Non-Orthogonal Multiple Access in Cognitive Relay Networks

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Non-Orthogonal Multiple Access in Cognitive Relay Networks

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Abstract—**In this paper, we present the optimal power allocation and relay precoder design for a cognitive relay network employing non-orthogonal multiple access (NOMA) technique. The secondary user (SU) nodes share the spectrum with the licensed primary user (PU) while keeping the interference at PU nodes below a permissible threshold. The cognitive nodes communicate with each other with the assistance of a cognitive amplify and forward (AF) relay equipped with multiple antennas. The proposed source power allocation and relay precoder design aim at maximizing the sum-rate of the cognitive destination nodes while maintaining the interference to the PU node below the specified threshold. Given that the SU transmit node has to restrict its transmit power in order to reduce the interference to the PU, use of NOMA can significantly improve the SU performance. The resulting optimization problem is non-convex in its original form. In order to circumvent the difficulties associated with the non-convexity, we employ minorization-maximization technique to obtain convex approximations to some of the non-convex functions appearing in the problem. We then solve this problem to obtain the optimal resource allocation policy at source and precoder design at the relay. Numerical results are provided to illustrate the performance of the proposed design for various operating conditions and parameters.**

I. Introduction

Several multiple access techniques have been developed over generations, e.g., time division multiple access (TDMA) in first generation (1G), frequency divisional multiple access (FDMA) in second generation (2G), and code division multiple access (CDMA) in third generation (3G). These techniques are categorized as orthogonal multiple access (OMA) techniques since the resource allocated to each user is orthogonal either in time, frequency and code. However, these techniques do not utilize spectrum efficiently and do not scale well with the predicted exponential increase in the mobile data and connectivity requirements. In this context, non-orthogonal multiple access (NOMA) technique has been projected as a promising candidate to enhance the spectrum efficiency and to effectively tackle the massive connectivity requirements [1]–[3] in the future wireless networks. Recent advancements in the microelectronics and device processing capabilities have made NOMA viable. The works cited above have shown that NOMA is superior over OMA in terms of spectral efficiency, user fairness and Quality of Service (QoS) [1]–[3].

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In NOMA, the transmitter employs superposition coding (SC) to multiplex messages and the users apply successive interference cancellation (SIC) to retrieve the intended messages [1]. The power allocated at the transmitter to the messages intended for different users depend on their channel gains. This power allocation policy helps NOMA to attain the same sum rate as OMA techniques but with lower transmit power [3].

Spectrum scarcity has been a long standing problem in wireless communication networks with tremendous increase in users of mobile data. Cognitive radio (CR) technology handles spectrum access dynamically and helps solving the spectrum scarcity problem. In underlay CR networks, secondary users (SUs) operate under a power constraint in order to limit the interference to the primary user (PU) below permissible level. Cooperative relaying techniques can improve the performance of the SU network in such scenarios [4]. Application of NOMA techniques in CR relay networks have been studied in [5], [6]. Outage probability in a large-scale CR network consisting of single-antenna SU and PU nodes is studied in [5] and diversity order achievable in cooperative CR networks is studied in [6].

In this paper, we consider the use of NOMA in an underlay cognitive AF relay network. Specifically, we study the optimal transmit power allocation and relay precoder design in a CR relay network consisting of an SU transmitter employing NOMA communicating with multiple SU receivers via a multi-antenna relay node. The proposed design is based on the maximization of the sum-rate under PU interference constraint. The requirement to keep the interference to the PU below a certain threshold in underlay CR network severely restricts the transmit power of the SU transmitter. NOMA can improve the sum-rate in this scenario. The problem in its original form turns out to be non-convex. However, we introduce convex approximations and employ minorization-maximization (MM) technique [7] to solve the problem efficiently. We first address the optimal power allocation problem and then optimize relay precoding matrix keeping the interference to primary node below a threshold.

