Low-Complexity Two-Way AF Relay Design for Millimeter Wave Communication Systems

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Low-Complexity Two-Way AF Relay Design for Millimeter Wave Communication Systems

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***Abstract*—In this paper, we consider the problem of hybrid multi-input multi-output (MIMO) transceiver design for a non-regenerative hybrid two-way amplify-forward (AF) relay based millimeter wave (mmWave) communication system. We propose two low complexity mmWave system designs based on the quality of available channel state information (CSI). We first propose the design of transceivers and a two-way AF relay, by minimizing sum-mean square error (SMSE) under the constraint of total relay transmit power while assuming the perfect channel knowledge. We later extend it to a robust design under the assumption that the transceivers posses only imperfect information about the channel state, where the CSI error is assumed to follow the Gaussian distribution. In both the designs, reduced hardware complexity (introduced due to large number of antenna elements in mmWave system) is achieved by analog-digital hybrid processing by using orthogonal matching pursuit (OMP)-based sparse signal processing. We present numerical results on the performance of both the designs over various dictionaries. The comparison results show that robust design is resilient to the presence of CSI errors. Furthermore, we also demonstrate the convergence of both the proposed algorithms to a limit even though global convergence is hard to prove due to non convex nature of overall optimization problem.**

I. Introduction

Millimeter wave (mmWave) communication, envisioned as one of the promising technologies for future wireless networks, has the potential to offer massive bandwidth for next generation mobile communication. With the incredible increase in wireless data traffic, the currently available spectrum at microwave frequencies (upto 6 GHz) would be totally exhausted in near future. Hence, the idea of exploiting the under-utilized mmWave spectrum (30-300 GHz) for cellular applications is gaining tremendous interest among researchers [1]. At mmWave frequencies, as large number of antennas can be packed in small volume due to small size of elements, it is possible to achieve high beamforming gain that overcomes significant path loss, making MIMO a key parameter. However, due to high cost and power consumption of RF chains, mmWave systems demand reduced hardware complexity. Hence, unlike traditional MIMO, where each antenna element is associated with a dedicated RF chain, it is important to investigate methods to reduce RF chains in mmWave communication systems. Hybrid MIMO architecture overcomes this limitation by processing the data in two sequential phases, i.e., digital baseband processing followed by analog RF beamforming. Various techniques to implement hybrid beamforming have been proposed [2]. Recently, hybrid design by alternating minimization and least square minimization for mmWave system has been discussed in [3]. mmWaves suffer severe losses such as rain fading, free space path loss and pronounced coverage and blockage holes [4]. The resulting performance degradation can be mitigated to some extent by co-operative relaying techniques. In literature, relays have been widely studied for conventional MIMO systems. Joint precoding/receive filter design in multi-user two-way MIMO relay systems is discussed in [5]. In [6], MMSE based MIMO half-duplex (HD) non-regenerative relay precoder design under channel imperfections has been discussed. Full-duplex (FD) relay based system design with self interference cancellation technique for MIMO system having two users has been discussed in [7] and [8]. Recently, studies on application of AF relay in mmWave system has also been reported in [9].

In this paper, we propose non-robust and robust hybrid two-way AF relay-assisted mmWave communication system design with all units operating in half duplex (HD) mode. We consider two users that can communicate with each other only via relay. We jointly obtain optimal filters by minimizing total SMSE under the constraint on total transmit power at the relay. We later decompose this optimal design to a low complexity analog-digital hybrid design by using OMP-based sparse approximation method. OMP is a well known signal processing method and has been extensively studied in literature [10]. We design this proposed system by considering the availability of perfect CSI.

However, the available CSI is not always perfect due to various factors such as estimation, feedback delays, quantization, etc., which can introduce errors. Hence, system designed assuming the perfect CSI are not likely to achieve desired performance in the presence of CSI errors. Effect of imperfect CSI on the performance of MIMO systems is discussed in [11]. Hence, we extend our design to a robust case, considering imperfect CSI and develop a system that is resilient to such errors. We evaluate the performance of both the proposed designs for various parameters and results are discussed later in this paper. The rest of the paper is organized as follows. Sec. II describes the system model. The proposed low complexity designs are discussed in Sec. III and Sec. IV presents the simulation results. Finally, the conclusion is given in Sec. V.

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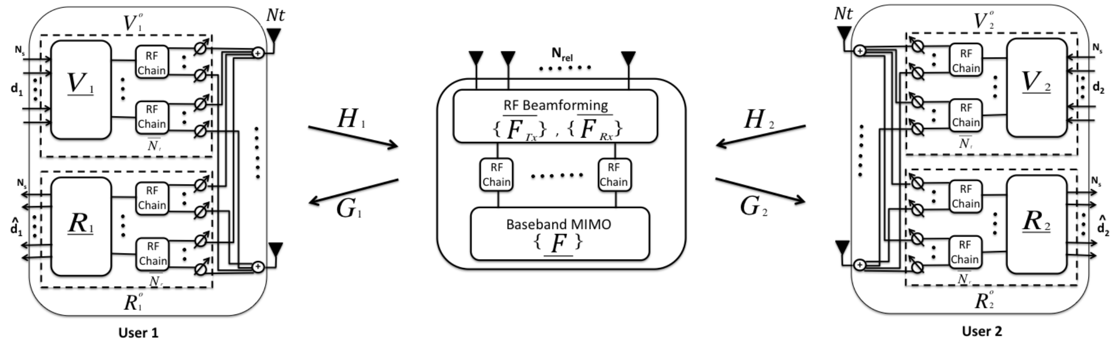

Fig. 1. Hybrid two-way AF relay-assisted system design for mmWave communication.

