Design of Micro-heaters Inspired by Space Filling Fractal Curves

by

Kumar Sai Charan Karnati, Seshadri Reddy Nagireddy, Rishabh Bhooshan Mishra, Aftab M Hussain

in

The IEEE Region 10 Symposium
(*TENSYMP-2019*)

Report No: IIIT/TR/2019/-1

Centre for VLSI and Embeded Systems Technology
International Institute of Information Technology
Hyderabad - 500 032, INDIA
June 2019

Design of Micro-heaters Inspired by Space Filling Fractal Curves

Kumar Sai Charan Karnati, Seshadri Reddy Nagireddy, Rishabh Bhooshan Mishra, Aftab M. Hussain*
Center for VLSI and Embedded System Technologies (CVEST)
International Institute of Information Technology (IIIT) – Hyderabad, Telangana-500032

Email – aftab.hussain@iiit.ac.in

***Abstract*—Micro-heating capabilities are important in applications such as thermotherapy, drug delivery, gas sensing and so on. Micro-heaters need to be designed such that they provide uniform temperature across a given surface with minimum metal consumption. In case of Joule heating based micro-heaters, the metal coil design plays an important role in determining its heating characteristics. This paper presents multiple designs of space filling curves that can be used as metal coils in micro-heaters to achieve uniform temperature distribution. We have modelled three types of fractal geometries of different orders - Peano, Hilbert and Moore. In order to compare the performance of these geometries with a typical double spiral heat-pad, we kept a fixed area of 30 mm × 30 mm and input power of 1 W. We have used PDMS (poly-dimethylsiloxane) of thickness 200 µm as a base for the heat-pad and copper is used for the geometries. Further, we maintained constant metal width and thickness of 0.375 mm and 100 µm respectively, across all geometries. Our finite element analysis (FEA) suggests that Peano order-2 & 3, Moore order-3 & 4, Hilbert order-3 & 4 outperformed the double spiral heat-pad in terms of temperature uniformity. The average temperature of all these micro-heaters are around 370 K for 1 W input power. Overall, Peano order-2 curve performed better compared to all geometries in terms of the metal coil length and temperature uniformity trade-off.**

Keywords—Fractal curves, Hilbert curve, Moore curve, Peano curve, temperature uniformity, Heat-pad, Micro-heater.

I. Introduction

Micro-heaters are of major importance in various applications such as thermotherapy, drug delivery, gas sensors and other MEMS applications [1-5]. Further, recent studies have shown the use of micro-heaters for targeted destruction of malign cancer cells [6, 7]. The key requirements of using a micro-heater are low power consumption, low areal footprint, low cost and fast response [8]. These properties mainly depend on the metal coil design and the backing material used to fabricate the heater. Some previous studies which tried to optimize metal coil geometry for better performance used meander, parallel meander, fan, double spiral and honey-comb curves [9-12]. These studies reported the maximum temperature obtained using these geometries. While a common strategy to compare micro-heaters is to operate all the geometries at the same voltage, or temperature, we argue that the ideal way to compare micro-heater designs is to fix a particular power density (power per unit area of the heater) and compare the obtained average temperature and temperature distribution.

In this work, we present a detailed comparison between micro-heater performance of using metal coils based on several orders of different fractal geometries such as Peano curve, Hilbert curve and Moore curve. For a fair comparison, we provide the same input power density to all the heaters and the heat dissipation constant is maintained the same for all heaters. We look to optimize the geometry in such a way that it provides better maximum temperature, average temperature and temperature variation with optimum use of metal.

