

# **Relay Selection and Residual Self-Interference Mitigation for Cognitive Full-Duplex Two-Way MIMO Relaying System**

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Nachiket Ayir, Ubaidulla P

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# Relay Selection and Residual Self-Interference Mitigation for Cognitive Full-Duplex Two-Way MIMO Relaying System

Nachiket Ayir and P. Ubaidulla

Signal Processing and Communication Research Center (SPCRC),  
International Institute of Information Technology (IIIT), Hyderabad, India.

Email: ayir.nachiket@research.iiit.ac.in, ubaidulla.p@iiit.ac.in

**Abstract**—In this paper, we consider the problem of relay selection in a cognitive full-duplex two-way multiple-input multiple-output relaying network. The cognitive nodes operate on the same frequency as the licensed primary user (PU). The end-to-end communication between the secondary users (SU) via a secondary relay (SR) takes two time slots. The SR operates on amplify-and-forward (AF) protocol. In the first time slot, the SR relay with the highest receive signal-to-noise ratio is selected as the optimal relay. At the same time, the precoders at the SU are designed to limit the interference to the PU below a certain threshold. All the nodes in the network are assumed to have imperfect knowledge of channel state information of their loopback self-interference channels and hence experience residual self-interference (RSI). After selecting the optimal SR, in the next time slot, we design the SR precoder and SU receive filters, to mitigate the RSI. We do so by minimizing the sum of mean square error of the end-to-end communication with a transmit power constraint at SR. To curb the effect of cumulative RSI caused by the AF operation of the SR, we update the SR precoder and SU receive filters at each time slot. The efficacy of our proposed designs is illustrated in the simulation results.

## I. INTRODUCTION

The traditional wireless communication systems are either half-duplex or out-of-band full-duplex (FD) [1] systems. In-band FD systems [1] are truly FD since they both transmit and receive at the same time, using the same frequency and the same antenna. As a result, in-band FD systems can significantly improve upon the spectral efficiency as compared to its traditional counterparts. This will not only enhance the data rates but also increase the capacity of the network by freeing up a lot of bandwidth. On account of the recent developments in fabricating a non-reciprocal circulator using transistors [2] and enhancements in self-interference mitigation techniques [3], in-band FD multiple-input multiple-output (MIMO) systems have become more practically feasible than ever. Relay networks have been explored and extensively studied as a means to facilitate better

quality of service, improved link capacity, and enhanced network coverage. Two-way in-band full-duplex relaying combines the benefits of both these technologies. Such a two-hop two-way FD amplify-and-forward (AF) MIMO relaying system was studied in [4], wherein the beamformer at relay and receive filters at transceivers were jointly designed for minimizing the sum of mean square error (SMSE) at the transceivers. The effect of path loss on system design was neglected in this and most of such works on two-way FD relaying.

In a cognitive radio environment, multiple unlicensed secondary users (SU) share the spectrum with a licensed primary user (PU). However, the SU need to operate under a transmit power constraint to limit the interference to PU. Thus, cognitive technology leverages the spectrum utilization in wireless communication. In such a scenario, FD two-way relaying, having better efficiency than one-way relaying, will be beneficial to enhance the range as well as rate of communication. In [5], a FD MIMO cognitive cellular system was studied where only the relay station was FD. The authors presented a MSE-based robust transceiver design for this system.

A relaying network with multiple relays is more robust to channel outages than a single relay network. In such a scenario, relay selection, wherein only one relay is selected, enhances the system performance while having low complexity [6]. Most of the previous works on relay selection for a two-way FD AF relaying network, including [7],[8], consider the outage probability, symbol error probability, average channel capacity, and/or the outage capacity as the criteria for optimal relay selection.

To the best of authors' knowledge, there is no previous work which considers the problem of relay selection and residual self-interference (RSI) mitigation for a cognitive two-way FD AF MIMO relaying network. In this paper, we begin by designing the transceiver precoders for SU and addressing the optimal relay selection problem. We then consider the design of relay precoder and transceiver receive filters to mitigate the RSI at the nodes. We evaluate the performance of the proposed designs in a LTE cellular cognitive network scenario.