Notations: Throughout this paper, column vectors and matrices are respectively denoted by bold-faced lowercase and uppercase letters. $\underline{\mathbf{X}}$ and $\overline{\mathbf{X}}$ implies that the variable $\mathbf{X}$ corresponds to the baseband and RF block, respectively. $\mathbf{C}_{k,t}$ denotes the matrix value for $(k)^{th}$ user at timeslot t. $\mathrm{tr}(\cdot)$, $\mathbb{E}\{\cdot\}$, $||\cdot||_F$, $vec(\mathbf{X})$ and $\otimes$ denotes the trace , expectation, frobenius norm, vectorization operation and Kronecker product respectively.

II. System Model

We consider a hybrid mmWave wireless communication system with two users (U1 and U2) and a non-regenerative relay as shown in Fig. 1. Both users communicate with each other only via a two-way relay assuming that there is no direct link between them. All the units are equipped with hybrid MIMO processors and operate in half-duplex mode. In the first time slot, both the sources simultaneously transmit N_s- dimensional column vector $\mathbf{d}_i$, $i \in \{1,2\}$ through its corresponding Nt antennas to the relay. In the next time slot, relay re-transmits the signal after multiplying by a relay gain matrix. $\mathbf{H}_{i,t}$ and $\mathbf{G}_{i,t}$ represent the channel gains from user i to relay (forward channel) and relay to user i (reverse channel) at time t for $i \in \{1,2\}$ respectively. The relay is assumed to have total of N_{rel} number of antennas from which Nr_{Tx} are used as transmit antennas and Nr_{Rx} as the receive antennas. The number of RF chains associated with each user and the relay unit are N_{RF} and Nr_{RF}, respectively, such that $N_s \leq N_{RF} \leq Nt$ and $Nr_{RF} \leq N_{rel}$. Let $\mathbf{V}_i^o,\ \mathbf{R}_i^o\ \ i \in \{1,2\}$, and $\mathbf{F^o}$ be the optimal precoder, receive filter matrix and relay gain matrix in the conventional fully - digital MIMO system at the i^{th} user terminal. Then, we design the hybrid filters to satisfy the following: $\mathbf{V}_i^o = \underline{\mathbf{V}}_i\overline{\mathbf{V}}_i,\ \mathbf{F}^o = \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}$, and $\mathbf{R}_i^o = \underline{\mathbf{R}}_i^H\overline{\mathbf{R}}_i^H$. The signal received by the relay in time slot $(t-1)$ is given by,

$$\mathbf{y}_{r,t-1} = \mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1}\mathbf{x}_{1,t-1} + \mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1}\mathbf{x}_{2,t-1} + \mathbf{n}_r, \quad (1)$$

where, $\mathbf{n}_r$ is additive white Gaussian noise with zero mean and variance $\sigma_n^2\mathbf{I}$ at the receiver, $\mathbf{n}_r \in \mathbb{C}^{Nt \times 1}$. In time slot t, the transmitted signal by the relay can be given by,

$$\mathbf{x}_{r,t} = \mathbf{F}_t\mathbf{y}_{r,t-1}. \quad (2)$$

Each user cancel out its own transmitted signal, which acts as self interference, from the overall received signal. Thus the resultant received signal for any user-k can be given by,

$$\mathbf{y}_{k,t} = \mathbf{G}_{k,t}\mathbf{F}_t\mathbf{H}_{\overline{k},t-1}\mathbf{V}_{\overline{k},t-1}\mathbf{x}_{\overline{k},t} + \mathbf{G}_{k,t}\mathbf{F}_t\mathbf{n}_r + \mathbf{n}_k, \quad (3)$$

where, $\overline{k}$ denotes other user i.e. when $k=1$, $\overline{k}=2$ and *vice-versa*. We first design the non-robust system assuming that the transceiver possesses perfect CSI. We later extend it to robust design by assuming channel imperfections. Specifically, the CSI in this case can be modeled as,

$$\mathbf{C} = \widehat{\mathbf{C}} + \boldsymbol{\Delta}, \quad (4)$$

where $\widehat{\mathbf{C}}$ is the estimated CSI knowledge, and $\boldsymbol{\Delta} \sim \mathcal{N}(0, \sigma_E^2\mathbf{I})$ denotes the corresponding error in the CSI [12].

III. Low-Complexity AF Two-Way Relay Design

In this section, we present the proposed non-robust and robust mmWave system designs with 2 users in a non-line of sight path, communicating only via relay. For each of them, we first design the fully-digital optimal filters by minimizing SMSE, constrained on total relay transmit power. Subsequently, we obtain the respective hybrid matrices with lower complexities using OMP-based sparse approximation technique.