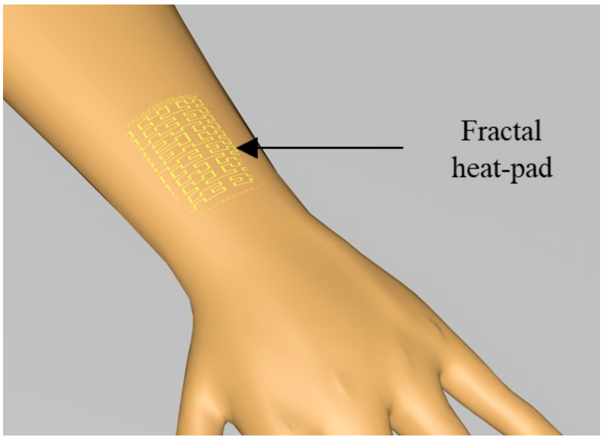

Fig. 1. Illustration of a flexible metallic heat-pad based on a fractal metal coil design.

II. Space Filling Curves

Space-filling curves have been studied as purely mathematical constructs since the end of the nineteenth century [13]. It is an intriguing idea that a one-dimensional geometric shape can “fill” a 2-D space. To this end, many recursive and self-similar curves have been designed, which leads to important locality properties. These curves are recursive such that a part of the curve is replaced by a mapping of the entire curve to obtain higher order curves. As the order of the curves approaches infinity, the curves tend to completely fill the given unit square. By their nature, space filling curves occupy the unit square uniformly. As their order is increases, the density of the filling increases. This is the motivation in using fractal geometry derived from space-filling curves to design metal coils for micro-heaters. Given the uniform distribution of this curves in 2-D space, lower temperature variation can be expected from such designs. In this work, we have used various orders of the Hilbert curve, the Moore curve and the Peano curve as inspiration to design micro-heaters.

In case of the Peano curve, we are working with a variant of the original Peano curve called the Wunderlich curve. The first, second and third order Peano curve is as shown in Fig. 2. It is constructed using a seed shape and for every successive iteration, a copy of the seed shape is rotated by 90° anticlockwise and placed inside the curve in such a way that the end of one copy is next to beginning of the next copy. These copies are subsequently joined together to form a continuous higher order curve.

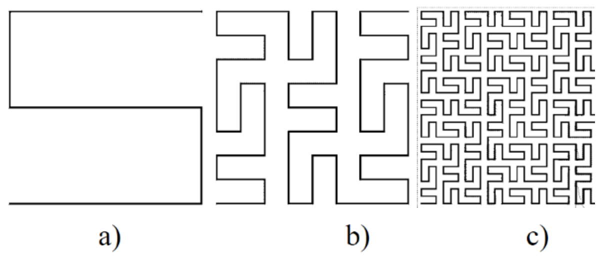

Fig. 2. Geometries of the Wunderlich variant of the Peano curve (a) order-1, (b) order-2, and (c) order-3.

The first, second, third and fourth order Hilbert and Moore curves are as shown in Fig. 3 and Fig. 4 respectively. These curves are based on the fact that a square can be divided into four smaller squares and the previous design can be placed in the smaller tiles. In case of the Hilbert curve, the original curve is positioned in the four furthest corners of the smaller square tiles and then interconnected using straight lines. The top two copies are maintained as original while the bottom two are rotated 90° each. In case of the Moore curve, the four furthest corners of the smaller tiles are filled such that both the top two and the bottom two copies are rotated 90° each. However, a major difference between these curves is that Hilbert's Order n curve is generated from Hilbert's Order n-1 curve whereas Moore's Order n curve is also generated from Hilbert's Order n-1 curve.

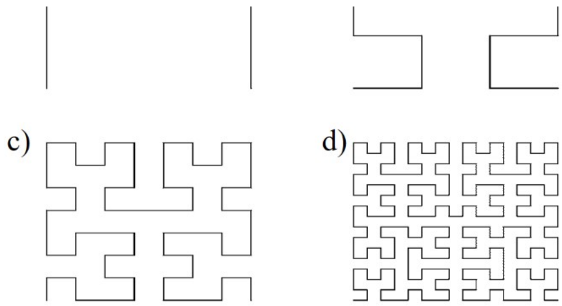

Fig. 3. Geometries of the Hilbert curve (a) order-1 (b) order-2 (c) order-3 and (d) order-4.