In detailing the contributions highlighted above, the rest of the paper is organized as follows. Section II describes the system model. The design of the proposed optimal source power allocation and relay precoder is discussed in Section III. Simulation results are presented in Sec. IV and the conclusions are presented in Section V.

Notations : We use bold lowercase and uppercase letters for vectors and matrices, respectively. The transpose, conjugate transpose, and trace of a matrix $\mathbf{M}$ are denoted by $\mathbf{M}^T$, $\mathbf{M}^H$, and $tr(\mathbf{M})$, respectively. The $vec(\mathbf{M})$ and $\otimes$ denote the vectorization of matrix $\mathbf{M}$ and the Kronecker product, respectively. Positive definite and positive semi-definite matrices are denoted by notations $\mathbf{M} \succ 0$ and $\mathbf{M} \succeq 0$. The symbols $\mathbb{C}^{N\times N}$, $\mathbb{R}^{N\times N}$ and $\mathbb{R}_+^{N\times N}$ are used for $N \times N$-dimensional complex, real and nonnegative real spaces, respectively. The $\mathbf{l_2}$ norm of a vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is denoted as $\|\mathbf{x}\|$ which is defined as $\|\mathbf{x}\| = \sum_{n=1}^{N} |x_n|^2$ where $|x_n|$ is the absolute value of n^{th} coordinate of vector $\mathbf{x}$. $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$ denotes circularly symmetric complex Gaussian vector with mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$. The minimum, maximum and statistical expectation, argument functions are denoted by min$(\cdot)$, max$(\cdot)$, $\mathbb{E}[\cdot]$ and $\arg_{\{\}}(\cdot)$, respectively.

II. System Model

We consider a CR network comprising a SU tranmitter T, which employs NOMA, communicating with K SU receive/destination nodes through a multi-antenna SU relay node while sharing spectrum with a PU node P. All the nodes except the relay are equipped with a single antenna each. The SU relay node is equipped with N_r antennas. Direct communication links between the T and K destination nodes are ignored as they undergo larger path loss, and the communication happens only via the R. The system operates in half-duplex mode, i.e., the communication takes place in two time slots. In the first time slot, source node encodes data $s_k \in \mathbb{C}^{1\times 1}, \forall\ k \in \mathbb{K} \triangleq \{1, 2, \ldots, K\}$ using superposition coding and tranmsits the resulting signal

$$x_s = \sum_{k=1}^{K} \sqrt{P_k}\, s_k, \tag{1}$$

where s_k (with $\mathbb{E}\{|s_k|^2\} = 1$) is the data symbol meant for the kth destination node and P_k is the transmit power allocated to this symbol. The signal received by the relay node R in the same time slot is given by

$$\mathbf{r} = \mathbf{h}x_s + \mathbf{w}, \tag{2}$$

where $\mathbf{h} \in \mathbb{C}^{N\times 1}$ is the channel gain vector from T to R and $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2\mathbf{I})$ is the additive white Gaussian noise vector at R. In the second time slot, R amplifies the received signal using precoding matrix $\mathbf{F} \in \mathbb{C}^{N\times N}$ and transmits the following signal:

$$\mathbf{x_r} = \mathbf{Fr}. \tag{3}$$

The signal received by the k^{th} receive node D is given by

$$y_k = \mathbf{g}_k^H\mathbf{Fh}x_s + \mathbf{g}_k^H\mathbf{Fw} + n_k, \tag{4}$$

where $\mathbf{g}_k \in \mathbb{C}^{N\times 1}$ is the channel gain vector from R to k^{th} D, and $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the additive white Gaussian noise at k^{th} D. All the destination nodes use SIC for retrieving the message signal. The optimal order of decoding is in the order of increasing channel strengths of the D nodes. Hence, we assume $\|\mathbf{g}_1\| \leq \|\mathbf{g}_2\| \leq \ldots \leq \|\mathbf{g}_K\|$. After SIC, the signal-to-interference-plus-noise ratio (SINR) $\Gamma_{k,j}$ at $\mathrm{D}_j,\ j \in \mathbb{J} \triangleq \{k, k+1, \ldots, K\}$, for decoding s_k is given by

$$\Gamma_{k,j} = \frac{P_k|\mathbf{g}_j^H\mathbf{Fh}|^2}{\left(\sum_{i=k+1}^{K} P_i\right)|\mathbf{g}_j^H\mathbf{Fh}|^2 + \sigma_w^2\mathbf{g}_j^H\mathbf{FF}^H\mathbf{g}_j + \sigma_j^2}. \tag{5}$$