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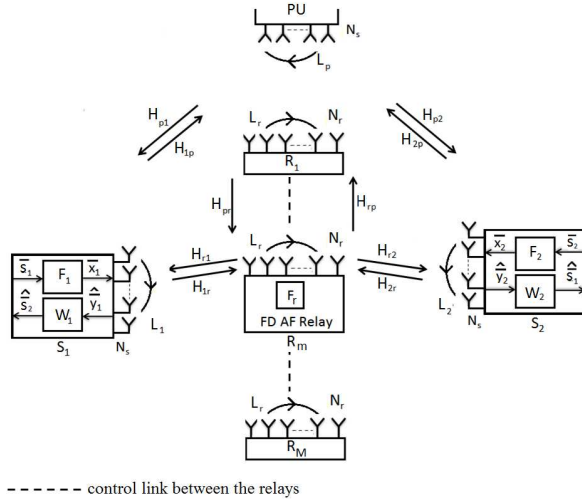


Fig. 1: System diagram of a cognitive FD two-way relaying network with  $M$  cognitive relays.

The main contributions of this paper are twofold: (i) PU interference constrained relay selection and transceiver precoder design, (ii) iterative computation of the feedback term required for RSI mitigation.

The rest of this paper is organized as follows. Section II describes the system model. The problems of precoder design for SU, and optimal secondary relay (SR) selection are considered in Section III. The design of SR precoder and SU receive filters is discussed in Section IV. Simulation results are presented in Section V and the conclusions are drawn in Section VI.

*Notations* : We denote a scalar, vector, and matrix by italic lowercase, boldface lowercase, and boldface uppercase, respectively. For a matrix  $\mathbf{A}$ , its transpose, conjugate, conjugate transpose, inverse, determinant, vectorization operation, trace, and Frobenius norm are denoted by  $\mathbf{A}^T$ ,  $\mathbf{A}^C$ ,  $\mathbf{A}^H$ ,  $\mathbf{A}^{-1}$ ,  $|\mathbf{A}|$ ,  $\text{vec}(\mathbf{A})$ ,  $\text{tr}(\mathbf{A})$ ,  $\|\mathbf{A}\|_{\mathbb{F}}$ , respectively.  $\mathbf{A} \otimes \mathbf{B}$  represents the kronecker product of matrices  $\mathbf{A}$  and  $\mathbf{B}$ .  $\mathbb{E}\{\cdot\}$  is the expectation operator,  $\|\mathbf{y}\|$  represents 2-norm of  $\mathbf{y}$ ,  $\text{mat}(\cdot)$  performs the inverse operation of  $\text{vec}(\cdot)$ . If a variable  $j = 1$ , then  $\underline{j} = 2$  and vice versa.  $x, \mathbf{x}, \mathbf{X}$  denote time-slot dependent variables denote in the current time slot  $t$ , while  $x^{(l)}, \mathbf{x}^{(l)}, \mathbf{X}^{(l)}$  denote variables in any time slot  $l$ .

## II. SYSTEM MODEL

The cognitive FD two-way relaying network shown in Fig. 1. consists of a PU, two secondary transceivers  $S_1, S_2$  and  $M$  secondary AF relays. We assume that all relays are centrally coordinated by a single control unit. Transceivers  $S_1, S_2$  and PU have  $N_s$  antennas each. Each SR has  $N_r$  antennas, out of which  $N_s$  are receiving antennas while all  $N_r$  are transmitting antennas, such that  $N_r \geq N_s$ . All the nodes in the network transmit and receive simultaneously on the same frequency which is allocated to the licensed PU. We assume that there is no

direct communication link between the SU due to high path loss and so they communicate via the optimal SR.

The matrix  $\mathbf{H}_{ij}$  represents the MIMO channel matrix between transmitter  $i$  and receiver  $j$ . So,  $\mathbf{H}_{kr} \in \mathbb{C}^{N_s \times N_r}$ ,  $\mathbf{H}_{rk} \in \mathbb{C}^{N_r \times N_s}$ ,  $\mathbf{H}_{lp} \in \mathbb{C}^{N_s \times N_s}$  and  $\mathbf{H}_{pl} \in \mathbb{C}^{N_s \times N_s}$ ,  $k \in \{1, 2, p\}$ ,  $l \in \{1, 2\}$ , represent the MIMO channels as shown in Fig. 1. All the channel links are modeled as independent and frequency-flat Rayleigh fading channels and are assumed to be static for one time slot. The knowledge of channel state information (CSI) for these channel matrices is assumed to be perfect. The matrices  $\mathbf{L}_r \in \mathbb{C}^{N_s \times N_r}$  and  $\mathbf{L}_k \in \mathbb{C}^{N_s \times N_s}$ ,  $k \in \{1, 2, p\}$ , represent the loopback self-interference (SI) MIMO channels. We consider the knowledge of CSI for the loopback channels to be imperfect, such that

