Relay Selection and Resource Allocation for Energy Harvesting Cooperative Networks

by

Prudhvi Deep Mutyala, Shubham Jain, Ubaidulla P

in

IEEE 85th Vehicular Technology Conference (IEEE VTC2017-Spring)

Report No: IIIT/TR/2017/-1

Centre for Communications
International Institute of Information Technology
Hyderabad - 500 032, INDIA
June 2017
Relay Selection and Resource Allocation for Energy Harvesting Cooperative Networks

M Prudhvi Deep, Shubham Jain, and P. Ubaidulla
Signal Processing and Communication Research Center (SPCRC),
International Institute of Information Technology (IIIT),
Hyderabad, India.

Abstract—In this paper, we consider a cooperative wireless network involving two transceiver nodes whose communication is assisted by energy-constrained two-way amplify-and-forward (AF) relay nodes. The transceiver nodes can simultaneously transmit information and energy, and the relay nodes harvest the energy from the received signal and use the harvested energy to amplify and retransmit the received signal to the transceiver nodes. We consider a power splitting-based relaying (PSR) protocol as well as a time switching-based relaying (TSR) protocol to effect simultaneous extraction of information and harvesting of energy at the relay. We address the problem of sum-rate maximization under constraints on the total transmit power and the harvested energy in both scenarios. In order to solve this problem, we propose an optimal resource allocation and relay selection scheme. The performance of the proposed PSR and TSR protocols is illustrated by numerical simulations.

I. INTRODUCTION

Wireless devices are typically powered by batteries to ensure their portability. The finite operational period of the batteries limits the lifetime of the network. In a practical scenario, this need for periodic replacement of energy sources is undesirable. A network can have access to a continuous supply of energy by harvesting energy from its surroundings. Energy harvesting from radio-frequency (RF) signals has received great attention recently [1], [2], [3]. RF signals can carry both energy and information at the same time, enabling communication node to harvest energy and extract information simultaneously. In [1], the authors propose capacity-energy function to outline the fundamental tradeoffs in simultaneous information and energy transfer. In [2], the work in [1] is extended to frequency-selective channels with additive white Gaussian noise (AWGN).

Recent advances in simultaneous information transfer and energy harvesting using RF signals generally follow two approaches. In the first approach, the receiver is able to process the information and extract energy from the same signal [1], [2]. This receiver architecture may not hold in practice [3], as circuits used for harvesting energy from RF signals are not yet able to decode the carried information directly. The second approach considers practical receiver architectures, namely, time switching (TS) and power splitting (PS), proposed in [4], [5], [6]. Cooperative relay techniques have been employed in wireless networks to achieve better network connectivity, efficient utilization of resources such as power and bandwidth. Depending on the ability of the relay to regenerate or decode the signals from the terminals, several transmission protocols have been studied. The regenerative relay adopts the decode-and-forward (DF) protocol and performs the decoding process at the relay [7]. The non-regenerative relay typically adopts a form of amplify-and-forward (AF) protocol and does not perform decoding at the relay, but amplifies the signals to retransmit them back to the terminals [8], [9]. In the context of simultaneous transfer of energy and information, TS-based relaying (TSR) and PS-based relaying (PSR) can be employed in the cooperative systems [10], [11].

In this paper, we consider an AF two-way relaying system with a pair of transceivers and a relay. Two-way relaying provides better spectral efficiency compared to one-way relaying [12]. The two-way relay harvests energy from the RF signals broadcasted by the transceiver nodes and uses the harvested energy to amplify and forward the signals to their destination. Using the TS and PS receiver architectures proposed in [3], we propose the TSR and PSR protocols to facilitate energy harvesting and information processing at the relay. For the TSR and PSR protocols, we derive expressions for the achievable signal to noise interference ratio (SINR) at the destination and address the problem of optimal resource allocation and relay selection to maximize the rate of communication between the transceiver nodes. For both protocols, we propose an optimal scheme for power allocation along with energy harvesting and optimal relay selection. In both the cases, we arrive at a mixed-integer programming optimization problem. This is simplified using high SINR approximation of the instantaneous sum-rate to obtain closed form solutions. The power allocation is done on the basis of solutions obtained using the Karush-Kuhn-Tucker (KKT) conditions.