A. Conventional Joint Transceivers and Relay Filter Design

We design the optimal system by minimizing the overall SMSE (ε) under the constraint on total relay transmit power. Thus, the optimization problem can be expressed as:

$$\min_{\mathbf{F},\mathbf{V}_i,\mathbf{R}_i} \varepsilon \qquad \text{subject to (s.t.): } ||\mathbf{x}_{r,t}||_F^2 \leq (Nr_{Tx}P_r), \quad (5)$$

where, P_r is the transmit power at each antenna at relay. We consider an extra variable α for derivational simplicity. For proposed system, SMSE at timeslot t can be formulated as:

$$\varepsilon_t = \mathbb{E}\big[||\mathbf{x}_{2,t-1} - \alpha^{-1}\mathbf{R}_{1,t}\mathbf{y}_{1,t}||^2\big] + \mathbb{E}\big[||\mathbf{x}_{1,t-1} - \alpha^{-1}\mathbf{R}_{2,t}\mathbf{y}_{2,t}||^2\big], \quad (6)$$

$$\begin{aligned} = \mathrm{tr}\Big(&\mathbf{I} - \alpha^{-1}\mathbf{R}_{1,t}\mathbf{G}_{1,t}\mathbf{F}_t\mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1} \\ &-\alpha^{-1}\mathbf{V}_{2,t-1}^H\mathbf{H}_{2,t-1}^H\mathbf{F}_t^H\mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H \\ &+\alpha^{-2}\mathbf{R}_{1,t}\mathbf{G}_{1,t}\mathbf{F}_t\mathbf{H}_{2,t-1}\mathbf{V}_{2,t-1}\mathbf{V}_{2,t-1}^H\mathbf{H}_{2,t-1}^H\mathbf{F}_t^H\mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H \\ &+\alpha^{-2}\sigma_r^2\mathbf{R}_{1,t}\mathbf{G}_{1,t}\mathbf{F}_t\mathbf{F}_t^H\mathbf{G}_{1,t}^H\mathbf{R}_{1,t}^H + \alpha^{-2}\sigma_1^2\mathbf{R}_{1,t}\mathbf{R}_{1,t}^H \\ &+\mathbf{I} - \alpha^{-1}\mathbf{R}_{2,t}\mathbf{G}_{2,t}\mathbf{F}_t\mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1} \\ &-\alpha^{-1}\mathbf{V}_{1,t-1}^H\mathbf{H}_{1,t-1}^H\mathbf{F}_t^H\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H \\ &+\alpha^{-2}\mathbf{R}_{2,t}\mathbf{G}_{2,t}\mathbf{F}_t\mathbf{H}_{1,t-1}\mathbf{V}_{1,t-1}\mathbf{V}_{1,t-1}^H\mathbf{H}_{1,t-1}^H\mathbf{F}_t^H\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H \\ &+\alpha^{-2}\sigma_r^2\mathbf{R}_{2,t}\mathbf{G}_{2,t}\mathbf{F}_t\mathbf{F}_t^H\mathbf{G}_{2,t}^H\mathbf{R}_{2,t}^H + \alpha^{-2}\sigma_2^2\mathbf{R}_{2,t}\mathbf{R}_{2,t}^H\Big). \end{aligned} \quad (7)$$

Optimization problem given in (5) after substituting (7), can be solved by using Karush-Kuhn-Tucker conditions. The Langrangian function $J(\alpha, \mathbf{V}_{1,t-1}, \mathbf{V}_{2,t-1}, \mathbf{R}_{1,t}, \mathbf{R}_{2,t}, \mathbf{F}_t)$ associated with the above optimization problem is given by,

TABLE I

Iterative algorithm computing $\mathbf{F}^0,\ \mathbf{V}^o,\ and\ \mathbf{R}^o$ for MSE optimization
1. Initialize $\mathbf{F}_t, \mathbf{V}_{k,t-1}$ and $\mathbf{R}_{k,t}$ $\quad \forall k \in \{1,2\}$
2. Do,
3. Update $\tilde{\mathbf{F}}_t$ by using $\mathbf{R}_{k,t}, \mathbf{V}_{k,t-1}$ $\quad \forall k \in \{1,2\}$
4. Update α_t by using $\tilde{\mathbf{F}}_t$
5. Update $\mathbf{F}_t$ by using $\tilde{\mathbf{F}}_t$ and α_t
6. Update $\mathbf{R}_{k,t}$ by using $\mathbf{F}_t$ and $\mathbf{V}_{k,t-1}$ $\quad \forall k \in \{1,2\}$
7. Update $\mathbf{V}_{k,t-1}$ by using $\mathbf{F}_t$ and $\mathbf{R}_{k,t}$ $\quad \forall k \in \{1,2\}$
5. Repeat until convergence.

$$J = \epsilon_t + \lambda\big(\alpha^2 \mathrm{tr}\big(\sum_{k=1}^{2} \tilde{\mathbf{F}}_t \mathbf{H}_{k,t-1} \mathbf{V}_{k,t-1} \mathbf{V}_{k,t-1}^H \mathbf{H}_{k,t-1}^H \tilde{\mathbf{F}}_t^H + \sigma_r^2 \tilde{\mathbf{F}}_t \tilde{\mathbf{F}}_t^H\big) + Nr_{Tx}P_r\big), \quad (8)$$

where, relay beamforming matrix is split as $\mathbf{F} = \alpha\tilde{\mathbf{F}}$, for simplicity. Formulated problem is not convex in optimization variables, thus global solution cannot be achieved. An iterative algorithm given in Table I is used to jointly obtain the sub-optimal solution for the proposed problem. Consider the minimization w.r.t. receive filter $\mathbf{R}_i$ while keeping other variables fixed, so for user 1, differentiating the lagrangian w.r.t. $\mathbf{R}_1^H$, and equating it to zero, we get,

$$\mathbf{R}_{1,t} = \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{1,t}^H \big(\mathbf{G}_{1,t}\tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{1,t}^H + \sigma_r^2 \mathbf{G}_{1,t}\tilde{\mathbf{F}}_t \tilde{\mathbf{F}}_t^H . \mathbf{G}_{1,t}^H + \alpha^2\sigma_r^2 \mathbf{I}\big)^{-1}. \quad (9)$$

