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Numerical Modelling of Radiating Boundary Conditions Combined with Modified Absorbing Boundary Condition for Viscoelastic Wave Propagation

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1. Introduction

Modelling the Sommerfeld radiating boundary is a major challenge appearing in the study of many engineering problems involving elastic wave propagation [1]. Particularly, in the finite element analysis of Structure-Soil-Structure-Interaction (SSSI) problems, where the soil modelling has to be truncated at a finite distance. This truncation of the model at finite boundary leads to reflection of radiating elastic waves. The reflected waves from the boundary will affect the solution and may lead to instabilities in the numerical analysis. Therefore, it is necessary to provide an artificial boundary condition that will transmit the outward propagating waves with minimum or negligible reflections. To address the radiating boundary condition problem, researchers have developed various kinds of formulations over few decades, such as: a). Local Absorbing Boundary Conditions (ABC) [2], b). Absorbing Layers techniques which includes Perfectly Matched Layers [3-4], Caughey Absorbing Layer Method (CALM) [5], Absorbing Layers by Increasing Damping (ALID) [6] c). Boundary element method [7], and d). Infinite elements [8-10].

ABC corresponds to the situation where the boundary is supported on infinitesimal dash-pots oriented normal and tangential to the boundary. Though these boundary conditions are very simple, the major drawback is they are inefficient when the wave impinges the boundary in inclined direction. The key property of a PML that distinguishes it from an ordinary absorbing material is that it is designed such that the propagating waves incident upon the PML from a bulk medium do not reflect on the interface. The CALM consists of defining the absorbing layers at the boundaries of the elastic medium under consideration but requires many absorbing layers which requires much computational time. The objective of this study is to present a method for radiating boundary conditions by using modified ABC to take the effect of viscous damping combining with ALID.

2. Methodology

The equation of motion of the system under dynamic equilibrium is defined as

$$[M]\ddot{u} + [C]\dot{u} + [K]u = F \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

Where $[M]$, $[C]$ and $[K]$ are the global mass, damping and stiffness matrices respectively. $\ddot{u}$, $\dot{u}$, u and F are the acceleration, velocity, displacement and external force vectors respectively. Using Rayleigh damping the damping matrix can be defined as

$$[C] = \alpha[M] + \beta[K] \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

Where α and β are mass and stiffness proportional damping coefficients. Rearranging equation (1) using Fourier transformation, complex mass $[\acute{M}] = [M]\left(1 - \frac{\alpha}{iw}\right)$ and complex stiffness $\left[\acute{K}\right] = [K](1 - iw\beta)$

$$\left[\acute{M}\right]\ddot{u} + [\acute{K}]u = F \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

The Absorbing Boundary Conditions correspond to a situation where the boundary is supported on infinitesimal dash-pots oriented normal and tangential to the boundary. The corresponding stress components are given by

$$\sigma = a \ \rho \ V_p \ \dot{u}$$

$$\tau = b \ \rho \ V_s \ \dot{v} \qquad \text{..........} \qquad (4)$$

Where, σ and τ are the normal and shear stresses, $\dot{u}$ and $\dot{v}$ are the normal and tangential velocities respectively; ρ is the mass density; V_s and V_p are S-wave and P-wave velocities respectively; a and b are dimensionless parameters. To define the absorbing boundary condition for viscoelastic wave propagation (VABC), it is necessary to include the effect of viscosity in equation (4). By applying Inverse Fourier transformation after replacing Young's modulus and density with complex Young's modulus and densities and ignoring stiffness proportional damping.

$$\sigma = a \ \rho \ V_p \ \dot{u} \ + 0.5a \ \rho \ V_p \ \alpha \ u \qquad \text{..........}$$

$$\tau = b \ \rho \ V_s \ \dot{v} \ + 0.5b \ \rho \ V_s \ \alpha \ v \qquad \text{..........} \qquad (5)$$

The equation (5) represents VABC which includes a dashpot with coefficient $a\rho AV_p$ and a spring with coefficient $0.5\alpha \ a\rho AV_p$. These computations are simple and the additional computational cost is negligible.

To achieve optimum performance when combining the ALID and VABC, it is necessary to avoid the impedance mismatch between the last absorbing layer and boundary conditions. This is achieved by replacing α with damping coefficient defined in last absorbing layer α_M.

3. Results

The efficiency of the proposed radiating boundary conditions is evaluated in two-dimensional scalar wave propagation. A pure P-wave propagation resulting from the explosive source in the infinite medium under plane strain condition as described in Fig. 1. Due to symmetry of the domain, only one fourth of the domain is considered in the modelling and in area of study is considered to be 4 times wavelength (λ) i.e. 4.0 m radius. Analysis has been carried out for three configurations a). ALID, b) ALID + ABC and c) ALID + VABC. Absorbing region (ALID) length is considered as 1.5λ for all the configurations. ABC and VABC boundary conditions are applied normal to the boundary.