In [10], the performances of PSR and TSR are compared by calculating the achievable throughput at the destination. However in our work comparison between the performance of PSR and TSR protocols is discussed in the context of sum rate maximization. We notice that PSR protocol outperforms TSR protocol in the context of sum rate maximization. This is supported by the plots illustrated in Section IV.

This work was supported in part by the Visvesvaraya Young Faculty Research Fellowship, Department of Electronics and Information Technology (DeitY), Government of India.
II. SYSTEM MODEL

We consider a wireless co-operative network consisting of two transceiver nodes, \( T_1 \) and \( T_2 \) and a set of two-way relays. The transceivers communicate with each other via a relay node selected from a set of \( L \) relay nodes. All the nodes are equipped with single antenna. Based on the time switching and the power splitting receiver architectures, we propose two relaying protocols to harvest energy from the RF signal, (i) PSR protocol and (ii) TSR protocol. We provide detailed analysis of the PSR and TSR protocols in the following sections.

A. Power Splitting-Based Relaying (PSR) Protocol

In case of PSR, communication takes place between the transceiver nodes over two time slots. In the first slot, the transmission of signal takes place at both transceiver nodes. The transmitted signals are then received by the relay. Let \( x_1 \in \mathbb{C} \) and \( x_2 \in \mathbb{C} \) be the transmit signals conveyed by the two transceiver nodes \( T_1 \) and \( T_2 \), respectively. We assume that \( \mathbb{E}\{|x_1|^2\} = \mathbb{E}\{|x_2|^2\} = 1 \), where \( \mathbb{E}\{\cdot\} \) is the expectation operator and \(|\cdot|\) denotes absolute value. The signal received by the \( i \)th relay, denoted by \( r_i \), can be expressed as

\[
r_i = \sqrt{P_i}h_ix_1 + \sqrt{P_2}g_ix_2 + \delta_i, \quad 1 \leq i \leq L,
\]

where \( P_1 \) and \( P_2 \) are the transmit powers of \( T_1 \) and \( T_2 \) respectively, \( h_i \) denotes the channel gain from \( T_1 \) to the \( i \)th relay, \( g_i \) denotes the channel gain from \( T_2 \) to the \( i \)th relay and \( \delta_i \sim \mathcal{CN}(0, \sigma_\delta^2) \) is the additive white Gaussian noise (AWGN) at the \( i \)th relay. In the second slot, the signal received by the \( i \)th relay, \( r_i \), is split into two parts for energy harvesting and information processing. Let \( \rho \in [0,1] \) be the power splitting ratio, meaning that \( \sqrt{\rho}r_i \) is used for energy harvesting and \( \sqrt{(1-\rho)}r_i \) is used for information processing. The total energy harvested at the \( i \)th relay, denoted by \( P_{EH}(\rho) \), can be expressed as

\[
P_{EH}(\rho) = \rho \eta \mathbb{E}\{|r_i|^2\} = \rho \eta |P_1|h_i|^2 + P_2|g_i|^2 + \sigma_\delta^2|,\]

where \( \eta \in [0,1] \) denotes the energy conversion efficiency. The remaining fraction of the received signal, \( \sqrt{1-\rho}r_i \), is transmitted by the relay after amplifying it by a complex number. Both the transceiver nodes receive the signal transmitted by the relay node. The signal received by the transceiver node \( T_1 \) can be written as

\[
y_1 = \sqrt{1-\rho}\left(\sqrt{P_1}w_1h_1x_1 + \sqrt{P_2}w_1g_1h_2x_2 + w_1h_1\delta_1\right) + \phi_1
\]

where \( w_i \in \mathbb{C} \) is the amplification factor or weight coefficient of the \( i \)th relay, \( \phi_1 \) represents the AWGN at transceiver \( T_1 \) with zero mean and variance \( \sigma_\phi^2 \). The signal received by the transceiver node \( T_2 \) is given by

\[
y_2 = \sqrt{1-\rho}\left(\sqrt{P_1}w_1h_1x_1 + \sqrt{P_2}w_1g_1h_2x_2 + w_1g_1\delta_1\right) + \phi_2,
\]

where \( \phi_2 \) represents the AWGN at transceiver \( T_2 \) with zero mean and variance \( \sigma_\phi^2 \). The terms \( \sqrt{P_1}w_1h_1x_1 \) and \( \sqrt{P_2}w_1g_1h_2x_2 \) in (4) and (5), respectively, represent the self-interference terms resulting from both transceivers own transmitted signals.