Similarly, for user 2,

$$\mathbf{R}_{2,t} = \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H (\mathbf{G}_{2,t}\tilde{\mathbf{F}}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H + \sigma_r^2 \mathbf{G}_{2,t}\tilde{\mathbf{F}}_t \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H + \alpha^2\sigma_r^2 \mathbf{I})^{-1}. (10)$$

Next, minimizing w.r.t. precoder matrix. Thus, for user 1,

$$\mathbf{V}_{1,t-1} = (\mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H \mathbf{R}_{2,t-1} \mathbf{G}_{2,t} \tilde{\mathbf{F}}_t \mathbf{H}_{1,t-1} + \lambda\alpha^2 \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \tilde{\mathbf{F}}_t \mathbf{H}_{1,t-1})^{-1} \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H. \quad (11)$$

Similarly, for user 2,

$$\mathbf{V}_{2,t-1} = (\mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H \mathbf{R}_{1,t} \mathbf{G}_{1,t} \tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1} + \lambda\alpha^2 \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1})^{-1} \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H. \quad (12)$$

Also, differentiating the lagrangian w.r.t. α and λ, we obtain their values as follows:

$$\frac{\partial J}{\partial \alpha} = 0,$$

olving, it can be written as,

$$\alpha^4 \lambda \mathrm{A}_3 = \sigma^2 A_1 \Rightarrow \lambda\alpha^4 = \frac{A_1}{A_3}, \quad (13)$$

where, $A_1 = \sigma^2 \mathrm{tr}\left(\mathbf{R}_{1,t}\mathbf{R}_{1,t}^H + \mathbf{R}_{2,t}\mathbf{R}_{2,t}^H\right), A_3 = \mathrm{tr}[\tilde{\mathbf{F}}_t \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H \tilde{\mathbf{F}}_t^H + \tilde{\mathbf{F}}_t \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H \tilde{\mathbf{F}}_t^H + \sigma_r^2 \tilde{\mathbf{F}}_t \tilde{\mathbf{F}}_t^H]$. Differentiating the lagrangian w.r.t. λ we get,

$$\frac{\partial J}{\partial \lambda} = 0 \Rightarrow \alpha^2 = \frac{\mathrm{Nr_{Tx}\ P_r}}{A_3}.$$

From (13), we have,

$$\lambda\alpha^2 = \frac{A_1}{\mathrm{Nr_T x P_r}}. \quad (14)$$

Solving for relay filter,

$$\frac{\partial J}{\partial \tilde{\mathbf{F}}_t^H} = -\mathbf{C} + \mathbf{D1}\tilde{\mathbf{F}}_t\mathbf{D2} + \mathbf{Q1}\tilde{\mathbf{F}}_t + \mathbf{S1}\tilde{\mathbf{F}}_t\mathbf{S2} + \mathbf{P1}\tilde{\mathbf{F}}_t + \mathbf{X1}\tilde{\mathbf{F}}\mathbf{X2} + \mathbf{L1}\tilde{\mathbf{F}}_t\mathbf{L2} + \mathbf{M1}\tilde{\mathbf{F}}_t = 0, \quad (15)$$

where, $\mathbf{C} = \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H \mathbf{V}_{2,t-1} \mathbf{H}_{2,t-1}^H + \mathbf{G}_{2,t}^H \mathbf{R}_{1,t}^H \mathbf{V}_{1,t-1}^H \mathbf{H}_{1,t-1}^H$, $\mathbf{D}1 = \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H \mathbf{R}_{1,t} \mathbf{G}_{1,t}$, $\mathbf{D}2 = \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H$, $\mathbf{Q}1 = \sigma_r^2 \mathbf{G}_{1,t}^H \mathbf{R}_{1,t}^H \mathbf{R}_{1,t} \mathbf{G}_{1,t}$, $\mathbf{S}1 = \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H \mathbf{R}_{2,t}$, $\mathbf{S}2 = \mathbf{H}_{1,t-1} \mathbf{V}_{1,t-1} \mathbf{V}_{1,t-1}^H \mathbf{H}_{t,t-1}^H$, $\mathbf{P}1 = \sigma_r^2 \mathbf{G}_{2,t}^H \mathbf{R}_{2,t}^H \mathbf{R}_{2,t} \mathbf{G}_{2,t}$, $\mathbf{X}1 = \lambda\sigma_r^2 \mathbf{I}_{Nr_{Rx}}$, $\mathbf{X}2 = \mathbf{H}_{1,t-1} \mathbf{V}_{1,t} \mathbf{V}_{1,t}^H \mathbf{H}_{1,t-2}^H$, $\mathbf{L}1 = \lambda\sigma_r^2 \mathbf{I}_{Nr_{Rx}}$, $\mathbf{L}2 = \mathbf{H}_{2,t-1} \mathbf{V}_{2,t-1} \mathbf{V}_{2,t-1}^H \mathbf{H}_{2,t-1}^H$, and $\mathbf{M}1 = \lambda\alpha^2\sigma_r^2 \mathbf{I}_{Nr_{Rx}}$. Performing the vectorization operation on (15) we get,

$$vec(\mathbf{D1}\tilde{\mathbf{F}}_t\mathbf{D2} + \mathbf{Q1}\tilde{\mathbf{F}}_t + \mathbf{S1}\tilde{\mathbf{F}}_t\mathbf{S2} + \mathbf{P1}\tilde{\mathbf{F}}_t + \mathbf{X1}\tilde{\mathbf{F}}\mathbf{X2} + \mathbf{L1}\tilde{\mathbf{F}}_t\mathbf{L2} + \mathbf{M1}\tilde{\mathbf{F}}_t) = vec(\mathbf{C}). \quad (16)$$