$$\mathbf{L}_l = \widehat{\mathbf{L}}_l + \boldsymbol{\Psi}_l, \quad l = 1, 2, p, r, \quad (1)$$

where  $\widehat{\mathbf{L}}_l$  is the available channel estimate and  $\boldsymbol{\Psi}_l$  is the error in the CSI with zero mean and covariance  $\mathbb{E}\{\boldsymbol{\Psi}_l \boldsymbol{\Psi}_l^H\} = N_s \sigma_{e_j}^2 \mathbf{I}_{N_s}$ . The symbol  $\alpha_{ij}$  represents the path loss between nodes  $i$  and  $j$ ;  $i, j \in \{1, 2, p, r\}$ , such that  $\alpha_{ij} = \alpha_{ji}$ . We consider the loopback path loss  $\alpha_{ii} = 1$  and it won't be explicitly mentioned henceforth.

The following events occur during each time slot  $t$ :

(i) The PU transmits signal  $\mathbf{x}_p \in \mathbb{C}^{N_s \times 1}$ . Since all nodes operate on the same frequency,  $\mathbf{x}_p$  will cause interference at the secondary nodes.

(ii) Based on the knowledge of transceiver - relay channels, an optimal relay is selected by the central control unit and notified to the SU using control channels.

(iii) Each of the SU precodes symbol vector  $\mathbf{s}_m \in \mathbb{C}^{N_s \times 1}$ , having covariance  $\mathbb{E}\{\mathbf{s}_m \mathbf{s}_m^H\} = \mathbf{I}_{N_s}$ , with matrix  $\mathbf{F}_m \in \mathbb{C}^{N_s \times N_s}$  and transmits  $\mathbf{x}_m$ ,  $m \in \{1, 2\}$  towards the selected relay. Using (1), the signal at the optimal SR after imperfect SI cancellation is

$$\mathbf{y}_r = \sqrt{\alpha_{1r}} \mathbf{H}_{1r} \mathbf{F}_1 \mathbf{s}_1 + \sqrt{\alpha_{2r}} \mathbf{H}_{2r} \mathbf{F}_2 \mathbf{s}_2 + \boldsymbol{\Psi}_r \mathbf{x}_r + \sqrt{\alpha_{pr}} \mathbf{H}_{pr} \mathbf{x}_p + \mathbf{n}_r, \quad (2)$$

where the noise  $\mathbf{n}_r$  is a circularly symmetric complex Gaussian random vector with zero mean and covariance  $\mathbb{E}\{\mathbf{n}_r \mathbf{n}_r^H\} = \sigma_{nr}^2 \mathbf{I}_{N_s}$ . The third term in (2) refers to the RSI due to imperfect SI cancellation as modeled in (1).

(iv) The selected SR precodes the signal received in the previous time slot,  $\mathbf{y}_r^{(t-1)}$ , using the precoding matrix  $\mathbf{F}_r \in \mathbb{C}^{N_r \times N_s}$  and transmits the resulting signal

$$\mathbf{x}_r = \mathbf{F}_r \mathbf{y}_r^{(t-1)}, \quad t \geq 2. \quad (3)$$

(v) Each of the SU receives signals from the selected SR and PU. After canceling its transmitted signal and imperfect loopback SI cancellation using (1), we have

$$\mathbf{y}_m = \sqrt{\alpha_{rm}} \mathbf{H}_{rm} \mathbf{F}_r \left\{ \sqrt{\alpha_{mr}} \mathbf{H}_{mr}^{(t-1)} \mathbf{F}_m^{(t-1)} \mathbf{s}_m^{(t-1)} + \mathbf{n}_r^{(t-1)} \right\} + \boldsymbol{\Psi}_r^{(t-1)} \mathbf{x}_r^{(t-1)} + \boldsymbol{\Psi}_m \mathbf{F}_m \mathbf{s}_m + \sqrt{\alpha_{pm}} \mathbf{H}_{pm} \mathbf{x}_p + \mathbf{n}_m, \quad m = 1, 2, \quad (4)$$

where noise  $\mathbf{n}_m$  is a circularly symmetric complex Gaussian random vector with zero mean and covariance  $\mathbb{E}\{\mathbf{n}_m \mathbf{n}_m^H\} = \sigma_{nm}^2 \mathbf{I}_{N_s}$ . The RSI is represented by the

terms containing  $\Psi_m, \Psi_r$ . The estimate of the data transmitted by the other secondary user,  $\widehat{\mathbf{s}}_m$ , is obtained by applying a receive filter  $\mathbf{W}_m \in \mathbb{C}^{N_s \times N_s}$  to  $\mathbf{y}_m$ ,

(vi) The signals transmitted by the selected SR and the SU cause interference at the PU given as

$$I_p = I_1 + I_2 + I_r, \quad (5)$$

where  $I_k = \alpha_{kp} \text{tr}(\mathbf{H}_{kp} \mathbf{x}_k \mathbf{x}_k^H \mathbf{H}_{kp}^H)$ ,  $k = 1, 2, r$ . (6)

The end-to-end communication requires two time slots.