The self-interference terms can be eliminated from the received signals using the knowledge of the channel state information (CSI), the amplification factor and each transceiver’s transmitted signal. The SINR at transceiver \( T_1 \) after self-interference cancellation is given by

\[
\text{SINR}_{i,1} = \frac{(1-\rho)[P_1|w_1|^2|h_i|^2 + P_2|w_1|^2|g_i|^2 + |w_1|^2\sigma_\delta^2]}}{(1-\rho)\sigma_\phi^2|h_i|^2 + \sigma_\delta^2}.
\]

and the SINR at transceiver \( T_2 \) is given by

\[
\text{SINR}_{i,2} = \frac{(1-\rho)[P_1|w_1|^2|h_i|^2 + P_2|w_1|^2|g_i|^2 + |w_1|^2\sigma_\delta^2]}}{(1-\rho)\sigma_\phi^2|g_i|^2 + \sigma_\delta^2}.
\]

The transmit power of the \( i \)th relay can be expressed as

\[
P_{R_i} = \mathbb{E}\{|w_i|\sqrt{1-\rho}r_i|^2\}
\]

\[
= (1-\rho)[P_1|w_1|^2|h_i|^2 + P_2|w_1|^2|g_i|^2 + |w_1|^2\sigma_\delta^2],
\]

from (9), we have

\[
|w_i| = \frac{\sqrt{P_{R_i}}}{\sqrt{(1-\rho)[P_1|h_i|^2 + P_2|g_i|^2 + \sigma_\delta^2]}}.
\]

Along the same lines, we can express the the SINR at the transceivers \( T_1 \) and \( T_2 \) in terms of \( P_{R_i} \) as follows:

\[
\text{SINR}_{1,i} = \frac{P_2P_{R_i}|h_i|^2|g_i|^2}{P_{R_i}\sigma_\delta^2|h_i|^2 + \sigma_\phi^2[P_1|h_i|^2 + P_2|g_i|^2 + \sigma_\delta^2]},
\]

\[
\text{SINR}_{2,i} = \frac{P_1P_{R_i}|h_i|^2|g_i|^2}{P_{R_i}\sigma_\delta^2|g_i|^2 + \sigma_\phi^2[P_1|h_i|^2 + P_2|g_i|^2 + \sigma_\delta^2]}.
\]

B. Time Switching-Based Relaying (TSR) Protocol

As shown in Fig. 1, the transmission block time \( T \) is divided mainly into two parts, \( \alpha T \) and \( (1-\alpha)T \). The relay node harvests energy from the received signals for a duration of \( \alpha T \), where \( 0 \leq \alpha \leq 1 \). The remaining block time, \( (1-\alpha)T \), is further divided into two equal halves, such that the first half \( (1-\alpha)/2 \), is used for information transmission from source to relay and the second half \( (1-\alpha)/2 \), is used for transmission of the processed information signal from the relay to destination.

Let \( x_1 \in \mathbb{C} \) and \( x_2 \in \mathbb{C} \) be the transmit signals conveyed by the two transceiver nodes \( T_1 \) and \( T_2 \), respectively. We assume
that $\mathbb{E}\{|x_1|^2\} = \mathbb{E}\{|x_2|^2\} = 1$. The signal received by the $i$th relay, denoted by $r_i$, can be expressed as

$$r_i = \sqrt{P_1} h_i x_1 + \sqrt{P_2} g_i x_2 + \delta_i, \quad 1 \leq i \leq L,$$  \hspace{1cm} (13)

Energy is harvested from the received signal for a duration of $\alpha T$, and the harvested energy at relay node can be expressed as

$$P_{EH}(\alpha) = \alpha \eta \mathbb{E}\{|r_i|^2\} = \alpha \eta |P_1| h_i^2 + P_2 |g_i|^2 + \sigma_k^2,$$ \hspace{1cm} (14)

where $\eta \in [0, 1]$ is the energy conversion efficiency of the energy harvesting receiver.