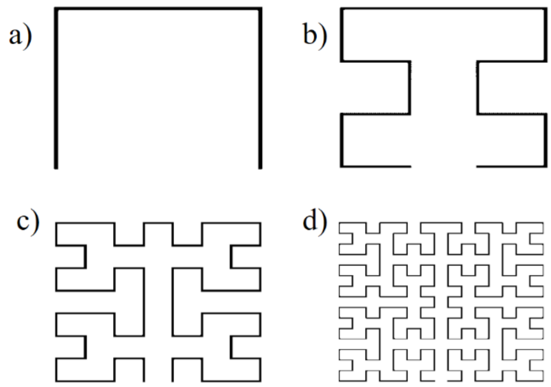

Fig. 4. Geometries of the Moore curve (a) order-1 (b) order-2 (c) order-3 (d) order-4.

III. MODELLING SPACE FILLING CURVES

A. *Space Filling Curves*

Before going into the modelling of space filling curves, let us consider the geometry of a typical double spiral heat-pad (Fig. 5a). To be able to deploy the curves for metal coil fabrication, we modified the geometries of the curves before we modelled them. To ensure a better current flow and manufacturability, we have smoothened the corners of the geometries. Here, the sharp bends are transformed into smooth arcs to improve the mechanics of all the geometries (Fig. 5 and Fig. 6) [14]. These designs can be fabricated using standard techniques of lithography, inkjet printing, roll-to-roll production and so on.

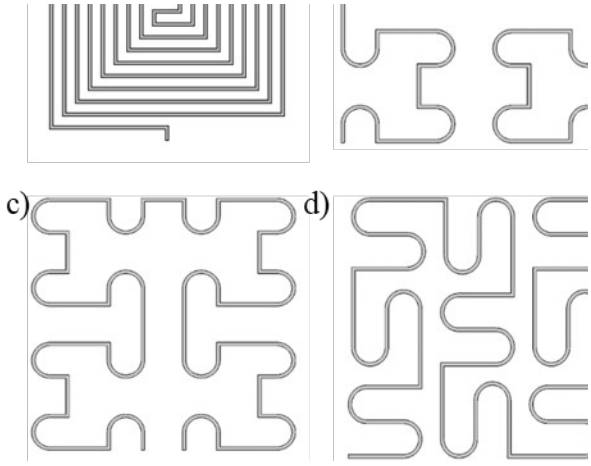

Fig. 5. Simulated metal coil geometries based on (a) double spiral, (b) Hilbert order-3, (c) Moore order-3 and (d) Peano order-2 curves.

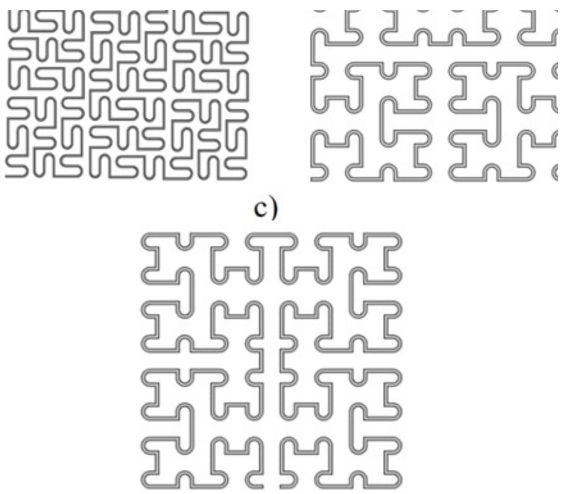

Fig. 6. Simulated metal coil geometries based on (a) Peano order-3, (b) Hilbert order-4 and (c) Moore order-4 curves.