Hence, the corresponding achievable rate can be expressed as

$$R_{k,j} = \frac{1}{2}log_2(1 + \Gamma_{k,j}). \tag{6}$$

For s_k to be decoded successfully at D_j, it is required that $\tilde{R}_k \triangleq min(R_{k,j})\ \forall\ j \in \mathbb{J}$ has to be at least equal to $R_{k,k}$. This is the rate with minimum SINR $\Gamma_{k,j}\ \forall\ j \in \mathbb{J}$. Based on this criterion, we can express the sum-rate $\tilde{R}_{sum}$ as

$$\tilde{R}_{\text{sum}} = \frac{1}{2}\sum_{k=1}^{K} \log_2\big(1 + min\big(\Gamma_{k,j}\big)\big)\ \forall\ j \in \mathbb{J}. \tag{7}$$

Moreover, we also impose the following ordering for boosting the SINR of destination nodes with weak channel gains and for ensuring non-zero rate allocation for such nodes:

$$P_1 \geq P_2 \ldots \geq P_K. \tag{8}$$

The total transmit power of T and R has to satisfy the following constraints:

$$\sum_{k=1}^{K} P_k \leq P_s\ , \tag{9a}$$

$$tr\big(\mathbf{F}\mathbb{E}\big[\mathbf{rr}^H\big]\mathbf{F}\big) \leq P_r, \tag{9b}$$

where P_s and P_r are power budgets at T and R, respectively. Based on (3), we can rewrite the relay power constraint as

$$\|\mathbf{Fh}\|_2^2 P_s + \sigma_w^2 tr\big(\mathbf{FF}^H\big) \leq P_r. \tag{10}$$

In both the time slots, the transmissions from the SU nodes cause interference to the PU node . The interference at P in the first time slot is given by

$$I_{p1} = \left(\sum_{k=1}^{K} P_k\right)|h_p|^2 + \sigma_p^2 \tag{11}$$

and that in the second time slot the interference is given by

$$I_{p2} = \left(\sum_{k=1}^{K} P_k\right)|\mathbf{g}_p^H\mathbf{Fh}|^2 + \sigma_w^2\mathbf{g}_p^H\mathbf{FF}^H\mathbf{g}_p + \sigma_p^2, \tag{12}$$

where $h_p \in \mathbb{C}^{1x1}$ is the channel gain from T to P in and $\mathbf{g}_p \in \mathbb{C}^{N\times 1}$ is the channel gain vector from R to P, and noise in both the slots at P are white Gaussian with variance σ_p^2. The interference resulting from cognitive nodes should be below a certain permissible threshold θ_p to ensure spectrum sharing between PU and SU nodes. Therefore, following conditions should hold:

$$\left(\sum_{k=1}^{K} P_k\right)|h_p|^2 + \sigma_p^2 \leq \theta_p, \tag{13a}$$

$$\left(\sum_{k=1}^{K} P_k\right)|\mathbf{g}_p^H\mathbf{Fh}|^2 + \sigma_w^2\mathbf{g}_p^H\mathbf{FF}^H\mathbf{g}_p + \sigma_p^2 \leq \theta_p. \tag{13b}$$