During the information receiving phase, the received signal is the same as (13), but received in a different time duration. The signal received by the relay is re-transmitted after amplifying it by a complex number. Both the transceiver nodes receive the signal transmitted by the relay node. The signal received by the transceiver $T_1$ can be written as

$$y_1 = \sqrt{P_1} w_i h_i x_1 + \sqrt{P_2} w_i g_i x_2 + w_i h_i \delta_i + \phi_1,$$ \hspace{1cm} (16)

where $w_i \in \mathbb{C}$ is the amplification factor or weight coefficient of the $i$th relay, $\phi_1$ represents the AWGN at transceiver $T_1$ with zero mean and variance $\sigma_1^2$. The signal received by the transceiver $T_2$ is given by

$$y_2 = \sqrt{P_1} w_i h_i x_1 + \sqrt{P_2} w_i g_i x_2 + w_i g_i \delta_i + \phi_2,$$ \hspace{1cm} (17)

The terms $\sqrt{P_1} w_i h_i x_1$ and $\sqrt{P_2} w_i g_i x_2$ in (16) and (17), respectively, represent the self-interference terms resulting from both transceivers’ transmitted signals.

The SINR at transceiver $T_1$ after self-interference cancellation is given by

$$SINR_{i,1} = \frac{P_2 |w_i|^2 |h_i|^2 |g_i|^2}{\sigma_1^2 |w_i|^2 |h_i|^2 + \sigma_i^2},$$ \hspace{1cm} (18)

and the SINR at transceiver $T_2$ is given by

$$SINR_{i,2} = \frac{P_1 |w_i|^2 |h_i|^2 |g_i|^2}{\sigma_2^2 |w_i|^2 |g_i|^2 + \sigma_i^2}. \hspace{1cm} (19)$$

The transmit power of the $i$th relay can be expressed as

$$P_{Ri} = \mathbb{E}\{|w_i|^2\} = P_1 |w_i|^2 |h_i|^2 + P_2 |w_i|^2 |g_i|^2 + |w_i|^2 \sigma_i^2,$$ \hspace{1cm} (20)

from (21), we have

$$|w_i| = \sqrt{\frac{P_{Ri}}{P_1 |h_i|^2 + P_2 |g_i|^2 + \sigma_i^2}}.$$ \hspace{1cm} (22)

Along the same lines, we can express the the SINR at the transceivers $T_1$ and $T_2$ in terms of $P_{Ri}$ as follows:

$$SINR_{1,i} = \frac{P_2 P_{Ri} |h_i|^2 |g_i|^2}{P_{Ri} \sigma_1^2 |h_i|^2 + \sigma_i^2 (P_1 |h_i|^2 + P_2 |g_i|^2 + \sigma_i^2)},$$ \hspace{1cm} (23)

$$SINR_{2,i} = \frac{P_1 P_{Ri} |h_i|^2 |g_i|^2}{P_{Ri} \sigma_2^2 |g_i|^2 + \sigma_i^2 (P_1 |h_i|^2 + P_2 |g_i|^2 + \sigma_i^2)}. \hspace{1cm} (24)$$

