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A Comparative Study of Absorbing Layer Methods to Model Radiating Boundary Conditions for the Wave Propagation in Infinite Medium

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Abstract:

Radiating boundary condition is an important consideration in the finite element modelling of unbounded media. Absorbing layer techniques such as Perfectly Matched Layers (PML) and Absorbing Layers by Increasing Damping (ALID) becoming popular as they are efficient in absorbing outward propagating waves energy. In this study, a comparative analysis has been carried out between PML and ALID+VABC (Absorbing Boundary conditions for Viscoelastic materials) methods. The methods are analyzed using LS-DYNA explicit solver and the efficiency is compared with standard solutions. The study concluded that PML requires less number of elements to model the boundary conditions when compared with ALID+VABC. But PML requires a smaller element length which increases overall computational time. Both the methods are efficient in absorbing the wave energy. However, PML requires additional implementation cost to solve the complex equations.

*Keywords**:** Radiating Boundary Conditions, Perfectly Match Layers, Absorbing Layer by Increase in Damping, Absorbing Boundary Conditions for Viscoelastic Wave Propagation, Soil-Structure Interaction.*

1. Introduction

Modelling the Sommerfield radiating boundary is a major challenge appearing in the study of many engineering problems involving elastic wave propagation [1]. Particularly, in the finite element analysis of Soil-Structure-Interaction (SSI) problems, where the soil modelling has to be truncated at a finite distance. This truncation of the model at finite boundary leads to reflection of radiating elastic waves. The reflected waves from the boundary will affect the solution and may lead to instabilities in the numerical analysis. Therefore, it is necessary to provide an artificial boundary condition that will transmit the outward propagating waves with minimum or negligible reflections.

To address radiating boundary conditions problem, researchers have developed various kinds of formulations over a few decades, such as a). Local Absorbing Boundary Conditions [2-4], b). Absorbing Layers techniques which include Perfectly Matched Layers [5-11], Caughey Absorbing Layer Method [12-13], Absorbing Layers by Increasing Damping [14-15] and Stiffness Reduction Method [16], c). Boundary element method [17], and d). Infinite elements [18-20].

Local Absorbing Boundary Conditions (ABC) proposed by Lysmer and Kuhlemeyer [2] correspond to a situation where the boundary is supported on infinitesimal dash-pots oriented normal and tangential to the boundary. These boundary conditions are extensively used in commercial software, since they are very easy to implement, and have negligible computational cost. Engquist and Majda [3] proposed a higher order local approximate boundary conditions. Badry and Ramancharla [4] proposed local Absorbing Boundary Conditions for viscoelastic wave propagation (VABC) to take the effect of mass proportional damping into consideration. Though ABC and VABC are very simple, the major drawback is that they are inefficient when the wave impinges the boundary other than normal direction.

Perfectly Matched Layer (PML) originally proposed by Berenger [5] is an artificial absorbing layer and has been widely used in recent years [6-10]. PML has been successfully implemented in time-domain for explicit dynamic solver by Basu in 2009 [11]. The basic idea of the PML is that the incident wave energy is absorbed inside the PML layers while matching impedance with adjacent layers. This property allows the PML to strongly absorb outgoing waves from the interior of a computational region without reflecting them back into the interior. However, the resulting finite element formulations are very complex and require much computational time.

The Caughey Absorbing Layer Method (CALM) was proposed by Semblat et al [12], is a simple and reliable alternative to the Perfectly Matched Layer (PML) like other absorbing layer methods [14-16]. The CALM consists of defining the absorbing layers at the boundaries of the elastic medium under consideration. These absorbing layers are modelled with the same elastic properties as the interior medium, but the Rayleigh damping is added to attenuate all waves that leave the interior domain. This method requires many absorbing layers to absorb the outward propagating waves and hence requires much computational time [16].

The Absorbing Layers by Increasing Damping (ALID) is similar to the CALM absorbing layers method. Rajagopal et al. [15] applied mass proportional damping part of Rayleigh damping to the absorbing region. As a result, this method does not affect the stable time step required for explicit finite element analysis scheme. The stiffness reduction method (SRM) was proposed Petit et al [16], in which Young’s modulus is reduced exponentially, in addition to increase in mass proportional damping.