### III. DESIGN OF OPTIMAL TRANSCIEVER PRECODERS AND RELAY SELECTION

In this section, we design the transceiver precoders  $\mathbf{F}_1, \mathbf{F}_2$  and select the optimal SR, both of which are addressed in the first time slot of communication.

#### A. Transceiver Precoder Design

The aim of designing precoders for SU is: (i) to nullify the effect of the SU – SR channel link, and (ii) to limit the interference to the PU below a threshold. To address the first sub-problem, we do the QR-decomposition of the hermitian of the channel matrix  $\mathbf{H}_{kr}$  as:  $\mathbf{H}_{kr}^H = \mathbf{Q}_k \mathbf{R}_k$ .  $\mathbf{Q}_k$  is orthogonal matrix,  $\mathbf{R}_k$  is upper triangular matrix. Thus, the precoder matrices are given as

$$\mathbf{F}_k = \mu_k \mathbf{Q}_k \mathbf{V}_k, \quad k = 1, 2. \quad (7)$$

and the corresponding transmit power is given as

$$P_k = \mathbb{E}\{\|\mathbf{F}_k \mathbf{s}_k\|^2\} = \mu_k^2 \text{tr}(\mathbf{V}_k \mathbf{V}_k^H), \quad k = 1, 2, \quad (8)$$

where, the scaling factor  $\mu_k$  addresses the second sub-problem. and its derivation is explained in the next subsection. The matrix  $\mathbf{V}_k$  is derived from  $\mathbf{R}_k$  such that

$$\mathbf{H}_{kr} \mathbf{F}_k = \mu_k \mathbf{I}_{N_s}. \quad (9)$$

#### B. Optimal Relay Selection

The cognitive relay network comprises of  $M$  FD AF relays and a central control unit (CU). The CU selects the SR with the highest receive signal-to-noise ratio (SNR) as the optimal SR, such that the total interference from the SU and the selected SR is below the interference tolerance limit,  $\theta$ , of the PU. Apart from the transceiver-relay channel conditions, the two main factors that affect the relay selection process are: (i) the distance of the SR from SU, PU, and (ii) threshold  $\theta$ . The CU has perfect knowledge of CSI for all the  $M$  transceiver-relay channel links. For each such link, the CU selects optimal scaling factors  $\mu_k, k \in \{1, 2\}$  such that they maximize the receive SNR at the corresponding SR while keeping the total interference power,  $I_p$ , at the PU below  $\theta$ . The control unit then selects the SR having the highest SNR and notifies it to SU using the control channel.

For a given interference threshold  $\theta$ , the constraint is

$$I_p \leq \theta, \quad (10)$$

Using (5), (6), (7), (8), (9), we express (10) as

$$\begin{aligned} \theta &\geq \alpha_{1p} \|\mathbf{H}_{1p} \mathbf{x}_1\|^2 + \alpha_{2p} \|\mathbf{H}_{2p} \mathbf{x}_2\|^2 + \alpha_{rp} \|\mathbf{H}_{rp} \mathbf{x}_r\|^2 \\ &= \alpha_{1p} \mu_1^2 d_1 + \alpha_{2p} \mu_2^2 d_2 + \alpha_{rp} p_r \|\mathbf{H}_{rp}\|_{\mathbb{F}}^2, \end{aligned} \quad (11)$$

where,  $d_k = \text{tr}(\mathbf{H}_{kp} \mathbf{Q}_k \mathbf{V}_k \mathbf{V}_k^H \mathbf{Q}_k^H \mathbf{H}_{kp}^H)$ ,  $k \in \{1, 2\}$  and  $\mathbb{E}\{\mathbf{x}_r \mathbf{x}_r^H\} = p_r \mathbf{I}_{N_s}$ , where,  $p_r$  is power transmitted by each antenna of the SR. We prove in next section that the SR always transmits at maximum power, such that

$$p_r = P_r^{max} / N_r = p_r^{max}. \quad (12)$$

As a consequence, equation (11) can be expressed as

$$\begin{aligned} \alpha_{1p} \mu_1^2 d_1 + \alpha_{2p} \mu_2^2 d_2 &\leq \theta - \alpha_{rp} p_r^{max} \|\mathbf{H}_{rp}\|_{\mathbb{F}}^2 \\ &= \theta'. \end{aligned} \quad (13)$$