For any three matrices, $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ of suitable dimensions, $\mathrm{vec}(\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3) = \mathbf{A_3}^T \otimes \mathbf{A}_1 \mathrm{vec}(\mathbf{A}_2)$. Applying this identity in (18), we get

$$\widehat{\mathbf{f}}_t = \mathbf{Z}^{-1}\mathbf{f}, \quad (17)$$

where $\widehat{\mathbf{f}}_t = vec(\tilde{\mathbf{F}}_t)$ and $\mathbf{f} = \mathbf{vec}(\mathbf{C})$ and $\mathbf{Z} = \mathbf{D}2^T \otimes \mathbf{D}1 + \mathbf{I}_{Nt_{Tx}}^T \otimes \mathbf{Q}1 + \mathbf{S}2^T \otimes \mathbf{S}1 + \mathbf{I}_{Nt_{Tx}}^T \otimes \mathbf{P}1 + \mathbf{X}2^T \otimes \mathbf{X}1 + \mathbf{L}2^T \otimes \mathbf{L}1 + \mathbf{I}_{Nr_{Tx}}^T \otimes \mathbf{M}1$. We can obtain $\tilde{\mathbf{F}}_t$ from $\widehat{\mathbf{f}}_t$ in (17) and having done that we can compute optimal relay filter as,

$$\mathbf{F}_t = \alpha\tilde{\mathbf{F}}_t. \quad (18)$$

TABLE II

OMP-based iterative algorithm for hybrid precoder and receive filter design
Require $\mathbf{P}_k^o, \mathbf{\Phi}, \mathbf{S}_{BF}$
1: $\overline{\mathbf{Q}}_k = [\ ]$
2: $\mathbf{A}_0 = \mathbf{P}_k^o$
3: for $i = 1$ to N_{RF} do
4: $\quad \mathbf{\Psi}_{i-1} = (\mathbf{\Phi}\mathbf{S}_{BF})^H(\mathbf{\Phi}\mathbf{A}_{i-1})$
5: $\quad l = arg\ max_{m=1\ldots M}(\mathbf{\Psi}_{i-1}\mathbf{\Psi}_{i-1}^H)_{m,m}$
6: $\quad \underline{\mathbf{Q}}_k = [\overline{\mathbf{Q}}_k \| \mathbf{\Phi}(:,k)]$
7: $\quad \underline{\mathbf{Q}}_k = (\overline{\mathbf{Q}}_k^H \overline{\mathbf{Q}}_k)^{-1} \overline{\mathbf{Q}}_k^H \mathbf{P}_k^o$
8: $\quad \mathbf{A}_i = \frac{\mathbf{P}_k^o - \overline{\mathbf{Q}}_k \underline{\mathbf{Q}}_k}{\|\mathbf{P}_k^o - \overline{\mathbf{Q}}_k \underline{\mathbf{Q}}_k\|_F}$
9: end for
10: $\underline{\mathbf{Q}} = \sqrt{N_s} \frac{\mathbf{Q}}{\|\overline{\mathbf{Q}}\underline{\mathbf{Q}}H\|_F}$
11: return $\overline{\mathbf{Q}}, \underline{\mathbf{Q}}$
Precoder: $\overline{N} = \overline{N}_t, \mathbf{P}^o = \mathbf{V}^o, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k^r}^{\frac{1}{2}}, \overline{\mathbf{Q}} = \overline{\mathbf{V}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{V}}$
Receive filter: $\overline{N} = \overline{N}_r, \mathbf{P}^o = \mathbf{R}^{oH}, \mathbf{\Phi} = \mathbf{\Gamma}_{\mathbf{y}_k}^{\frac{1}{2}}, \overline{\mathbf{Q}} = \overline{\mathbf{R}}$, and $\underline{\mathbf{Q}} = \underline{\mathbf{R}}$

B. OMP-Based Low Complexity Hybrid Solution

In this section, we design hybrid matrices for proposed communication model. We decompose the previously obtained optimal matrices into their corresponding hybrid processors. To achieve this, we adopt the orthogonal matching pursuit (OMP)-based sparse approximation technique. We first formulate the OMP sparse signal processing problem for all these filters, solving which results in the corresponding hybrid matrices. The design of the hybrid system is discussed in subsequent subsections.