### III. OPTIMAL RELAY SELECTION AND POWER ALLOCATION

The overall problem formulation and the proposed solutions are detailed in the following. The objective of the problem is to maximize the overall rate under energy-harvesting and transmit power constraints. This objective is achieved by choosing the best relay, which operates with the energy provided by the transceivers, and by optimal resource allocation at the transceivers and the relay node. This problem can be formulated as

$$\max \quad R_k, \hspace{1cm} \text{subject to}$$

where $R_k$ is the achievable rate when the $k$th relay is operating, and $\mathcal{I}$ is the set of relay indices. The feasible set $\Lambda$ is defined by the power and energy harvesting constraints. This is a joint optimization over the relay indices, the transmit powers, and the variable $\xi$, $\xi$ represents $\rho$ in case of PSR and $\alpha$ in case of TSR. As such it is a mixed-integer program and hard to solve. However, since only one relay will be operating at any given time, this optimization can be performed in two simple steps; first over the transmit powers and the variable $\xi$ and then over the relay indices, as given below:

$$\max_{k \in \mathcal{I}, P_1, P_2, P_{R_{k,e}} \in \Lambda} R_k.$$ \hspace{1cm} (25)

Consequently, first we maximize the rate with respect to the variable $\xi$ and transceiver transmit powers. We consider this rate maximization problem in both PSR and TSR protocols in the following sections.

#### A. Power Splitting-Based Relaying (PSR) Protocol

The instantaneous achievable rate of the wireless cooperative network with two-way relaying is given as

$$R_i = \frac{1}{2} \log_2 (1 + SINR_{i,1}) + \frac{1}{2} \log_2 (1 + SINR_{i,2}). \hspace{1cm} (27)$$

The factor of $\frac{1}{2}$ used in (27), results from the two time slots required to complete the information exchange between the transceivers. In order to make the mathematical analysis more viable, we follow the following high-SINR approximation of the rate:

$$R_i \approx \frac{1}{2} \log_2 \left( \frac{P_2 P_{Ri} |h_i|^2 |g_i|^2}{P_{Ri} \sigma_1^2 |h_i|^2 + \sigma_i^2 (P_1 |h_i|^2 + P_2 |g_i|^2 + \sigma_i^2)} \right)$$

$$+ \frac{1}{2} \log_2 \left( \frac{P_1 P_{Ri} |h_i|^2 |g_i|^2}{P_{Ri} \sigma_2^2 |g_i|^2 + \sigma_i^2 (P_1 |h_i|^2 + P_2 |g_i|^2 + \sigma_i^2)} \right)$$

$$= \frac{1}{2} \log_2 \left( \frac{P_1 P_2 P_{Ri}^2 |h_i|^4 |g_i|^4}{X_i Y_i} \right) , \hspace{1cm} (28)$$

where

$$X_i = P_{Ri} \sigma_1^2 |h_i|^2 + \sigma_i^2 (P_1 |h_i|^2 + P_2 |g_i|^2 + \sigma_i^2), \hspace{1cm} (29)$$

$$Y_i = P_{Ri} \sigma_2^2 |g_i|^2 + \sigma_i^2 (P_1 |h_i|^2 + P_2 |g_i|^2 + \sigma_i^2). \hspace{1cm} (30)$$
This high SINR approximation can significantly simplify the optimal power allocation and PS ratio computation, which can now be reformulated as

$$
\begin{align*}
\max & \quad \frac{1}{2} \log \left( \frac{P_1 P_2 P_{Ri}^2 |h_i|^4|g_i|^4}{X_i Y_i'} \right) \\
\text{s.t.} & \quad P_1 + P_2 \leq P_T, \\
& \quad P_{EH}(\rho) \geq \theta, \\
& \quad 0 \leq \rho \leq 1,
\end{align*}
$$

where $P_T$ is the total transmit power of the two transceiver nodes $T_1$ and $T_2$ and $\theta$ is the minimum amount of the energy harvested at the relay. Note that $\theta$ is also the upper limit for the transmit power at the relay, meaning that the relay uses only the harvested energy to amplify and re-transmit the received signal. The transmit power constraint and energy harvesting constraint acts as a safeguard such that the transmit power at the relay does not exceed the harvested power.