The time variation of the displacement at explosion source is considered using a Ricker wavelet as defined in equation (6) with parameters used in this problem are A_f = 10 mm, f_p= 2Hz and t_s = 1.5 sec. A 1.0 m radius cavity is created around the source to apply the pressure wave and also to avoid numerical instabilities due to meshing near the origin.

$$U(0,t) = A_f\left[1 - 2\pi^2 f_p^2 (t - t_s)^2\right] e^{-\pi^2 f_p^2 (t-t_s)^2} \qquad \text{..........} \qquad (6)$$

To obtain the appropriate source field for use in numerical method, analytical simulation to the infinite media is carried out using

$$u(r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{-i\pi H_0^{(2)}(k*r)F_w\right\} e^{+iwt} dw \qquad \text{..........} \qquad (7)$$

Where, r is the radial distance from the source, F_w is fourier transform of Ricker wavelet (6), w is the frequency, $k = w/c_p$ is the wave number. The inverse Fourier transform in equation (7) is performed numerically.

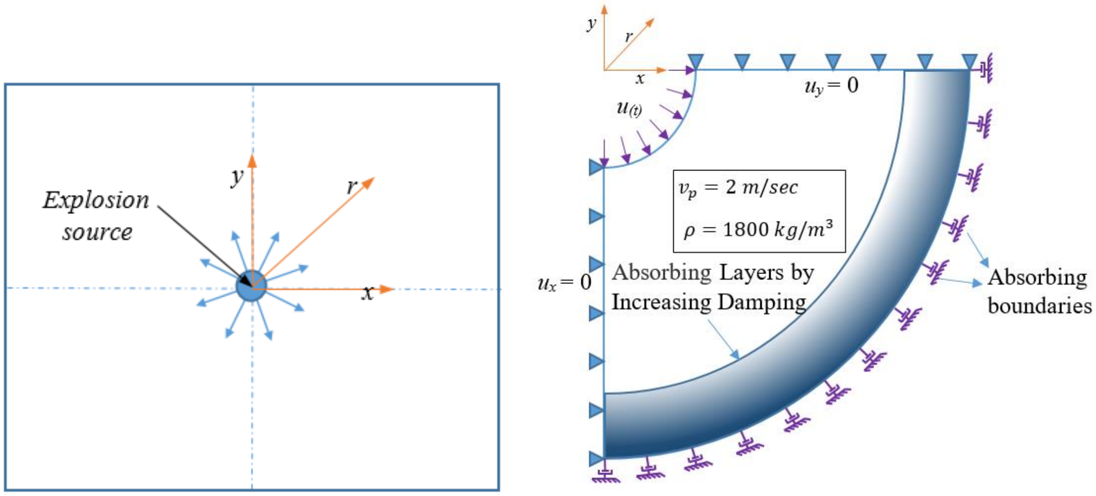

Figure 1. 2D Plane strain model for pure P-Wave propagation

The absolute maximum percentage of reflection is calculated for different loading frequencies. Fig. 2 shows percentage of reflections for loading frequencies ranging from 1Hz to 10 Hz for when the model is created for 2Hz designed loading frequency. It is noted again that the reflections in VABC with ALID are around 10% for 1Hz loading frequency reducing to less than 0.1% for load frequencies of 7Hz and above.

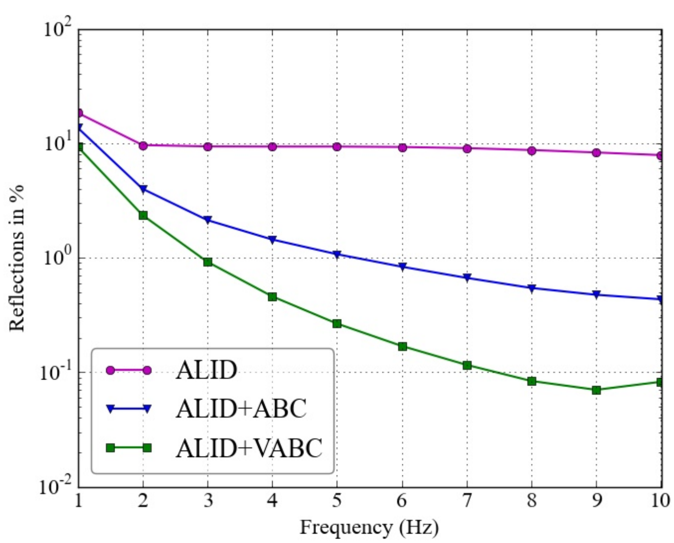

Figure 2: Percentage of reflection for three different configurations against design load frequency

4. **Conclusions**

The following conclusions were drawn when using proposed method for radiating boundary conditions by combining the absorbing layers and modified absorbing boundary conditions:

1. The performance and efficiency of the proposed method is much improved when compared with ALID with minimum additional cost in implementation.
2. The performance degrades when the model is subjected to incident wave at a lower frequency than the designed frequency, but performs excellently at higher incident frequencies.
3. The study noted that, combining ALID with original ABC also yields reasonably good performance over ALID alone.
4. The proposed approach uses features that are readily available in many commercial programs, such as Oasys GSA used in this investigation, but the novel feature here is assigning the correct set up and damping properties.
5. It is also noted that, if the loading frequency increases compared to the design load frequency, there is a small degradation in the performance of the proposed boundary condition. However, this is small enough that it would not normally be a concern.

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