B. *Modelling and Simulation*

We have modelled the geometries above in COMSOL Multiphysics. In order to do a fair comparison among the heat-pads, we constructed all the fractal design and control models with the same area, metal coil thickness and width. The curves were designed with a width of 0.375 mm in an area of 30 mm × 30 mm. The geometries were then extruded to a thickness of 100 µm to model a copper thin film. A block of polymer PDMS (poly-dimethylsiloxane) and size 30 mm × 30 mm × 200 µm was constructed as a base for the heat-pads. The heat pads are based on the principle of Joule heating.

When a current is passed through one end of the heat-pad, there is heat dissipation due to the resistivity of metal. In order to effectively compare the performance of various designs of heat-pads we delivered the same power density to all of them. This was done by calculating the resistance from the length and area of the curves and driving a particular current to each of the designs. We chose to deliver 1 W power to all the geometries considering the fact at such heaters are generally drivable using simple micro-controller circuits. The Joule heating module in COMSOL Multiphysics was utilized to control the currents and heat dissipation properties of the model. Copper was taken as the material of the metal coils (resistivity, 1.72E-8 Ω/m) and PDMS as material of the base (resistivity 1E+12 Ω/m). The outward heat flux was modelled as convective heat transfer from the top surface of the heat-pad to air. The heat transfer coefficient (h) as 10 W/m^{2}K.

IV. Results and Discussion

To complete the comparison and establish a trend, we simulated a wire-line going from one end of the square heat pad to another as the simplest form of “micro-heater” (Fig. 7). This metal line was also delivered the same power density that was used for other heaters. Because the total length of copper in this case was very less, the resistance of the wire-line was much smaller than other heat-pads. Thus, the current through this heater was much larger than the others, to maintain the same power density. The temperature distribution of the wire-line heater for 1 W input power is shown in Fig. 7.

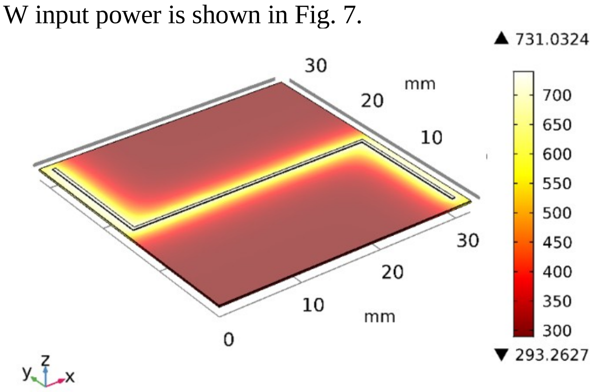

Fig. 7. 3D surface temperature plot of wire-line heat-pad for 1 W input power.

The high current through the wire-line heater resulted in a maximum temperature of 731 K throughout the path of the metal geometry and an average temperature of around 350 K for the rest of the heat-pad. Because the simple wire-line heater does not cover a significant area on the heat-pad, this non-uniformity was expected.

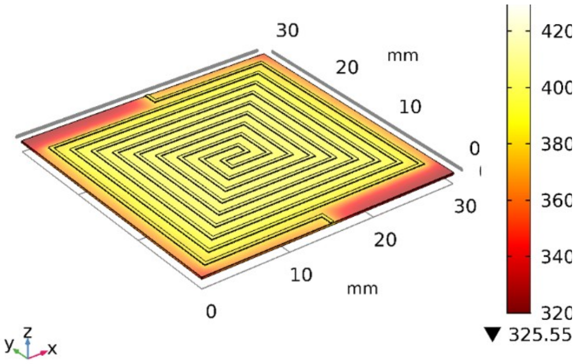

Fig. 8. 3D surface temperature plot of a double spiralheat-pad with 1 W input power.