III. Cognitive Source Power Allocation and Relay Precoder Design

Now we consider the problem of maximizing the achievable sum rate given in (7) by optimal allocation of power at the source T and optimal design of relay precoder under transmit power and PU interference constraints . The resulting optimization problem can be formulated as

$$\max_{\mathbf{F},\{P_k\}} \quad \frac{1}{2}\sum_{k=1}^{K}\log_2\left(1+\min_{j}\left(\Gamma_{k,j}\right)\right), \forall\, j\in\mathbb{J}, \tag{14a}$$

$$\text{s.t.} \quad P_k \le min\left(P_1...P_{k-1}\right), \forall\, k\in\mathbb{K}, \tag{14b}$$

$$\sum_{k=1}^{K}P_k \le P_s, \tag{14c}$$

$$\|\mathbf{Fh}\|_2^2 P_s + \sigma^2 tr\big(\mathbf{FF}^H\big) \le P_r\ , \tag{14d}$$

$$I_{p1}\le\theta_p,\ I_{p2}\le\theta_p. \tag{14e}$$

This problem is difficult to solve due to non-convex nature of the objective function. Hence, the globally optimal solution is hard to obtain. However, we reformulate the problem into a more tractable form using certain approximations as detailed below.

The objective function in problem (14) in its present form is difficult to deal with as it involves SINR terms, which are non-convex. It can be equivalently re-written as

$$\max_{\mathbf{F},\{P_k\},\{r_k\}} \quad \left(\prod_{k=1}^{K} r_k\right)^{\frac{1}{2K}}, \tag{15a}$$

$$s.t. \quad r_k - 1 \le \min\left(\Gamma_{k,j}\right), \tag{15b}$$

$$\text{(14b), (14c), (14d), \&(14e)} \tag{15c}$$

$$\forall\, k\in\ \mathbb{K},\ j\ \in\ \mathbb{J},$$

where $r_k \in \mathbb{R}_+^{1\times 1}\ \forall\, k\in\mathbb{K}$ and objective (15a) is obtained by considering the fact that $\log(\cdot)$ is a non-decreasing function and the geometric mean of r_k for $\forall\ k\in\mathbb{K}$ is concave and increasing. Geometric mean can be expressed as system of second-order conic (SOC) constraints [8]. Hence, the objective function is now convex.

Next, we turn our attention to the new constraint (15b), which is non-convex. We can rewrite the constraint (15b) using (5) as

$$r_k - 1 \le \frac{P_k|\mathbf{g}_j^H\mathbf{Fh}|^2}{\left(\sum_{i=k+1}^{K}P_i\right)|\mathbf{g}_j^H\mathbf{Fh}|^2+\sigma_w^2\mathbf{g}_j^H\mathbf{FF}^H\mathbf{g}_j+\sigma_j^2}, \tag{16}$$

This constraint is a function of three optimization variables, namely, $\mathbf{F},\ \{P_k\},\ \{r_k\}$, and it is not jointly convex in these variables. However, we can obtain convex approximations of this constraint for fixed values of $\mathbf{F}$ or $\{P_k\}$. Hence, we can solve the overall optimization problem by solving two convex subproblems alternately: one for a given value of $\mathbf{F}$ and the other for a given value of P_k. In the following, we detail the development of these two subproblems.

A. *Convex Formulation for Given Relay Precoding Matrix* $\mathbf{F}$

The constraint (16) can be written as

$$r_k - 1 \le \frac{P_k\vartheta_j}{\left(\sum_{i=k+1}^{K}P_i\right)\vartheta_j+\omega_j},\ \forall\ k\in\mathbb{K},\ j\in\mathbb{J}, \tag{17}$$

where $\vartheta_k = |\mathbf{g}_k^H\mathbf{Fh}|^2$ and $\omega_k = \sigma_w^2\mathbf{g}_k^H\mathbf{FF}^H\mathbf{g}_k+\sigma_k^2$. For a given value of $m_k\in\mathbb{R}_+^{1\times1}\ \forall\ k\in\mathbb{K}$, it holds that

$$r_k m_k - m_k \le P_k\vartheta_j, \tag{18a}$$

$$\left(\sum_{i=k+1}^{K}P_i\right)\vartheta_j+\omega_j \le m_k,\ \forall\ k\ \in\mathbb{K},\ j\ \in\mathbb{J}. \tag{18b}$$