The boundary element method [17] consists in solving an equation on the boundary of the domain only and the radiation conditions are taken into account analytically. It also reduces the dimension of the problem to a surface in 3D and to a curve in 2D thus decreasing the size of the problem to solve. However, the final problem involves the full matrix which is generally nonsymmetrical. There are also singularities in the integrals which need special attention for the numerical integrations. It is mainly limited to linear problems and to homogeneous domains or otherwise one has to introduce special and complex techniques to deal with nonlinear or non-homogeneous situations.

Apart from ABC and Absorbing Layers (PML, CALM and etc.), techniques have been proposed to map the semi-infinite domain onto a finite domain using Infinite element [18-20]. The accuracy of the infinite elements depends upon the choice of the shape functions and the order of approximation. The dynamic Infinite elements [20] includes the effect of wave propagation into the unbounded domain by using frequency dependent mass and stiffness matrices. This method requires complex transformations of mass and stiffness matrices from the frequency domain to time domain.

Israeli and Orszag [14] presented a technique to combine the absorbing layers and local Absorbing Boundary Conditions, in which absorbing layer properties are modified such that the damping coefficient is zero at the boundary. The damping properties are gradually increased from zero at the beginning to a maximum at the middle of the layers and then reduced to zero at the end of the absorbing region. Therefore, absorbing layer properties are matched with the Absorbing Boundary Conditions. However, this method needs twice the number of layer compared with that required for ALID. Badry and Ramancharla [22] have addressed the issue of impedance mismatch between ALDI and ABC by adjusting the viscosity property of local absorbing boundary conditions for viscoelastic wave propagation (ALID+VABC). Therefore, this method requires less number of elements compared to ALID alone.

It is also a well-known fact that absorbing layer techniques such PML and ALID+VABC are efficient in absorbing the wave energy. However, the computational and implementation costs are not well studied. Therefore, in this study, a comparison between PML and ALID+VABC is carried to verify the efficiency of these methods in absorbing the wave energy, the mesh requirements and the effect of loading frequency. Finally, computation time and implementation cost are also discussed about.

2. ALID+VABC Theory

In this section, we first briefly discussed ALID and VABC boundary conditions and then explained about how these two methods were combined to get proposed boundary conditions.

2.1 Absorbing Layers by Increasing Damping (ALID)

The equation of motion of the system under dynamic equilibrium is defined as

$$[M]\ddot{u} + [C]\dot{u} + [K]u = F \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

Where$[M]$, $[C]$ and $[K]$ are the global mass, damping and stiffness matrices respectively. $\ddot{u}$, $\dot{u}$, u and F are the acceleration, velocity, displacement and external force vectors respectively. Damping matrix can be defined using Rayleigh damping coefficients as

$$[C] = \alpha[M] + \beta[K] \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

Where α and β are mass and stiffness proportional damping coefficients.

The ALID method uses a material with gradually increasing damping in successive layers such that they absorb incident wave energy and the reflections due to impedance mismatch in successive layers is minimized. At the beginning of the absorbing region, the damping is kept equal to the damping in Area of Study (AoS), zero in most cases and maximum at the end of the absorbing region. Thus the wave entering the absorbing region is gradually damped in the absorbing layers.

To define an effective absorbing region, it is necessary to optimize variables such as damping coefficients α and β, absorbing region length and number of elements in ALID. It is noted that introducing damping into the model can decrease the value of the stable time increment within explicit schemes, thereby reducing computational efficiency. However, a high value of α causes only a small decrease in the stable increment as compared to that of β [15, 21]. It is therefore, avoided using β in the absorbing region to eliminate the time stepping the issue and also allows efficient computation, since the mass matrix in explicit schemes is usually diagonal.

Rajagopal et al [15] and Petit et al [16] provided the guidelines to define the optimal values for absorbing region. This summarises to

- The total thickness of the absorbing layer, L = 1.5 times incident wavelength
- The maximum damping coefficient $\alpha_M = \omega$, where, ω is incident wave frequency
- The damping co coefficient, $\alpha = \alpha_M (x/L)^p$, where, attenuation factor, p = 3 and x is the distance from AoS. $x = 0$ at the beginning of the ALID and $x = L$ at the end of ALID.
- The mesh density i.e. a minimum number of elements should be 20 per wavelength.