1) Hybrid OMP-Based Precoding/Receive Filter design: Given the optimal filter in (9), (10), and (11), (12). We obtain the proposed hybrid processors by decomposing it to RF/baseband filters, using OMP sparse approximation technique. OMP algorithm has been extensively studied in literature and used for various signal processing applications [2], [10]. OMP is a greedy approach which iteratively computes the RF and baseband matrices. It selects the RF beamforming vectors from dictionary that is most strongly correlated to the

TABLE III

OMP based Hybrid Relay for MIMO

Require $\mathbf{F}^o, \mathbf{S}_{Rx-BF}, \mathbf{S}_{Tx-BF}$,

Initialize: $\mathbf{F}_{Rx-RF} = \mathbf{F}_{Tx-RF} = [\ ]$, $\mathbf{F}_{res} = \mathbf{F}^o$

for $i = 1$ to Nr_{RF} do

$\quad \boldsymbol{\Psi}^{i-1}_{Rx} = \mathbf{F}_{res}\mathbf{S}^H_{Rx-BF}$

$\quad q = arg\ max_{m=1 \dots M}(\boldsymbol{\Psi}^{i-1}_{Tx}(\boldsymbol{\Psi}^{i-1}_{Tx})^H)_{m,m}$

$\quad \overline{\mathbf{F}}_{Rx} = [\overline{\mathbf{F}}^T_{Rx}|(\mathbf{S}^T_{Rx-BF})(:,k)]^T$

$\quad \boldsymbol{\Psi}^{i-1}_{Tx} = \mathbf{S}^H_{Tx-BF}\mathbf{F}_{res}\overline{\mathbf{F}}^H_{Rx}$

$\quad l = arg\ max_{m=1 \dots M}(\boldsymbol{\Psi}^{i-1}_{Rx}(\boldsymbol{\Psi}^{i-1}_{Rx})^H)_{m,m}$

$\quad \overline{\mathbf{F}}_{Tx} = [\mathbf{F}_{Tx-RF}|\mathbf{S}_{Tx-BF}(:,k)]$

$\quad \underline{\mathbf{F}} = (\overline{\mathbf{F}}^H_{Tx}\overline{\mathbf{F}}_{Tx})^{-1}\overline{\mathbf{F}}^H_{Tx}\mathbf{F}^o\ \overline{\mathbf{F}}^H_{Rx}(\overline{\mathbf{F}}_{Rx}\overline{\mathbf{F}}^H_{Rx})$

$\quad \mathbf{F}_{res} = \frac{\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}}{||\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}||_F}$

end for

return $\overline{\mathbf{F}}_{Tx}, \overline{\mathbf{F}}_{Rx}$, and $\underline{\mathbf{F}}$

residue computed at each iteration. On the other hand, the baseband matrices are computed by solving the least square problem. In general for any user k, considering the respective optimal MMSE receive filter solution given in (9) or (10), the optimal MMSE solution can be rewritten as,

$$\mathbf{R}^o_k = \boldsymbol{\Gamma}_{\mathbf{y}_k}\boldsymbol{\Gamma}^{-1}_{\mathbf{y}_k\mathbf{s}_k}, \tag{19}$$

where $\boldsymbol{\Gamma}_{\mathbf{y}_k} = \mathbb{E}[\mathbf{y}_k\mathbf{y}^H_k]$ and $\boldsymbol{\Gamma}_{\mathbf{y}_k\mathbf{d}_k} = \mathbb{E}[\mathbf{y}_k\mathbf{d}^H_k]$, and the baseband receive filter matrix can be written as,

$$\underline{\mathbf{R}}^o_k = \boldsymbol{\Gamma}^{-1}_{\mathbf{z}_k}\boldsymbol{\Gamma}_{\mathbf{z}_k\mathbf{d}_k} = \left(\overline{\mathbf{R}}^H_k\boldsymbol{\Gamma}_{\mathbf{y}_k}\overline{\mathbf{R}}_k\right)^{-1}\overline{\mathbf{R}}^H_k\boldsymbol{\Gamma}_{\mathbf{y}_k\mathbf{d}_k}, \tag{20}$$

where, $\boldsymbol{\Gamma}_{\mathbf{z}_k} = \mathbb{E}[\mathbf{z}_k\mathbf{y}^H_k]$ and $\boldsymbol{\Gamma}_{\mathbf{z}_k\mathbf{d}_k} = \mathbb{E}[\mathbf{z}_k\mathbf{d}^H_k]$. The OMP sparse problem for receiver can be formulated as,

$$\overline{\mathbf{R}}^o_k = \underset{\overline{\mathbf{R}}_k}{\operatorname{argmin}}\,\mathbb{E}\|\mathbf{d}_k - \underline{\mathbf{R}}^{o\,H}_k\overline{\mathbf{R}}_k\|^2, \tag{21}$$

which can be rewritten as,

$$\overline{\mathbf{R}}^o_k = \underset{\overline{\mathbf{R}}^o_k}{\operatorname{argmin}}\,\|\boldsymbol{\Gamma}^{\frac{1}{2}}_{\mathbf{y}_k}\mathbf{R}^o_k - \boldsymbol{\Gamma}^{\frac{1}{2}}_{\mathbf{y}_k}\overline{\mathbf{R}}_k\underline{\mathbf{R}}^o_k\|^2_F. \tag{22}$$