The double spiral heat-pad was simulated using the same power density and area. The maximum temperature in this case was found to be 402 K while the minimum temperature was 325 K (Fig. 8). While this is an improvement over the performance of a simple wire-line heater, the double spiral heat-pad deisgn still shows large temperature non-uniformity and variance. We then simulated the Peano order-2, Hilbert order-3 and Moore order-3 curves. The temperature distribution for these curves is shown in Fig. 9. We have maintianed the same temperature color scale in order to allow for a quick comparison between the three curves. The length of copper used in these curves is approximately the same, thus, the perfomance difference is purely based on the geometry and metal distribution. In this case as well, the input power was fixed at 1 W with the heat-pad area fixed at 30 mm × 30 mm. By observation, it can be clearly seen that Peano order-2 curve provides a better temperature uniformity comapred to the other two curves.

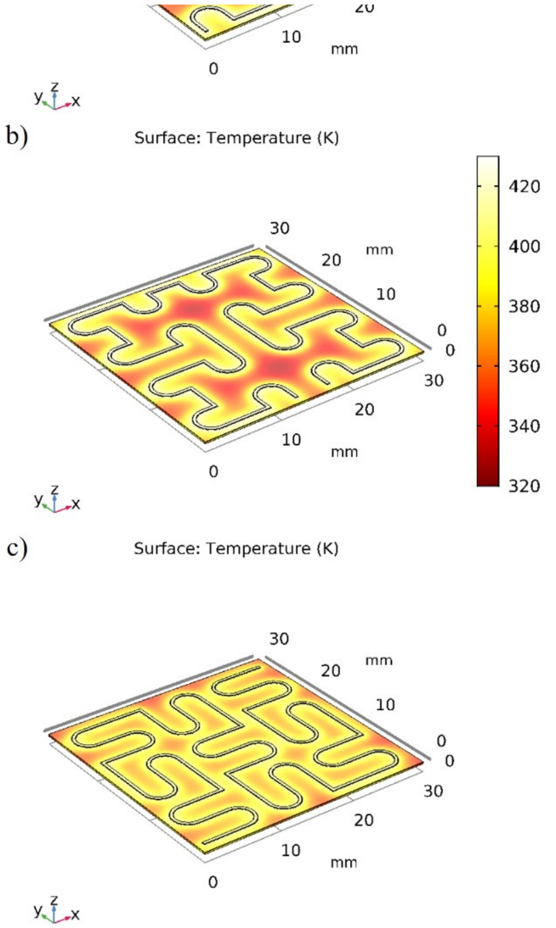

Fig. 9. 3D surface temperature plots of (a) Hilbert order-3 (b) Moore order-3 and (c) Peano order-2 curves. Color legend for all the temperature plots is common.

The designs were then self-replicated and formed into Peano order-3, Hilbert order-4 and Moore order-4 curves. The connections between the smaller copies of the lower order designs where made using smooth curves. The final designs were simulated as micro-heaters using the same power density as for the lower order designs. Fig. 10 shows the temperature distribution of these curves in steady state. In case as well, the temperature scale is maintained the same as in Fig. 9 so that temperature distribution can be visually compared across all designs. It can be clearly seen that there is an improvement in temperature uniformity with higher order fractal deisgns. However, because the curves are higer order iterations of a fractal, the total length of copper line is significantly larger than in the previous case. While this can result in a marginal improvement in temperature uniformity, the cost of fabricating the heater significantly increases. Hence, these designs may only be applicable in very specific applications where temperature uniformity is of primary concern. Further, the temperature achieved by Peano order-3 was observed to be lower than that from the other curves. This can be attributed to the large length of the curve because of which the temperature is much more uniform. Also, there is dissipation of heat from the copper surface and the sidewall that can reduce the overall steady state temperature.