2.2 Viscoelastic Local Absorbing Boundary Conditions (VABC)

The Absorbing Boundary Conditions for wave propagation in Maxwell type viscoelastic material [22] corresponds to a situation where the boundary is supported on infinitesimal dash-pots and spring oriented normal and tangential to the boundary. The corresponding stress components are given by

$$\sigma = a \ \rho \ V_p \ \dot{u} \ + 0.5a \ \rho \ V_p \ \alpha \, u \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

$$\tau = b \ \rho \ V_s \ \dot{v} \ + 0.5b \ \rho \ V_s \ \alpha \, v \qquad \ldots\ldots\ldots\ldots \qquad (4)$$

Where σ and τ are the normal and shear stresses, $\dot{u}$ and $\dot{v}$ are the normal and tangential velocities respectively; ρ is the mass density; V_s and V_p are the velocities of S-waves and P-waves respectively; a and b are dimensionless parameters; u and v are the normal and tangential displacements respectively; α is mass proportional damping coefficient.

2.3 Combining ALID and VABC

To achieve optimum performance when combining the ALID and VABC, it is necessary to avoid the impedance mismatch between the last absorbing layer and VABC [22]. Replacing α and β with the maximum damping coefficients α_M and β_M respectively in equation 3 and 4 yields

$$\sigma = a \ \rho \ V_p \ \dot{u} \ + 0.5a \ \rho \ V_p \ \alpha_M \, u \qquad \ldots\ldots\ldots.. \qquad (5)$$

$$\tau = b \ \rho \ V_s \ \dot{v} \ + 0.5b \ \rho \ V_s \ \alpha_M \, v \qquad \ldots\ldots\ldots.. \qquad (6)$$

From the above equations, it can be observed that combining VABC and ALID requires modelling a dashpot with coefficient $a\rho A V_p$ and a spring with coefficient $0.5\alpha_M \, a\rho A V_p$. The spring constant $0.5\alpha_M \, a\rho A V_p$ is added in addition to the standard ABC to match the impedance between the ABC and last absorbing layer. The spring computations and dashpot computations are simple and requires one-time calculations at the beginning of the analysis. Therefore, the additional computational cost to combine VABC with ALID is negligible. The major computation time is due to several elements required in ALID regions to absorb energy. It is also worth noting that the features such as springs and damper are readily available in almost all FE codes. Therefore, there is no additional implementation cost for this method.

3. Perfectly Matched Layers (PML)

The formulations for Perfectly Matched Layers (PML) for time domain explicit finite element analysis are given by Basu [11]. The equation of motion of the system under dynamic equilibrium in PML can be defined as

$$[M]\ddot{u} + [C]\dot{u} + [k]u + [K]U = F \qquad \text{…………} \qquad (7)$$

Where, $\ddot{u}$, $\dot{u}$, u and F are the acceleration, velocity, displacement and external force vectors respectively; $[M]$, $[C]$, $[k]$ and $[K]$ are the system matrices derived from the elements.

$$m = \int_{V_e} \rho f_M N^T N \, dv \,, \quad c = \int_{V_e} \rho \frac{c_s}{L} f_C N^T N \, dv,$$

$$k = \int_{V_e} \rho \left(\frac{c_s}{L}\right)^2 f_K N^T N \, dv, \quad K = \int_{V_e} \rho \left(\frac{c_s}{L}\right)^3 f_H N^T N \, dv \qquad \text{…………} \qquad (8)$$

$$U = \int_0^t u \, d\tau,$$

where, c_s is the shear wave speed L is the length of PML; The factors f_M, f_M, f_M and f_M are the loading frequency parameters; N is the shape function. For Non-PML region equation (7) becomes equation (1). From the inspection of (1) and (7), it can be observed that the method requires additional computation cost and additional memory. Also, this method is not readily available in many of the commercial software.