The bilinear term in (18a) is not convex since the corresponding Hessian matrix is not positive semidefinite. But, we can develop a convex approximate to it as follows. We begin by observing that

$$r_k m_k = 0.25\left(r_k+m_k\right)^2 - 0.25\left(r_k-m_k\right)^2. \tag{19}$$

As both terms in the right side of equality are convex functions, we can approximate the bilinear term on the left hand side by employing the MM technique [9].

As per MM technique we can maximize a convex function by maximizing its surrogate function. As the linear surrogate functions are computationally less complex, we employ this approach in our problem. Now, the term $\left(r_k-m_k\right)^2$ in (19) can be approximated by its first-order Taylor series $\mathcal{L}_k$ around $r_k^t,\ m_k^t$ as

$$r_k m_k = 0.25\left(r_k+m_k\right)^2 - 0.25\mathcal{L}_k, \tag{20}$$

where, $\mathcal{L}_k = \left[\left(r_k^t-m_k^t\right)^2+2\left(r_k^t-m_k^t\right)\left(r_k-r_k^t-m_k+m_k^t\right)\right]$ Based on this observation, we can reformulate the constraints in (18) as the following convex constraints:

$$0.25\left(r_k+m_k\right)^2 - 0.25\mathcal{L}_k - m_k \le P_k\vartheta_j, \tag{21a}$$

$$\left(\sum_{i=k+1}^{K}P_i\right)\vartheta_j+\omega_j \le m_k,\ \forall\ k\ \in\mathbb{K},\ j\ \in\mathbb{J}. \tag{21b}$$

Moreover, the constraints in (14e) can be written as

$$\left(\sum_{k=1}^{K}P_k\right)|h_p|^2+\sigma_p^2 \le I_p, \tag{22a}$$

$$\left(\sum_{k=1}^{K}P_k\right)\vartheta_p+\omega_p \le I_p. \tag{22b}$$

These constraints are linear in the optimization variables and hence convex.

Next, we consider the convex approximations for the case of fixed values of power allocations $\{P_k\}$.

B. *Convex Formulations for Given Source Power Allocation* $\{P_k\}$

We now look into the convex reformulation of the constraints (16), and (14d) that depend on $\mathbf{F}$. Using the following equality [10]

$$vec\left(\mathbf{M}_1\mathbf{M}_2\mathbf{M}_3\right) = \left(\mathbf{M}_3^T\otimes\mathbf{M}_1\right)vec\left(\mathbf{M}_2\right),$$

where $\mathbf{M}_1, \mathbf{M}_2$ and $\mathbf{M}_3$ are arbitrary matrices, we can rewrite the constraints (16), and (14d) as

$$r_k - 1 \leq \frac{\mathbf{f}^H \mathbf{B}_{k,j} \mathbf{f}}{\mathbf{f}^H \mathbf{C}_{k,j} \mathbf{f} + \sigma_j^2}, \ \forall\, k \in \ \mathbb{K}, \ j \ \in \ \mathbb{J}, \tag{23a}$$

$$\mathbf{f}^H \mathbf{D} \mathbf{f} \leq P_r, \ \mathbf{f}^H \mathbf{Q}_p \mathbf{f} \leq I_p, \tag{23b}$$

where, $\mathbf{f} = vec\left(\mathbf{F}\right),$

$$\mathbf{B}_{k,j} = P_k \left(\mathbf{h}^* \mathbf{h}^T \otimes \mathbf{g}_j \mathbf{g}_j^H\right),$$

$$\mathbf{C}_{k,j} = \left(\mathbf{h}^* \mathbf{h}^T \otimes \mathbf{g}_j \mathbf{g}_j^H\right) \left(\sum_{i=k+1}^{K} P_i\right) + \sigma_w^2 \mathbf{I} \otimes \mathbf{g}_j \mathbf{g}_j^H,$$

