F}$ can be any random $N_{\mathrm{s}}\times N_{\mathrm{s}}$ complex matrix.

IV. Simulation Results

For simulations, we consider the following parameters: $N_{\mathrm{s}}=2$, $N_{\mathrm{r}}=4$, $p_{\mathrm{r}}^{\max}=1,P_{\mathrm{j}}^{\min}=0.6,P_{\mathrm{j}}^{\max}=2,j\in\{1,2\}$. $\sigma_{em}^2=\frac{INR\,\sigma_{nm}^2}{P_i},m\in\{1,2,\mathrm{r}\}$. $\sigma_{nr}^2=\frac{\alpha_{1\mathrm{r}}\beta_1^2+\alpha_{2\mathrm{r}}\beta_2^2}{SNR}$, $\sigma_{ni}^2=\frac{\alpha_{\mathrm{r}i}p_{\mathrm{r}}\|\mathbf{G}_{\mathrm{r}i}\|_{\mathbb{F}}^2}{N_{\mathrm{s}}\,SNR},i\in\{1,2\}$. We consider the path loss as defined by the 3GPP LTE for outdoor macro cells [11], viz., $\alpha=15.3+37.6\log_{10}(d)\,\mathrm{dB}$, where d is the distance between nodes, in metres, at an operating frequency of 2 GHz. We consider $\alpha_{1\mathrm{r}}=\alpha_{2\mathrm{r}},\alpha_{1\mathrm{p}}=\alpha_{2\mathrm{p}}=\alpha_{\mathrm{rp}}$. We assume the MIMO channel matrices $\mathbf{G}_{m\mathrm{r}},\mathbf{G}_{\mathrm{r}m},m\in\{1,2,\mathrm{p}\},\mathbf{G}_{\mathrm{np}},\mathbf{G}_{\mathrm{pn}},\mathrm{n}\in\{1,2\}$ and channel estimation error matrices $\phi_j,j\in\{1,2,r,p\}$ to follow Rayleigh fading and their elements to be independent and identically distributed complex Gaussian random variables, each with zero mean and unit variance. Unless specified otherwise, we consider $\alpha_{1\mathrm{r}}$ = -92dBm and $\alpha_{1\mathrm{p}}$ = -101dBm. Around 1000 Monte Carlo simulations were averaged to arrive at each of the results.

Fig. 2 shows the variation in transmit power of the transceiver versus primary user interference threshold θ for

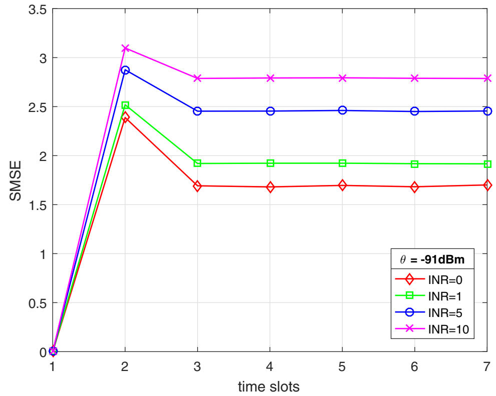

Fig. 4: Variation of SMSE over time, for different values of INR, at θ = -91dBm and SNR = 10dB.

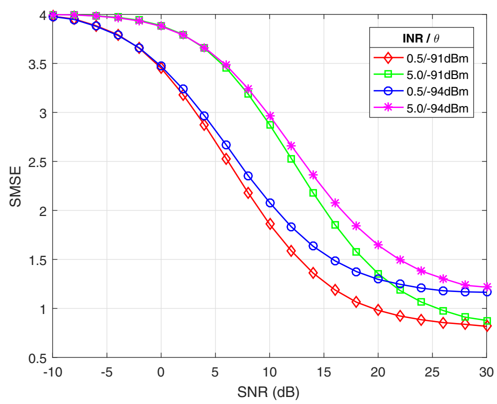

Fig. 5: SMSE, in the 7^{th} time slot, versus SNR, for different values of INR and θ.

different values of path loss $\alpha_{1\mathrm{p}}$. As expected, the transmit power increases with increase in tolerance limit of P.

Fig. 3 illustrates the number of iterations required, in the 7^{th} time slot, to obtain the optimal value of $\mathbf{F}$ and $\mathbf{V}_m, m \in \{1,2\}$, at SNR = 5dB, at different INR values and θ = -94dBm. We observe that beginning with any random complex 2x2 matrix $\mathbf{V}_m$, the SMSE converges after 6 iterations for all the INR values.This makes the design of optimal filters viable.

Fig. 4 describes the performance of the system over time. It shows the variation of SMSE , for different INR values, with θ = -91dBm. The SMSE stabilizes from 3^{th} time slot due to the effect of feedback term $\mathbf{J}_q$, which starts from $t = 3$.

We evaluated the SMSE and sum-rate performance of the system versus SNR for different values of INR and θ and the results are as shown in Fig. 5 and Fig. 6, respectively. The equation for sum-rate is similar to that in [5], but with path loss included and efficient computation. The proposed designs exhibit good SMSE and sum-rate performance at low INR values. This highlights the need for precoding with multiple antennas to suppress the residual SI. As seen in Fig. 6, the sum-rate tends to saturate above SNR = 25dB. Also, as expected, the system performs better at higher values of θ.

V. Conclusion

We considered a full-duplex two-way MIMO relaying system in a cognitive radio network, operating under transmit power constraints. We designed the optimal precoder matrices

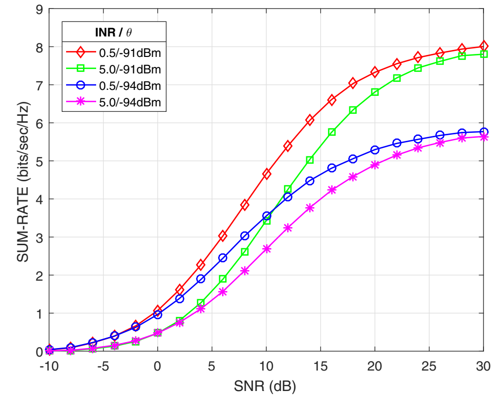

Fig. 6: Sum-rate, in the 7^{th} time slot, versus SNR, for different values of INR and θ.

for the cognitive transceivers, based on SNR maximization at the cognitive relay, to restrict primary user interference. We also proposed the optimal design of cognitive relay precoder and receive filters for the cognitive transceivers, based on SMSE minimization, to curb residual SI. To compensate for the continuously varying SI at the nodes, the relay precoder and receive filter matrices need to be updated in each time slot with the feedback from all the previous time slots. We presented an iterative technique for this update, having low memory requirement and high computational efficiency. The performance of the proposed designs was illustrated in the simulation results for different operating conditions. The proposed designs provide power efficiency with high sum-rate.

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