Since the SR will always transmits full power, we can only optimize the transmit power of the SU to limit  $I_p$ . Hence, we choose SNR at the SR, which is dependent on the transmit powers  $P_1, P_2$ , as the function to be maximized. From (2), (9), the SNR at the  $i^{th}$  SR is

$$\text{SNR}_{R_i} = \frac{\alpha_{1r_i} \mu_{1r_i}^2 N_s + \alpha_{2r_i} \mu_{2r_i}^2 N_s}{\sigma_{nr_i}^2 N_s}, \quad i = 1, 2, \dots, M, \quad (14)$$

where  $\mu_{jr_i}, j \in \{1, 2\}$ , is the scaling factor for  $i^{th}$  SU – SR channel. From (14), we observe that maximizing the SNR at SR is same as maximizing the signal power, for a given noise power. We assume the noise power at all relays to be the same. So, to obtain the optimal  $\mu_1, \mu_2$ , SR, we formulate the optimization problem as

$$\max_{k \in \mathcal{I}, (\mu_{1r_k}^2, \mu_{2r_k}^2) \in \Omega} \alpha_{1r_k} \mu_{1r_k}^2 + \alpha_{2r_k} \mu_{2r_k}^2, \quad (15)$$

where  $\mathcal{I}$  is the set of relay indices. The feasible set  $\Omega$  is defined by the interference threshold and transmit power constraints for SU. This is a joint optimization problem over relay indices and scaling factors. As such it is a mixed-integer program and hard to solve. However, since only one relay will operate at any given time, this optimization can be performed in two steps; first over the scaling factors  $\mu_{1r_k}^2, \mu_{2r_k}^2$ , then over the relay indices as

$$\max_{k \in \{1, 2, \dots, M\}} \max_{(\mu_{1r_k}^2, \mu_{2r_k}^2) \in \Omega} \alpha_{1r_k} \mu_{1r_k}^2 + \alpha_{2r_k} \mu_{2r_k}^2 \quad (16)$$

We begin with the inner optimization problem given as

$$\begin{aligned} \max_{\mu_{1r_k}^2, \mu_{2r_k}^2} &\alpha_{1r_k} \mu_{1r_k}^2 + \alpha_{2r_k} \mu_{2r_k}^2 \\ \text{subject to} &\alpha_{1p} \mu_{1r_k}^2 d_1 + \alpha_{2p} \mu_{2r_k}^2 d_2 \leq \theta' \end{aligned} \quad (17)$$

$$P_m^{min} \leq P_m \leq P_m^{max}, \quad m = 1, 2.$$

The optimization problem in (17) is a linear optimization problem in  $\mu_{1r_k}^2, \mu_{2r_k}^2$  and can be solved by a optimization tool such as *linprog* in MATLAB. The optimal values thus obtained are  $\mu_{1r_k}^{2*}, \mu_{2r_k}^{2*}$ .

We now proceed to the problem of optimal SR selection. The optimization problem can be formulated as

$$k^* = \underset{k \in \mathcal{I}}{\text{argmax}} \alpha_{1r_k} \mu_{1r_k}^{2*} + \alpha_{2r_k} \mu_{2r_k}^{2*}, \quad (18)$$

where the function to be maximized is the signal power, with optimal scaling factors  $\mu_{1r_k}^{2*}, \mu_{2r_k}^{2*}$ . Denoting the scaling factors corresponding to the optimal SR as  $\mu_1^*, \mu_2^*$ , the information conveyed to the SU is  $(k^*, \mu_1^*, \mu_2^*)$ . The SU then transmit their data to the optimal SR in the first time slot itself using the data channel. The corresponding optimal precoder matrices, transmit powers are given by (7), (8), respectively.

#### IV. DESIGN OF SECONDARY RELAY PRECODER AND TRANSCEIVER RECEIVE FILTERS

In the second time slot of communication, the selected SR precodes the signal received from the SU with  $\mathbf{F}_r$  and forwards it, from which the SU extract the required data using receive filters  $\mathbf{W}_1, \mathbf{W}_2$ . Henceforth, we will denote the optimal scaling factors as simply  $\mu_1, \mu_2$ .