Introducing dictionary $\mathbf{S}_{BF}$, the optimization problem can be rephrased as,

$$\tilde{\underline{\mathbf{R}}}^o_k = \underset{\tilde{\underline{\mathbf{R}}}_k}{\operatorname{argmin}}\,\|\boldsymbol{\Gamma}^{\frac{1}{2}}_{\mathbf{y}_k}\mathbf{R}^o_k - \boldsymbol{\Gamma}^{\frac{1}{2}}_{\mathbf{y}_k}\mathbf{S}_{BF}\tilde{\underline{\mathbf{R}}}^o_k\|^2_F, \tag{23}$$

$$s.t.\ \|\text{diag }(\tilde{\underline{\mathbf{R}}}^o_k\tilde{\underline{\mathbf{R}}}^{o\,H}_k)\|_0 = \overline{N}_r.$$

Similarly, the optimization problem for designing sparse precoder matrix can be written as,

$$\overline{\mathbf{V}}^o_k = \underset{\tilde{\mathbf{V}}_k}{\operatorname{argmin}}\,\|\boldsymbol{\Gamma}^{\frac{1}{2}}_{\mathbf{y}^r_k}\mathbf{V}^o_k - \boldsymbol{\Gamma}^{\frac{1}{2}}_{\mathbf{y}^r_k}\mathbf{S}_{BF}\tilde{\mathbf{V}}^o_k\|^2_F, \tag{24}$$

$$\text{s.t.}\quad \|\text{diag }(\tilde{\underline{\mathbf{V}}}^o_k\tilde{\underline{\mathbf{V}}}^{o\,H}_k)\|_0 = \overline{N}_t \text{ and } \|\tilde{\underline{\mathbf{V}}}_k\|^2_F = \|\mathbf{V}^o_k\|^2_F.$$

We obtain hybrid $\underline{\mathbf{V}}_k, \overline{\mathbf{V}}_k$ and $\underline{\mathbf{R}}_k, \overline{\mathbf{R}}_k$ matrices using OMP-based iterative algorithm as shown in Table-II.

2) Hybrid OMP based Relay Filter design: The optimal relay gain matrix in (18) is decomposed into hybrid matrices such that, $||\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}||_F$ is minimized. Following the approach in [9], OMP sparse problem can be formulated as,

$$(\overline{\mathbf{F}}^o_{Tx}, \underline{\mathbf{F}}^o, \overline{\mathbf{F}}^o_{Rx}) = \arg\min_{\overline{\mathbf{F}}_{Tx},\underline{\mathbf{F}},\overline{\mathbf{F}}_{Rx}} ||\mathbf{F}^o - \overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}||_F,$$

$$\text{s.t.:}\quad \overline{\mathbf{F}}^{(j)}_{Tx} \in \mathbf{S}_{Tx-BF}, (\overline{\mathbf{F}}_{Rx})^{(j)} \in \mathbf{S}_{Rx-BF}, and$$

$$||\overline{\mathbf{F}}_{Tx}\underline{\mathbf{F}}\overline{\mathbf{F}}_{Rx}||^2_F = P_r. \tag{25}$$

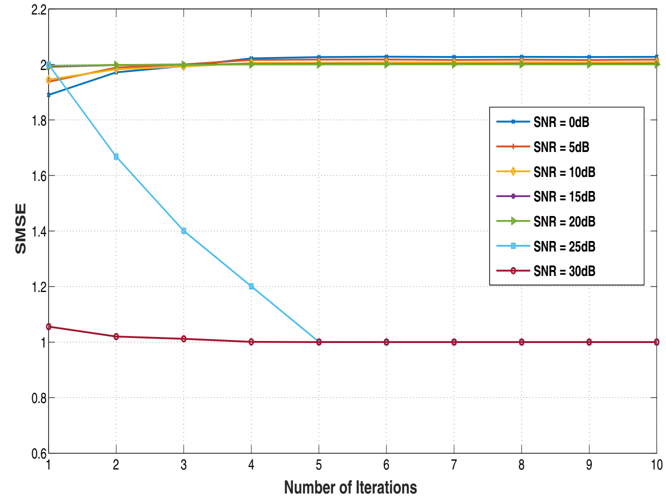

Fig. 2. MSE convergence plot for the proposed algorithm.

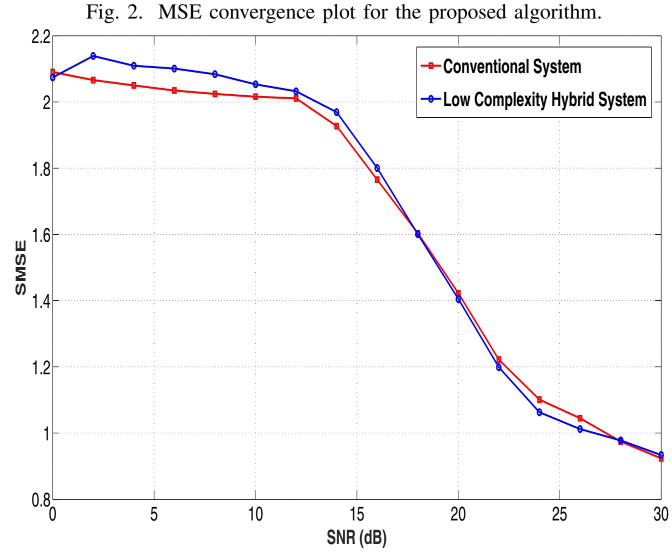

Fig. 3. SMSE for conventional (solid line) and Hybrid (dashed line) system.