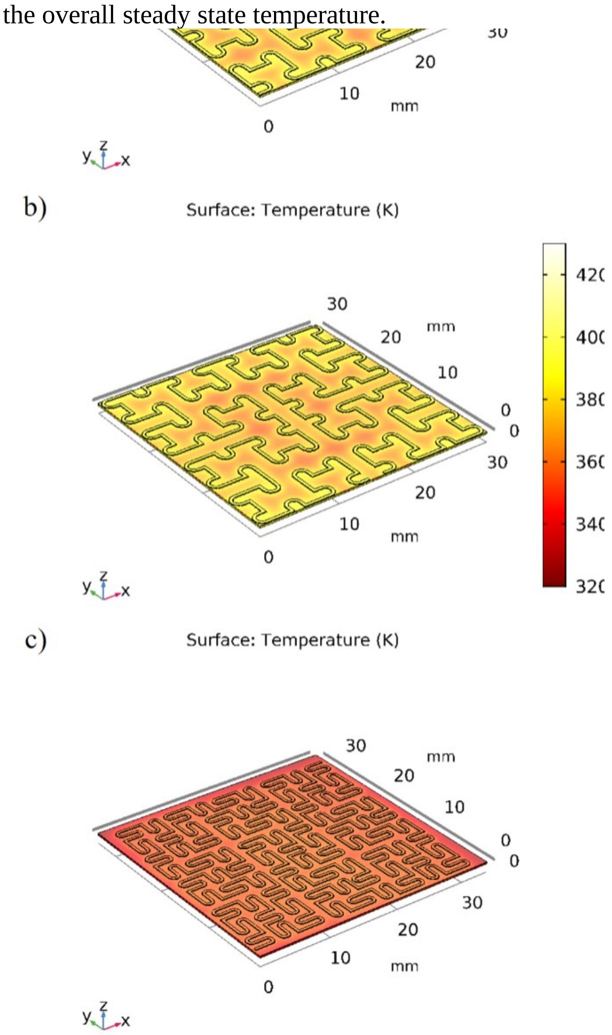

Fig. 10. 3D surface temperature plots of (a) Hilbert order-4 (b) Moore order-4 (c) Peano order-3 curves. Color legend for all the temperature plots is common.

Because of the nature of the curves, Hilbert and Moore order-4 span the exact same length where as Peano order-3 spans almost double the length. The lengths of all the curves simulated in this study are given in Table I. Clearly, there is a large difference between the lengths of the curves which leads to a large difference in the temperature distribution obtained from them. The main trade-off here is that use of more metal (by the way of employing higer order fractal designs) improves temperature uniformity at steady state, but increases the cost and complexity of fabrication.

TABLE I. LENGTHS OF VARIOUS FRACTAL GEOMETRIES SIMULATED IN THIS STUDY

Heat-pad Geometry	***Length (mm)***
Double spiral	522.90
Hilbert Order-3	224.52
Hilbert Order-4	455.06
Moore Order-3	224.52
Moore Order-4	455.06
Peano Order-2	273.09
Peano Order-3	927.70
Wire-line	60.00

Table II presents the average, maximum, minumum and variance (difference between maximum and minimum) for the temperature distribution of all the heat-pads simulated. Here, we have defined $\Delta T = max T - min T$. From Table II, it can be clearly seen that Peano order-2, Hilbert order-3 and Moore order-3 curves have similar average temperature and uniformity as compared to the typical double sprial heat-pad. However, the third order Hilbert and Moore curves use only 42% of the copper used by the double spiralheat-pad whereas Peano order-2 uses 52%. On closer analysis, it is observed that ΔT is 92.98 K and 91.40 K in third order Hilbert and Moore curves respectively, whereas it dropsto 61.60 K in Peano order-2 curve. Hence, given a fixed area, Peano order-2 curve uses only 52% of the copper used by a typical double spiral heat-pad and gives a higher average temperature and better temperature uniformity.