Due to the FD mode of operation, the AF relay receives its own signal as RSI, represented by  $\Psi_r \mathbf{x}_r$  in (2). This RSI also gets propagated to the SU as  $\Psi_r^{(t-1)} \mathbf{x}_r^{(t-1)}$  in (4) and keeps accumulating over time. To mitigate it, we design precoder  $\mathbf{F}_r$  and receive filters  $\mathbf{W}_1, \mathbf{W}_2$  by minimizing the SMSE of end-to-end communication, with a constraint on the transmit power of SR. To obtain closed form solution, matrix  $\mathbf{F}_r$  is decomposed as  $\mathbf{F}_r = \beta \bar{\mathbf{F}}_r$ , where  $\beta$  is a positive scaling factor and  $\|\bar{\mathbf{F}}_r\|_{\mathbb{F}} = 1$ . For the same reason, term  $\beta^{-1}$  is introduced in (20). Thus, the SMSE =  $f(\beta, \bar{\mathbf{F}}_r, \mathbf{W}_1, \mathbf{W}_2)$ . The following optimization problem gives the optimal design:

$$\begin{aligned} \min_{\beta, \bar{\mathbf{F}}_r, \mathbf{W}_1, \mathbf{W}_2} \quad & f(\beta, \bar{\mathbf{F}}_r, \mathbf{W}_1, \mathbf{W}_2) \\ \text{subject to} \quad & \mathbb{E}\{\|\mathbf{x}_r\|^2\} \leq P_r^{max}. \end{aligned} \quad (19)$$

For convenience we'll denote  $(\alpha_{1r}\mu_1^{2(j)} + \alpha_{2r}\mu_2^{2(j)} + \sigma_{nr}^2)\mathbf{I}_{N_s} + \alpha_{pr}p_p^{(j)}\mathbf{H}_{pr}^{(j)}\mathbf{H}_{pr}^{(j)H}$  by  $\Phi^{(j)}$  throughout this paper, where  $p_p$  is the per antenna transmit power at the PU. The SMSE of the two transceivers is given by

$$\begin{aligned} f(\beta, \bar{\mathbf{F}}_r, \mathbf{W}_1, \mathbf{W}_2) &= \sum_{i=1}^2 \mathbb{E}\{\|\mathbf{s}_i^{(t-1)} - \beta^{-1}\hat{\mathbf{s}}_i\|^2\} \\ &= \sum_{i=1}^2 \mathbb{E}\{\|\mathbf{s}_i^{(t-1)}\|^2\} + \beta^{-2}\mathbb{E}\{\|\mathbf{W}_i^H \mathbf{y}_i\|^2\} - \\ &\quad \beta^{-1}[\text{tr}(\mathbb{E}\{\mathbf{W}_i^H \mathbf{y}_i \mathbf{s}_i^{(t-1)H}\}) + \text{tr}(\mathbb{E}\{\mathbf{s}_i^{(t-1)} \mathbf{y}_i^H \mathbf{W}_i\})]. \end{aligned} \quad (20)$$

We now express each term of SMSE equation in terms of the relevant optimization variables. Thus,

$$\begin{aligned} \mathbb{E}\{\mathbf{s}_i^{(t-1)} \mathbf{y}_i^H \mathbf{W}_i\} &= \mu_i^{(t-1)} \sqrt{\alpha_{ir}\alpha_{ir}} \text{tr}(\mathbf{F}_r^H \mathbf{H}_{ri}^H \mathbf{W}_i), \\ \mathbb{E}\{\|\mathbf{s}_i^{(t-1)}\|^2\} &= N_s. \end{aligned} \quad (21)$$

Further, using (4) and (9), we have

$$\begin{aligned} \mathbb{E}\{\|\mathbf{W}_i^H \mathbf{y}_i\|^2\} &= \alpha_{ir} \text{tr}[\mathbf{W}_i^H \mathbf{H}_{ri} \mathbf{F}_r (\mathbb{E}\{\Lambda_0\} + \alpha_{ir}\mu_i^{(t-1)^2} \mathbf{I}_{N_s} + \sigma_{nr}^2 \mathbf{I}_{N_s} \\ &\quad + \alpha_{pr}p_p^{(t-1)} \mathbf{H}_{pr}^{(t-1)} \mathbf{H}_{pr}^{(t-1)H}) \mathbf{F}_r^H \mathbf{H}_{ri}^H \mathbf{W}_i] + \sigma_{ni}^2 \times \\ &\quad \text{tr}(\mathbf{W}_i \mathbf{W}_i^H) + \mathbb{E}\{\|\mathbf{W}_i^H \Psi_i \mathbf{F}_i \mathbf{s}_i\|_{\mathbb{F}}^2\} + \\ &\quad \alpha_{pi}p_p \|\mathbf{W}_i^H \mathbf{H}_{pi}\|_{\mathbb{F}}^2, \quad i = 1, 2, \end{aligned} \quad (22)$$