$$\mathbf{D} = \left(\mathbf{h}^* \mathbf{h}^T \otimes \mathbf{I}\right) P_s + \sigma_w^2 \mathbf{I},$$

$$\mathbf{Q}_p = \left(\mathbf{h}^* \mathbf{h}^T \otimes \mathbf{g}_p \mathbf{g}_p^H\right) \left(\sum_{i=1}^{K} P_i\right) + \sigma_w^2 \mathbf{I} \otimes \mathbf{g}_p \mathbf{g}_p^H.$$

We can now reformulate the constraints in (23b) as the following second-order conic constraints:

$$\|\tilde{\mathbf{D}}\mathbf{f}\|_2 \leq \sqrt{P_r} \tag{24a}$$

$$\left\| \begin{pmatrix} \tilde{\mathbf{Q}}_p \mathbf{f} \\ \sigma_p \\ (I_p - 1)/2 \end{pmatrix} \right\|_2 \leq \ (I_p + 1)/2 \tag{24b}$$

where, $\mathbf{D} = \tilde{\mathbf{D}}\tilde{\mathbf{D}}^H, \mathbf{Q}_p = \tilde{\mathbf{Q}}_p \tilde{\mathbf{Q}}_p^H, \because \mathbf{D} \succ 0, \mathbf{Q}_p \succeq 0$

Next, we will consider the remaining non-convex constraint (23a). For a given $m_k \in \mathbb{R}_+^{1\times 1}$, it holds that

$$r_k m_k - m_k \leq \mathbf{f}^H \mathbf{B}_{k,j} \mathbf{f}, \tag{25a}$$

$$\mathbf{f}^H \mathbf{C}_{k,j} \mathbf{f} + \sigma_j^2 \leq m_k, \ \forall\ k \ \in \mathbb{K}, \ j \ \in \mathbb{J}. \tag{25b}$$

We approximate the bilinear term similar to (20) as

$$0.25 \left(r_k + m_k\right)^2 - 0.25 \mathcal{L}_k - m_k \leq \mathbf{f}^H \mathbf{B}_{k,j} \mathbf{f}, \tag{26a}$$

$$\mathbf{f}^{\ H} \mathbf{C}_{k,j} \mathbf{f} + \sigma_j^2 \leq m_k. \tag{26b}$$

The term $\mathbf{f}^H \mathbf{B}_{k,j} \mathbf{f}$ in (26a) can be linearized to its surrogate function using Taylor's first-order approximation around the vector $\mathbf{f}^t$. Since this term is a real-valued function of complex vector variable, we make use of Writinger's calculus [11] for calculating the relevant derivatives. Taylor's first-order approximation of the function $f(\mathbf{z})$ around point $\mathbf{z}_0$ is given by

$$f(\mathbf{z}, \mathbf{z}_0) = f(\mathbf{z}_0) + 2\Re\{\left(\frac{\partial f}{\partial \mathbf{z}}\right)^T (\mathbf{z} - \mathbf{z}_0)\}. \tag{27}$$

Using this result and observing that $\mathbf{C}_{k,j}$ admits the decomposition $\mathbf{C}_{k,j} = \tilde{\mathbf{C}}_{k,j} \tilde{\mathbf{C}}_{k,j}^H$ since it is positive semi-definite, we can express the constraints (26) in the following SOC form:

$$0.25 \left(r_k + m_k\right)^2 - 0.25 \mathcal{L}_k - m_k \leq \tilde{\mathcal{L}}_{k,j}, \tag{28a}$$

$$\left\| \begin{pmatrix} \tilde{\mathbf{C}}_{k,j} \mathbf{f} \\ \sigma_j \\ (m_k - 1)/2 \end{pmatrix} \right\|_2 \leq \ (m_k + 1)/2, \tag{28b}$$

$$\text{where,}\ \tilde{\mathcal{L}}_{k,j} = 2\Re\left\{\left(\mathbf{f}^{\ t}\right)^H \mathbf{B}_{k,j} \mathbf{f}\right\} - \left\{\left(\mathbf{f}^{\ t}\right)^H \mathbf{B}_{k,j} \mathbf{f}^{\ t}\right\}. \tag{28c}$$