To solve this optimization problem, the RF beamforming matrices $\overline{\mathbf{F}}_{Tx}$ and $\overline{\mathbf{F}}_{Rx}$ are jointly selected from the set of candidate vectors $\mathbf{S}_{Tx-BF}$ and $\mathbf{S}_{Rx-BF}$. When both the vectors are selected, the effective residue is updated at each step, and corresponding baseband filter is obtained by least square solution. Hence, the overall error is minimized with each iteration. The OMP algorithm for obtaining hybrid relay processor is given in Table III.

C. Robust Hybrid Two-way AF Relay based mmWave System

In this section, we extend the above proposed design to a robust system by considering imperfections in CSI. We proceed with the similar approach as discussed in previous subsections III-A and III-B by incorporating the imperfections in the channel model, with known error variance, as given in (4). Further, following the similar approach, we jointly design optimal filters by considering expected values of the optimization objective and constraints. We further decompose and obtain the hybrid filters by using the OMP algorithm given in Table II and III. The comparison results for both the proposed designs is given in simulation results.

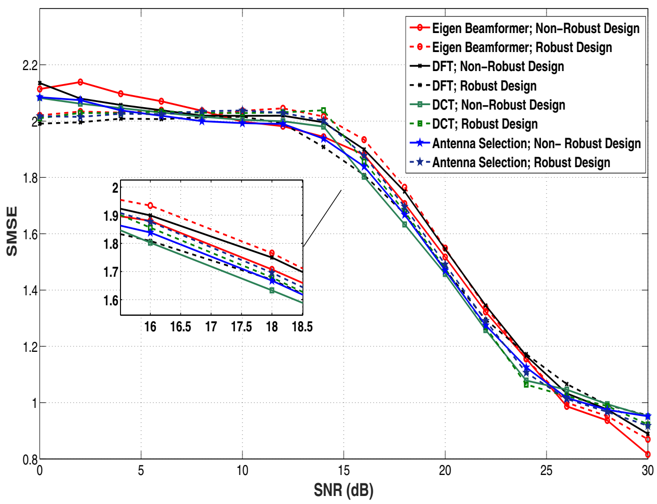

Fig. 4. SMSE for Non-robust (solid line) and robust (dashed line) hybrid system for multiple dictionaries.

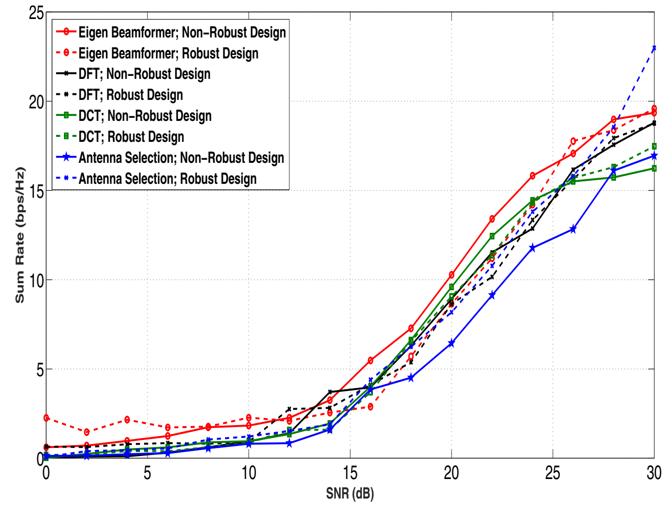

Fig. 5. Sum-rate for non-robust (solid line) and robust (dashed line) hybrid system for multiple dictionaries.

IV. Simulation Results

This section presents the simulation results for proposed non-regenerative HD two-way relay assisted mmWave communication systems. We compare the performance of both the proposed non-robust and robust design and with conventional two-way MIMO relay from the literature. Simulations are performed for $Nt = 6$, $Nr_{Tx} = Nr_{Rx} = 16$, and with number of RF chains $N_{RF} = 3$ for the users and $Nr_{RF} = 14$ at relay. The channel assumed is quasi-static flat fading Rayleigh channel with stochastic errors in CSI, modeled as Gaussian random vectors. We observe that the proposed algorithm converges to an optimal point as shown in Fig. 2 even though global convergence is hard to prove due to the non convex nature of optimization problem The comparison results for SMSE using eigen beamformer is plotted against varying SNR for proposed hybrid and conventional system as shown in Fig. 3. It is observed that proposed hybrid design achieves reduced hardware complexity without compromising on the performance. SMSE and sum rate plots are shown in Fig. 4 and 5 respectively considering various dictionaries, that includes, eigen beamforming, discrete fourier transform (DFT), discrete cosine transform (DCT) and antenna selection. It has been observed that SMSE decreases with increasing SNR, however there is no significant difference in the error for different dictionaries. While in Fig. 5, higher sum-rate is obtained with increasing SNR. Even though eigen beamformer performs best among all, any of the dictionaries can be selected, where selection may vary according to the application and implementation simplicity.

V. Conclusion

This paper investigated two-way AF relay-assisted mmWave communication systems. We proposed joint optimal filter design by minimizing SMSE constrained on total relay transmit power. The obtained optimal filters were further decomposed into analog-digital hybrid processors by OMP-based sparse signal processing technique and achieved low-complexity system. The design was developed by assuming perfect CSI knowledge and was also extended to a robust design by assuming erroneous CSI. We showed that the proposed algorithm converges to a limit. The performance of both the designs was evaluated on various parameters and was demonstrated with simulation results. Results show that the robust design performs better as it was resilient to the erroneous CSI.

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