where,

$$\Lambda_0 = \sum_{k=0}^{t-2} \prod_{l=1}^{t-k-1} \Psi_r^{(t-l)} \mathbf{F}_r^{(t-l)} \Phi^{(k)} \prod_{l=1}^{t-k-1} \mathbf{F}_r^{(k+l)H} \Psi_r^{(k+l)H} \quad (23)$$

for  $t \geq 3$  and  $\mathbf{0}_{N_s}$  else. We assume that  $\prod_{k=a}^b (\cdot) = 1$ , if  $b < a$ . Using Lemma 1 from [9] and proceeding as in Theorem 1 from [4], we obtain  $\mathbb{E}\{\Lambda_0\}$  as

$$\begin{aligned} \Lambda_f &= \mathbb{E}\{\Lambda_0\} = \sum_{k=1}^{t-1} (\sigma_{er}^2)^k \text{tr}(\mathbf{F}_r^{(t-k)} \Phi^{(t-k-1)} \mathbf{F}_r^{(t-k)H}) \\ &\quad \times \mathbf{I}_{N_s} \prod_{l=1}^{k-1} \text{tr}(\mathbf{F}_r^{(t-l)} \mathbf{F}_r^{(t-l)H}), \end{aligned} \quad (24)$$

The term  $\Lambda_f$  represents the contribution of all relay precoders,  $t = 2$  onwards, required to suppress the SI. Since, the first  $\mathbf{F}_r$  is computed at  $t = 2$ , its value is  $\mathbf{0}_{N_s}$  for  $t = 1, 2$ . After few algebraic manipulations on (24),  $\Lambda_f$  can be recursively computed as

$$\begin{aligned} \Lambda_f &= \sigma_{er}^2 [\text{tr}(\mathbf{F}_r^{(t-1)} \Phi^{(t-2)} \mathbf{F}_r^{(t-1)H}) \mathbf{I}_{N_s} \\ &\quad + \Lambda_f^{(t-1)} \text{tr}(\mathbf{F}_r^{(t-1)} \mathbf{F}_r^{(t-1)H})]. \end{aligned} \quad (25)$$

Due to the recursive structure of  $\Lambda_f$ , the nodes need not store all the previous relay precoder matrices, but only  $\mathbf{F}_r^{(t-1)}$  to compute  $\mathbf{F}_r$ . This greatly reduces the memory requirement at the relay and also results in low complexity and reduced computation time for  $\Lambda_f$ . Moreover, the precoder designed using (25) will lead to better performance than that proposed in [4] where only the  $n$  latest time slots are used for computing  $\Lambda_f$ .

Using Lemma 1 from [9] and (8), we can express

$$\mathbb{E}\{\|\mathbf{W}_i^H \Psi_i \mathbf{F}_i \mathbf{s}_i\|_{\mathbb{F}}^2\} = \sigma_{ei}^2 P_i \text{tr}(\mathbf{W}_i \mathbf{W}_i^H). \quad (26)$$

Using (2), (3), (9), we express relay's transmit power as  $\mathbb{E}\{\|\mathbf{x}_r\|^2\} = \text{tr}[\mathbf{F}_r (\Lambda_f + \Phi^{(t-2)}) \mathbf{F}_r^H] = \text{tr}(\mathbf{F}_r \mathbf{B}_r \mathbf{F}_r^H)$ , (27)

Having expressed the terms of SMSE and relay transmit power in terms of the optimization variables, we now turn to the solution of the problem in (19). The Lagrangian corresponding to this problem is given by

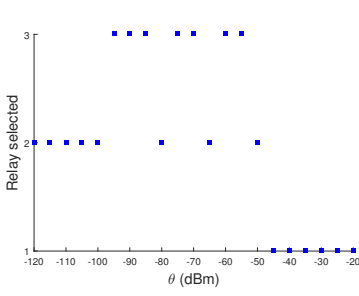
$$\begin{aligned} \mathcal{L}(\beta, \bar{\mathbf{F}}_r, \mathbf{W}_1, \mathbf{W}_2, \lambda) &= f(\beta, \bar{\mathbf{F}}_r, \mathbf{W}_1, \mathbf{W}_2) + \lambda [q^2 + \\ &\quad \mathbb{E}\{\|\mathbf{x}_r\|^2\} - P_r^{max}], \end{aligned} \quad (28)$$

where  $\lambda$  is the lagrangian variable and  $q$  is the slack variable. On substituting the results of (20), (21), (22), (27), (24), (26) in (28) and putting  $\mathbf{F}_r = \beta \bar{\mathbf{F}}_r$ , we get