4. Numerical examples

The objective of this section is to compare both the methods through various numerical examples and the results are validated with standard solutions. The elastic wave propagation problems are analysed using LS-DYNA [23]. A one-dimensional model is created to model the elastic wave propagation as shown in Fig. 1. The model is created using 8 noded brick element and restrained all degrees of freedoms except axial direction, replicating a 1D wave propagation problem. The 8 noded solid elements were chosen to model this problem as LS-DYNA currently supports only this element type for PML. The material is considered to be elastic, with Young's modulus, Poisson Ratio, and density as 1200e4N/m^2, 1/3 and 1800kg/m^3 respectively. The primary wave velocity, V_p is estimated as 100m/sec. The model is assumed to be subjected with predominant wave frequency, f = 5 Hz. The wavelength, λ is estimated as 20m for the incident wave. The length of the model is considered as 3λ i.e. 60m. The size of each element is taken as 1 m.

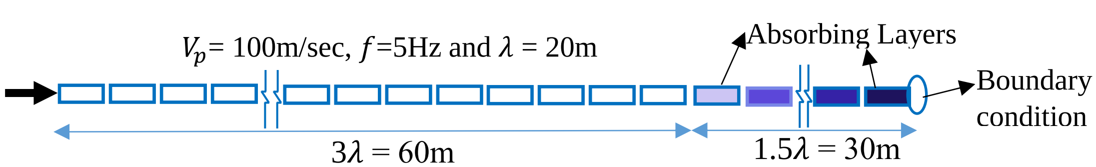

Figure 1: Configuration for one dimensional wave propagation model

4.1.1 Element Size and Time Step

In this section, the element size requirement is analyzed for both PML and ALID. The size of the element for ALID is fixed by defining as 20 elements for wavelength and the length of the absorbing region is taken as 1.5λ i.e. 30m. Same number of elements i.e. 30 elements are provided in the absorbing region in both cases to compare the efficiency. In the case of PML models are created with various element sizes viz 1.0m, 0.5m and 0.25m to compare the effect of element size. Results from all models are compared against the infinitely extended model, where the model length is such that the reflection from the boundary will not reach the Area of Study (AoS) during the analysis.

The load is applied at the left end of the member and wave travelling time to reach absorbing layers is 0.6 sec. The analysis has been carried out for 3 seconds to study the effects of reflection of the wave. Ricker wavelet load is chosen to apply as the predominant frequency

$$F(0,t) = A_f\left[1 - 2\pi^2 f_p^2 (t - t_s)^2\right] e^{-\pi^2 f_p^2 (t-t_s)^2} \quad \ldots\ldots\ldots.. \quad (16)$$

Where A_f is the amplitude, f_p is the peak frequency and t_s is time shift and the parameters used in this problem are A_f = 1 N, t_s = 0.2 sec. Peak frequency, $f_p$is the designed load frequency 5Hz.

The wave propagation forces at the middle of AoS region i.e. at 30m from loading point are shown in the Fig. 2. The reflections in case of ALID+VABC are about 2.8% and in case of PML the reflections are 8.77%, 1.83 and 1.5% for element lengths 1.0m, 0.5m and 0.25m respectively. In addition to the above, there are reflections around 1.0% and 0.5% in the case of PML with 0.25m and 0.5m due to a sudden change in element size. It is also noted that the reflections in case of PML are fictitious and continuously effecting the results.