As the objective function of (31) is a monotonically increasing function of the transmit power of the relay, $P_{Ri}$, it is self-evident that its optimal value is given by $P_{Ri} = \theta$. Having realized the optimal value of $P_{Ri}$, we proceed to compute the optimal value of $(P_1, P_2, \rho)$. Then, the problem in (31) can be stated as follows:

$$
\begin{align*}
\max & \quad \frac{1}{2} \log \left( \frac{P_1 P_2 P_{Ri} |h_i|^4|g_i|^4}{X_i Y_i'} \right) \\
\text{s.t.} & \quad P_1 + P_2 \leq P_T, \\
& \quad P_{EH}(\rho) \geq \theta, \\
& \quad 0 \leq \rho \leq 1,
\end{align*}
$$

where $X_i'$ and $Y_i'$ are obtained from $X_i$ and $Y_i$, respectively, by replacing $P_{Ri}$ with $P_{Ri}^*$ in (29) and (30). Since the logarithmic function is a monotonically increasing function, the maximization in (32) is equivalent to the minimization problem as follows:

$$
\begin{align*}
\min & \quad \frac{X_i' Y_i'}{P_1 P_2 P_{Ri} |h_i|^4|g_i|^4} \\
\text{s.t.} & \quad P_1 + P_2 \leq P_T, \\
& \quad P_{EH}(\rho) \geq \theta, \\
& \quad 0 \leq \rho \leq 1,
\end{align*}
$$

We describe a closed form optimal solution to this problem.

1) Optimal Scheme: We obtain optimal power allocation and power splitting ratio using the Karush-Kuhn-Tucker conditions. In order to facilitate further analysis, we rewrite (33) as follows:

$$
\begin{align*}
\min & \quad \frac{P_1 P_2}{(a_1 + b_1 P_1 + c_1 P_2)(a_2 + b_2 P_1 + c_2 P_2)} \\
\text{s.t.} & \quad P_1 + P_2 \leq P_T, \\
& \quad \rho(m P_1 + n P_2 + q) \geq \theta, \\
& \quad 0 \leq \rho \leq 1,
\end{align*}
$$

where

$$
\begin{align*}
a_1 &= \frac{\sigma_0^2}{P_{Ri}^* |g_i|^2} \left(1 + \frac{\sigma_0^2}{|h_i|^2} \right), \\
b_2 &= \frac{\sigma_0^2}{P_{Ri}^* |g_i|^2} \left(1 + \frac{\sigma_0^2}{|h_i|^2} \right), \\
b_1 &= \frac{\sigma_0^2}{P_{Ri}^* |g_i|^2}, \\
c_1 &= \frac{\sigma_0^2}{P_{Ri}^* |g_i|^2}, \\
c_2 &= \frac{\sigma_0^2}{P_{Ri}^* |g_i|^2}.
\end{align*}
$$

Then, the Lagrangian associated with the problem in (34), is given by

$$
\mathcal{L}(P_1, P_2, \rho, \mu, \nu) = f + \mu_1[\theta - \rho(m P_1 + n P_2 + q)] + \mu_2(P_1 + P_2 - P_T) + \mu_3(\rho - 1) - \mu_4 \rho,
$$

where $f = (a_1 + b_1 P_1 + c_1 P_2)(a_2 + b_2 P_1 + c_2 P_2)$ and $\{\mu_i\}_{i=1}^4$ are the Lagrangian multipliers. The optimal solution satisfies the following KKT conditions:

$$
\begin{align*}
\frac{\partial f}{\partial P_1} - \mu_1 P_1 - \mu_2 P_2 &= 0, \\
\frac{\partial f}{\partial P_2} - \mu_1 P_1 - \mu_2 P_2 &= 0, \\
\mu_1[\theta - \rho(m P_1 + n P_2 + q)] &= 0, \\
\mu_2(P_1 + P_2 - P_T) &= 0, \\
\mu_3(\rho - 1) &= 0, \\
\mu_4 \rho &= 0, \\
\mu_i &\geq 0, \quad i = 1, 2, 3, 4, \\
\text{and the constraints in (34)}.
\end{align*}
$$

By solving the set of KKT conditions given above, we obtain the optimal values of $P_1, P_2,$ and $\rho$ as follows:

$$
\begin{align*}
P_1' &= \frac{P_T}{r_3 \sqrt{1 + \frac{r_1^2 + r_2^2}{r_3^2}} - (r_2 + r_3)}, \\
P_2' &= \frac{P_T}{r_3 \sqrt{1 + \frac{r_1^2 + r_2^2}{r_3^2}}}, \\
\rho' &= \frac{\theta}{\eta(P_1' |h_i|^2 + P_2' |g_i|^2 + \sigma_0^2)},
\end{align*}
$$

where $r_1 = b_1 b_2 + a_1 b_2 + a_1 b_2 + a_1 b_2$, $r_2 = c_1 c_2 + a_1 c_2 + a_1 c_2$, and $r_3 = a_1 a_1$.