TABLE II. TEMPERATURE DATA FOR ALL SIMULATED GEOMETRIES. ALL THE TEMPERATURE READINGS ARE IN KELVIN

Heat-pad	***Avg T***	***Max T***	***Min T***	***ΔT***
Double spiral	384.40	401.32	325.52	75.80
Hilbert Order-3	384.68	425.74	332.76	92.98
Hilbert Order-4	377.01	395.08	356.76	38.32
Moore Order-3	384.90	423.97	332.57	91.40
Moore Order-4	377.08	393.02	356.41	36.61
Peano Order-2	386.36	410.53	348.93	61.60
Peano Order-3	355.26	361.86	328.12	33.74
Wire-line	373.68	724.36	293.26	431.10

To have a visual comparison of the trade-off in length and device performance, we plotted the average temperature (Fig. 11) and temperature variance (Fig. 12) with the length of the metal used.

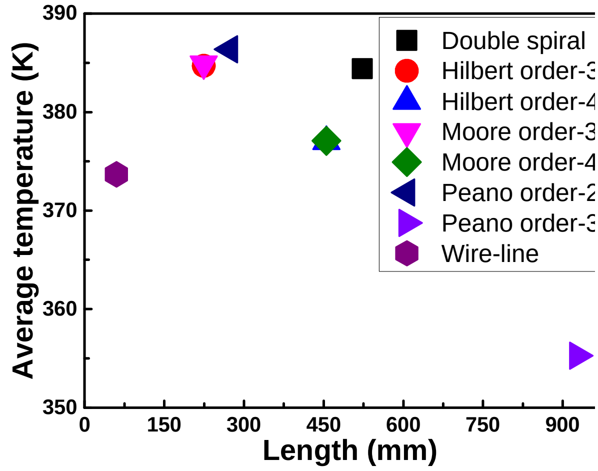

Fig. 11. Plot of average temperature versus length of a metal coil for 1 W input power.

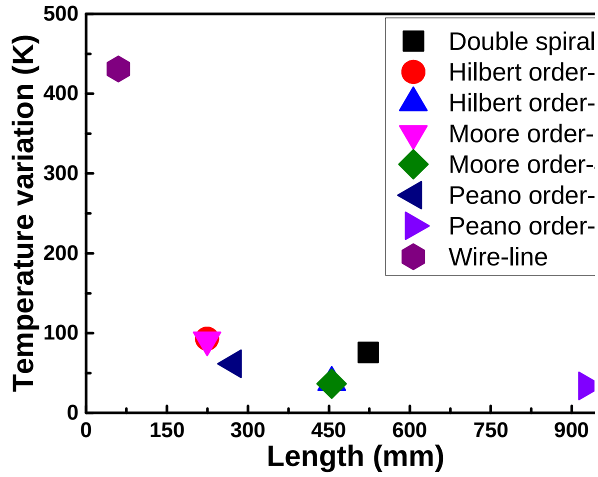

Fig. 12. Plot of temperature variance versus the length of a metal coil for 1 W input power.

The fourth order Hilbert and Moore curves also perform better comapred to the double spiral curve. While they use 87% of the copper used by the double spiral curve, they have a reduction in ΔT by 52%, which provides a much better temperature distribution. However, they provide a lower average temperature compared to the double spiral heat-pad. As expected, off all the designs simulated in this study, the Peano order-3 provides the least difference between the maximum and minimum temperature at 33.74 K. However, it consumes about 77% more copper than a typical double spiral heat-pad, and more than 3× compared with third order Hilbert and Moore curves. The compromise between the length of the heat-pad metal coil and the temperature uniformity is clear from Fig. 12. As the length of the metal coil is increased, there is a clear reduction in the temperature variance, however, the rate of reduction is very slow for higher order fractal curves.

V. Conclusion

We conclude that the design of the metal coil in a micro-heater depends on the intended application, uniformity requirements and cost constraints. If temperature uniformity is of primary importance, one can use the fourth order Hilbert and Moore curves. These curves provide a high temperature uniformity without increasing metal coil length substantially. Interestingly, although Hilbert and Moore curves have the same basic structure and span the same length, Moore curves were observed to have higher uniformity compared to Hilbert curves.