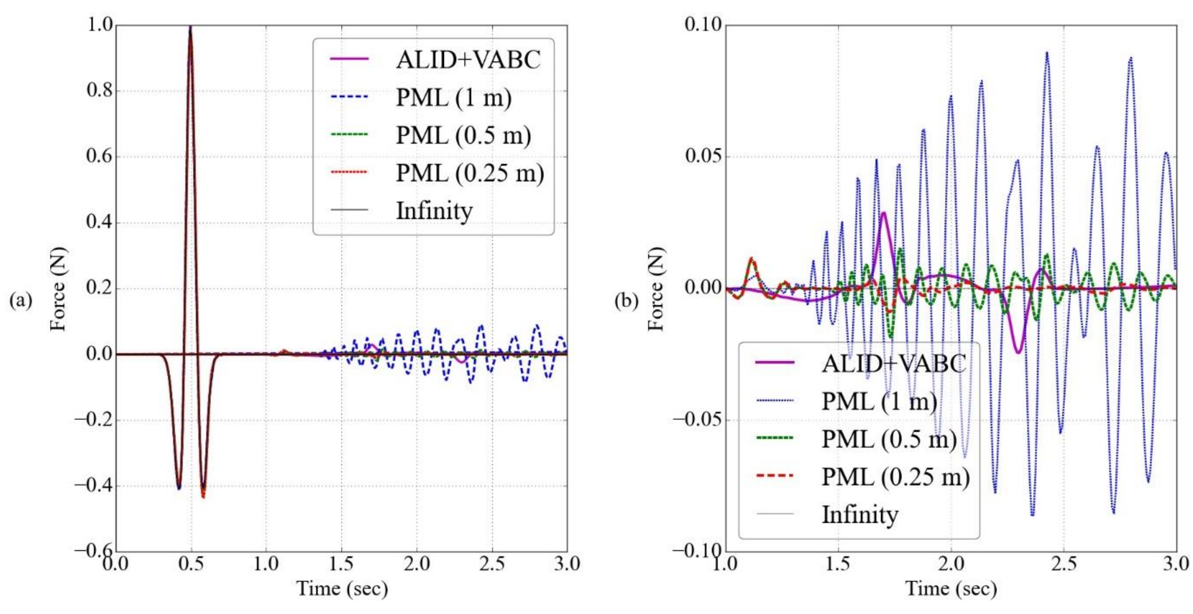

Figure 2: Forces in the element at the middle of AoS i.e. at 30m: a) full-time record and b) zoomed response

The finite element formulations in PML region and the length of the elements affect the computational efficiency. Table 1 shows the time step required for all the configurations to evaluate the effect on computational time step. From the table, it is noted that the PML element formulations affect the computation time step of about 0.55% which is negligible. But, the element length requirement reduces the computational time step at least 50%. Therefore, computational time for the entire model increased to twice.

Table 1: Time step required for boundary conditions

S.No	Boundary Condition Type	Element Length (m)	Time Step (sec)	Reduction in Time Step (%)
1	ALID+VABC	1.00	8.5285E-03	0.00
2	PML	1.00	8.4813E-03	0.55
3	PML	0.50	4.2407E-03	50.28
4	PML	0.25	2.1203E-03	75.14
5	Infinite	1.00	8.5285E-03	--

4.1.2 Number of elements required

Several elements required in the absorbing region affect the overall size of the model, especially in 3D finite element analysis. In this section, the efficiency of the boundaries is analyzed by changing the number of elements. Similar to the previous section, number of elements in ALID+VABC is fixed to 30 and varied from 5 to 30 elements in case of PML.

Table 2: Percentage of reflections

S.No	Element Length (m)	ALID	PML			
		30 elements	30 elements	15 elements	10 elements	5 elements
1	1.00	2.8	8.77	8.77	8.77	10.25
2	0.50		1.83	1.50	1.26	2.61
3	0.25		1.13	1.06	1.19	2.47

Maximum percentage of reflections are computed in the middle of the AoS region and the results are shown in Table. 2. PML produces three times higher reflections compared to ALID+VABC when the element length is equal to the mesh size (in the AOS). However, PML performs slightly better when the element size is reduced to half or beyond. We noted that the reflections in case of PML when elements are more than 5 is mainly due to a sudden change in the element size between AoS and PML regions. We also noted that reducing the number of elements in the PML region increases the performance which indicates that the optimal performance of PML is sensitive to sudden change in element size and number of elements provided.

Therefore, it is concluded that PML requires fewer elements compared with ALID+VABC and the reduction in the number of elements are much higher. However, it requires a smaller element size compared to that of AoS region for a given loading frequency which reduces the computation time step by more than 50% and therefore increases overall computation time. It is also noted that the performance advantage in terms of absorbing outward propagating energy is negligible.

5. Conclusions

In this study, PML and ALID+VABC method are used as absorbing boundaries and the wave propagation problems are solved. The results are compared and the following conclusions were drawn.

1. PML requires smaller element size for a given loading frequency when compared to ALID+VABC.
2. PML requires smaller computation time step and therefore is computationally expensive.
3. ALID+VABC requires a large number of elements compared to the PML. The number of degrees of freedom is high in 3D wave propagation problem.
4. Both the methods provide very efficient boundary conditions in absorbing boundary conditions.
5. PML equations are complex and are not available in most of the existing commercial FE codes. The implementation cost is high. However, ALID+VABC method is readily available in almost all the FE codes.

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