Based on the convex approximations, we can now reformulate the second subproblem as the following convex optimization problem:

$$\max_{\mathbf{F}, \{P_k\}, \{r_k\}, \{m_k\}} \quad \left(\prod_{k=1}^{K} r_k\right)^{\frac{1}{2K}} \tag{29a}$$

$$\text{s.t. (14b), (14c), (22) (21) for given } \mathbf{F} \tag{29b}$$

$$\text{(24), (28) for given } \{P_k\} \tag{29c}$$

1) Algorithm for the Optimal Design: Putting together the convex reformulation of the two subproblems, we now present the proposed algorithm to solve the original rate maximization problem formulated in (14). The **Algorithm1** solves the original non-convex problem by alternately solving the convex reformulations of the two subproblems developed in the preceding subsections.

Algorithm 1 Algorithm for optimal source allocation and relay precoder design

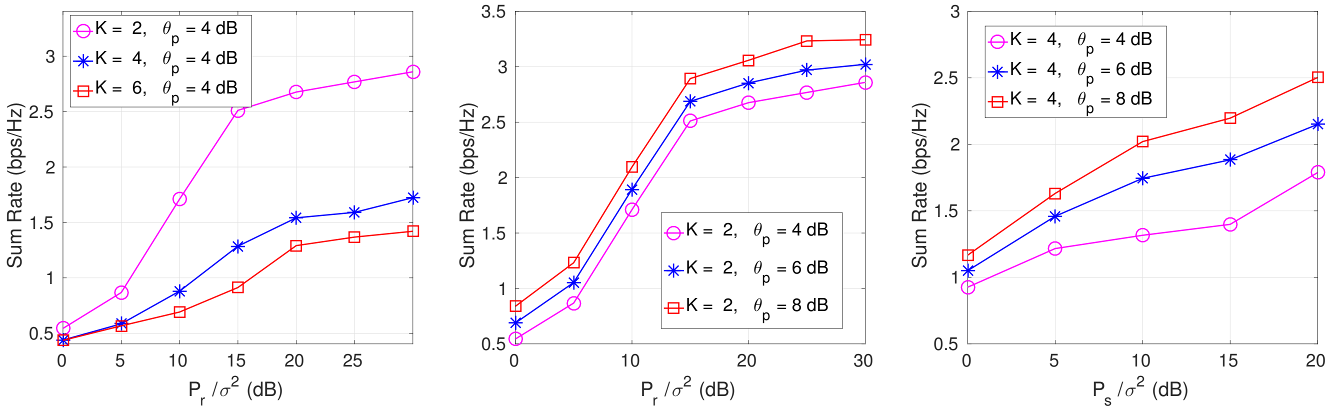
1: **initialize** $\left[\ r_k^0, m_k^0, P_k^0, \mathbf{F}^{\ 0}\ \right]$ from feasibility set of (14), n=0;
2: **repeat**
3: Given $\mathbf{F}$ solve (29) iteratively for $\{r_k\}, \{m_k\}, \{P_k\}$
4: Given P_k solve (29) iteratively for $\{r_k\}, \{m_k\}, \{\mathbf{F}\}$
5: $n = n + 1;$
6: **until** Convergence

Remarks : The $(n+1)^{th}$ iteration in algorithm produces a solution set from feasibility set of problem (14) because the initial feeding set is taken from feasible set. The **Algorithm 1** returns increasing sequence of objective values because the surrogate functions used for approximating non-convex functions are increasing with each iteration. As the feasibility set is convex and compact (feasibility set is bounded by power budgets at source and relay), the algorithm converges to a finite value.