B. Time Switching-Based Relaying (TSR) Protocol

The instantaneous achievable rate of the wireless cooperative network with two-way relaying is given as

$$
R_i = \frac{1 - \alpha}{2} \log(1 + SINR_{i,1}) + \frac{1 - \alpha}{2} \log(1 + SINR_{i,2}).
$$

The factor of $\frac{1 - \alpha}{2}$ used in (38), results from the two time slots required to complete the information exchange between the transceivers. We follow the high-SINR approximation of the rate:
where

\[
X_i = P_Ri \sigma_i^2 |h_i|^2 + \sigma_i^2 (P_1|h_i|^2 + P_2|g_i|^2 + \sigma_i^2), \quad (40)
\]

\[
Y_i = P_Ri \sigma_i^2 |g_i|^2 + \sigma_i^2 (P_1|h_i|^2 + P_2|g_i|^2 + \sigma_i^2). \quad (41)
\]

As in the case of PSR protocol, we can use high SINR approximation to simplify the optimal power allocation and TS ratio computation, which can now be reformulated as

\[
\max_{\alpha, P_1, P_2, P_{Ri}} \quad \frac{1 - \alpha}{2} \log \left( \frac{P_1 P_2 P_{Ri}^2 |h_i|^4 |g_i|^4}{X_i Y_i} \right) \]  
\text{s.t.} \quad P_1 + P_2 \leq P_T, \quad \rho_{EH} \geq \rho, \quad P_{Ri} \leq P_T, \quad 0 \leq \alpha \leq 1, \quad (42)
\]

where \( P_T \) is the total transmit power of the two transceiver nodes \( T_1 \) and \( T_2 \) and \( \rho \) is the minimum amount of the energy harvested at the relay.

As the objective function of (42), is a monotonically increasing function of the transmit power of any relay, \( P_{Ri} \), it is self-evident that its optimal value is given by \( P_{Ri}' = \rho \). Having realized the optimal value of \( P_{Ri} \), we proceed to compute the optimal value of \((P_1, P_2, \alpha)\). Then, the problem in (42) can be stated as follows:

\[
\max_{\alpha, P_1, P_2} \quad \frac{1 - \alpha}{2} \log \left( \frac{P_1 P_2 P_{Ri}'^2 |h_i|^4 |g_i|^4}{X_i Y_i} \right) \]  
\text{s.t.} \quad P_1 + P_2 \leq P_T, \quad \rho_{EH} \geq \rho, \quad P_{Ri}' \leq P_T, \quad 0 \leq \alpha \leq 1, \quad (43)
\]

1) Optimal Scheme: If \( \alpha \) in (43) is a constant, we can solve this problem in the same lines as (32). We perform a linear search over the range of \( \alpha \) and select the value which maximizes both the objective and satisfies the energy harvesting threshold constraint. With \( \alpha = 0 \) the energy harvesting threshold of (43) cannot be satisfied for a nonzero value of \( \theta \). So, we initialize the linear search with a value of \( \alpha \) close to zero. Let us denote the objective of (43) by \( R \) and \( R' \) represents the maximum value of this objective. \( P_1', P_2' \) and \( \alpha' \) represents the optimal values corresponding to \( R' \). The proposed solution can be outlined as follows:

1) Initialize: \( \alpha; P_1' = 0; P_2' = 0; R' = 0; P_T; \theta; \delta \).
2) while \( \alpha \leq 1 \)
3) solve (43) exactly like we solved (32) and obtain the values of \( P_1 \) and \( P_2 \).
4) Calculate \( R \) using \( P_1 \) and \( P_2 \).
5) if \( R \geq R' \)
   i) Set \( \alpha' = \alpha; P_1' = P_1; P_2' = P_2; R' = R \).
   ii) \( \alpha = \alpha + \delta \).
6) else
   i) \( \alpha = \alpha + \delta \).