If the amount of copper used, and hence the cost of the device is an important constraint, one can use Peano order-2 or third order Hilbert and Moore curves. These curves provide a reasonable temperature uniformity compared to the typical double spiral structure, while consuming almost half the copper. Indeed, we observed that higher metal length (with higher order fractals) provide a better temperature uniformity. However, the gain in uniformity for higher order fractals was observed to be limited.

References

[1] J.-G. Kang et al., "Temperature control of micro heater using Pt thin film temperature sensor embedded in micro gas sensor," Micro and Nano Systems Letters, journal article vol. 5, no. 1, p. 26, 2017.

[2] T. Li, L. Xu, and Y. Wang, "Micro-Heater-Based Gas Sensors," in Micro Electro Mechanical Systems, Q.-A. Huang, Ed. Singapore: Springer Singapore, 2018, pp. 717-752.

[3] W.-J. Hwang, K.-S. Shin, J.-H. Roh, D.-S. Lee, and S.-H. Choa, "Development of micro-heaters with optimized temperature compensation design for gas sensors," Sensors (Basel, Switzerland), vol. 11, no. 3, pp. 2580-2591, 2011.

[4] A. M. Hussain, E. B. Lizardo, G. A. Torres Sevilla, J. M. Nassar, and M. M. Hussain, "Ultrastretchable and Flexible Copper Interconnect-Based Smart Patch for Adaptive Thermotherapy," Advanced Healthcare Materials, vol. 4, no. 5, pp. 665-673, 2015.

[5] A. M. Hussain and M. M. Hussain, "CMOS-Technology-Enabled Flexible and Stretchable Electronics for Internet of Everything Applications," Advanced Materials, vol. 28, no. 22, pp. 4219-4249, 2016.

[6] R. Pimentel-Domínguez, P. Moreno-Álvarez, M. Hautefeuille, A. Chavarría, and J. Hernández-Cordero, "Photothermal lesions in soft tissue induced by optical fiber

microheaters," Biomedical optics express, vol. 7, no. 4, pp. 1138-1148, 2016.

[7] K. Visvanathan and Y. B. Gianchandani, "Ultrasonic microheaters using piezo-ceramics for cauterization and other applications," TRANSDUCERS - International Solid-State Sensors, Actuators and Microsystems Conference, pp. 2421-2424, 2009.

[8] Y. Xian, L. Jianjun, L. Huafeng, and Z. Xiaofeng, "Fabrication of a MEMS micro-hotplate," Journal of Physics: Conference Series, vol. 276, no. 1, p. 012098, 2011.

[9] S. Joy and J. K. Antony, "Design and Simulation of a Micro Hotplate Using COMSOL Multiphysics for MEMS Based Gas Sensor," Fifth International Conference on Advances in Computing and Communications (ICACC), pp. 465-468, 2015.

[10] H. Liu, L. Zhang, K. Li, and O. Tan, "Microhotplates for Metal Oxide Semiconductor Gas Sensor Applications—Towards the CMOS-MEMS Monolithic Approach," Micromachines, vol. 9, no. 11, p. 557, 2018.

[11] H. Kumar, K. K. Singh, N. Sood, A. Kumar, and R. K. Mittal, "Design and simulation of a Micro Hotplate for MEMS based integrated gas sensing system," IEEE Sensors Applications Symposium (SAS), pp. 181-184, 2014.

[12] O. Sidek, M. Z. Ishak, M. A. Khalid, M. Z. A. Bakar, and M. A. Miskam, "Effect of heater geometry on the high temperature distribution on a MEMS micro-hotplate," 3rd Asia Symposium on Quality Electronic Design (ASQED), pp. 100-104, 2011.

[13] M. Bader, Space-Filling Curves: An Introduction with Applications in Scientific Computing. Springer Publishing Company, Inc., 2012, p. 291.

[14] J. A. Fan et al., "Fractal design concepts for stretchable electronics," Nature Communications, vol. 5, p. 3266, 2014.