IV. Simulation Results

For all the simulations, we assume that all the channels undergo Rayleigh fading. We consider the gain vector from T to R to be $\mathbf{h} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}\right)$ and Channel gain vectors from R to Ds to be $\mathbf{g}_k \sim \mathcal{CN}\left(\mathbf{0}, \sqrt{d_k^{-\alpha}}\mathbf{I}\right)\ \forall\ k \in \mathbb{K}$ and $\mathbf{g}_p \sim \mathcal{CN}\left(\mathbf{0}, \sqrt{d_p^{-\alpha}}\mathbf{I}\right)$, where d_k, d_p are the the normalized distance from R to the k^{th} D and T/R to the P, respectively. The path loss exponent is denoted by α. Path loss exponent considered is $\alpha = 2$. We consider $d_1 = 100$, $d_K = 1$ and all other Ds are equally spaced between d_K and d_1. This ensures the assumption of channels to be $\|\mathbf{g}_1\|_2 \leq \|\mathbf{g}_2\|_2 \ \ldots \ \leq \|\mathbf{g}_K\|_2$. We consider $d_p = 100$. We assume R has N = 4 antennas. We consider all the noise components w and $n_k\ \forall\ k \in \mathbb{K}$ to be i.i.d and have unit variance. The noise variance at P is considered to be unit variance. Simulation results are averaged over 500 realizations.

Fig. 1(a) shows variation of sum rate with SNR at R P_r/σ^2 with different number of users K = 2, 4 and 6 for permissible



(a) Achievable sum rate vs P_r/σ^2 for given permissible interference θ_p at P.

(b) Achievable sum rate vs P_r/σ^2 for given number of Ds K.

(c) Achievable sum rate vs P_s/σ^2 for given number of Ds K.

Fig. 1: Achievable sum rate under various conditions.

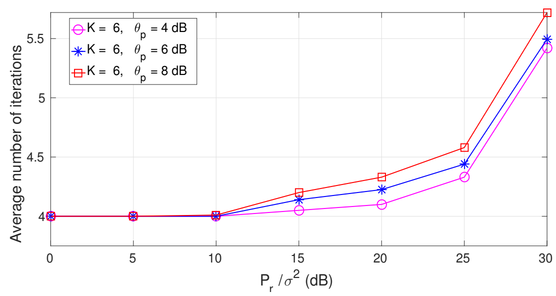


Fig. 2: Average number of iterations for convergence vs P_r/σ^2 for given number of Ds K.

interference $\theta_p = 4$ dB at P. Source SNR P_s/σ^2 is considered to be 20 dB. It can observed that sum rate increases with increase in SNR at R as it is expected. It can be also observed that with increase in number of Ds sum rate is decreasing.

Fig. 1(b) shows variation of sum rate with SNR at R P_r/σ^2 with different interference permissible levels θ_p = 4, 6, and 8 dB for given number of Ds $K = 2$. Source SNR P_s/σ^2 is considered to be 20 dB. It is observed that with increase in permissible interference level sum rate is increasing. Increase in threshold of interference at P allows more power to be allocated to the messages for Ds which in turn increases the sum rate as shown in the simulation.

Fig. 1(c) shows variation of sum rate with SNR at T P_s/σ^2 for different interference permissible levels θ_p = 4, 6, and 8 dB. Here, we consider number of Ds $K = 4$. Similarly,

Fig. 2 shows the average number of iterations required for algorithm convergence. It can be seen that, average number of iterations for convergence is increasing with increase in permissible levels θ_p for given K. It is also increasing with increase in R SNR.

V. Conclusion

We presented the optimal SU source power allocation and relay precoder design in a cognitive AF relay network employing NOMA. The optimal design maximized the sum-rate while maintaining interference to the primary node below a predefined threshold. Even though the original problem turned out to be a non-convex problem, we proposed an algorithm to obtain the optimal design using MM techniques and convex conic approximations.

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