The value of the increment \( \delta \) can be chosen so as to ensure the required accuracy. Following the procedure illustrated above, we can arrive at \( P_1', P_2' \) and \( \alpha' \), the optimal values of the rate maximization problem.

C. Optimal Relay Selection

Having solved the optimal solutions for the power allocation and PS ratio and TS ratio for PSR and TSR protocol respectively, we now address the problem of selecting the relay that results in the highest achievable rate from the set of \( L \) relays. This problem can be formulated as

\[
k' = \arg\max_{k \in \mathbb{Z}} R_k', \quad (44)
\]

where \( R_k' \) is the rate achieved when the \( k \)th relay is operating under optimal conditions described earlier. The optimal relay is selected as the one that leads the highest \( R_k' \). Combining the results, we can represent the optimal solution to the problem in (32) as \((k', P_1', P_2', P_{R_{k'}}, \rho')\) and in (43) as \((k', P_1', P_2', P_{R_{k'}}, \alpha')\).

IV. SIMULATION RESULTS

In this section, we illustrate the performance of the proposed joint relay selection and optimal power allocation scheme with energy harvesting through numerical simulations. We draw comparisons between the performances of both PSR and TSR protocols. We assume that all the channels undergo Rayleigh fading.

![Fig. 2. Rate versus energy harvesting threshold (\( \theta \)) for different values of \( \rho \).](image-url)
The energy harvesting threshold \( \theta \) of the transceivers. The results are provided for energy harvesting and information processing at the relay. We proposed two relaying protocols, namely, PSR protocol and TSR protocol, to enable wireless energy harvesting and information processing at the relay. We addressed the problem of optimal relay selection and resource allocation in this network. The optimal relay is selected so as to maximize the communication rate between the transceiver nodes. We proposed an optimal scheme for power allocation, relay selection, and computation of PS ratio and TS ratio in PSR and TSR protocols respectively. The performance of the proposed schemes are demonstrated via numerical simulations. From these simulations, we concluded that PSR scheme outperforms TSR scheme.

Fig. 3. Rate versus total transmit power \( (P_T) \) for different values of \( \theta \)

Firstly, we study the performance of the proposed schemes in terms of maximum achievable rate versus energy harvesting threshold. The corresponding results are provided for total transmit power limits of \( P_T = 30 \) dB and \( P_T = 20 \) dB. The energy harvesting threshold \( (\theta) \) ranges from 1 mW to 10 mW. The performance results are displayed in Fig. 2. We can observe that the PSR scheme outperforms the TSR scheme. The rates obtained in Fig. 2 are high, thus validating the high-SINR approximation used above.

Secondly, we demonstrate the performance of the proposed schemes in terms of the rate versus total transmitted power of the transceivers. The results are provided for energy harvesting threshold \( (\theta) \) values of 15 dBm and 10 dBm. The total transmitted power \( P_T \) ranges from 0dB to 30dB. The performance results are displayed in Fig. 3. Here, as expected we can observe that the rates are increasing with increasing transmit power. From the Fig. 3, we can conclude that PSR scheme outperforms TSR scheme.

V. CONCLUSION

We considered an amplify-and-forward wireless cooperative network, where an energy constrained two-way relay node harvests energy from the received RF signal and utilizes the harvested energy to amplify-and-forward the signal to the destination node. We proposed two relaying protocols, namely, PSR protocol and TSR protocol, to enable wireless energy harvesting and information processing at the relay. We addressed the problem of optimal relay selection and resource allocation in this network. The optimal relay is selected so as to maximize the communication rate between the transceiver nodes. We proposed an optimal scheme for power allocation, relay selection, and computation of PS ratio and TS ratio in PSR and TSR protocols respectively. The performance of the proposed schemes are demonstrated via numerical simulations. From these simulations, we concluded that PSR scheme outperforms TSR scheme.

REFERENCES