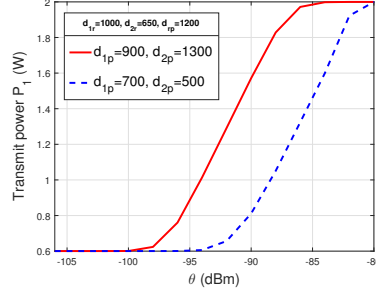
$$\begin{aligned} \mathcal{L} &= \sum_{i=1}^2 \{N_s + \beta^{-2}(\sigma_{ni}^2 + \sigma_{ei}^2 P_i) \text{tr}(\mathbf{W}_i \mathbf{W}_i^H) - \mu_i^{(t-1)} \\ &\quad \times \sqrt{\alpha_{ir}\alpha_{ir}} \text{tr}(\mathbf{W}_i^H \mathbf{H}_{ri} \bar{\mathbf{F}}_r + \bar{\mathbf{F}}_r^H \mathbf{H}_{ri}^H \mathbf{W}_i) + \alpha_{pi}p_p \times \\ &\quad \|\mathbf{W}_i^H \mathbf{H}_{pi}\|_{\mathbb{F}}^2 + \alpha_{ir} \text{tr}(\mathbf{W}_i^H \mathbf{H}_{ri} \bar{\mathbf{F}}_r \mathbf{B}_i \bar{\mathbf{F}}_r^H \mathbf{H}_{ri}^H \mathbf{W}_i)\} \\ &\quad + \lambda [q^2 + \beta^2 \text{tr}(\bar{\mathbf{F}}_r \mathbf{B}_r \bar{\mathbf{F}}_r^H) - P_r^{max}], \end{aligned} \quad (29)$$

where  $\mathbf{B}_i = \Lambda_f + (\alpha_{ir}\mu_i^{(t-1)^2} + \sigma_{nr}^2)\mathbf{I}_{N_s} + \alpha_{pr}p_p^{(t-1)}\mathbf{H}_{pr}^{(t-1)}\mathbf{H}_{pr}^{(t-1)H}$ .

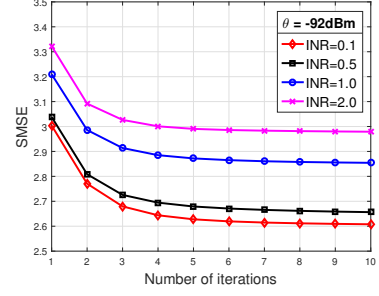
The optimization problem in (29) can be solved using the Karush-Kuhn-Tucker (KKT) conditions [10]. Since the SMSE function is not jointly convex in the optimization variables, we use the coordinate descent method.



(a) Optimal relay selection for varying  $\theta$ .



(b) Variation of transmit power versus  $\theta$ .



(c) Convergence of SMSE at each time slot

Fig. 2

Thus, the optimal values of  $\mathbf{F}_r, \mathbf{W}_i, i \in \{1, 2\}$ , are obtained iteratively. First, keeping  $\mathbf{W}_i$  fixed, we apply the KKT conditions  $\frac{\partial \mathcal{L}}{\partial \beta} = 0, \frac{\partial \mathcal{L}}{\partial \bar{\mathbf{F}}_r^c} = \mathbf{0}_{N_r \times N_s}, \frac{\partial \mathcal{L}}{\partial \lambda} = 0, \frac{\partial \mathcal{L}}{\partial z} = 0$  to respectively get

$$\Rightarrow \lambda \beta \text{tr}(\bar{\mathbf{F}}_r \mathbf{B}_r \bar{\mathbf{F}}_r^H) = \beta^{-3} (c_1 + c_2), \quad (30)$$

$$\begin{aligned} \Rightarrow & \sqrt{\alpha_{r1} \alpha_{r2}} (\mu_1^{(t-1)} \mathbf{H}_{r2}^H \mathbf{W}_2 + \mu_2^{(t-1)} \mathbf{H}_{r1}^H \mathbf{W}_1) \\ & = \lambda \beta^2 \bar{\mathbf{F}}_r \mathbf{B}_r + \alpha_{r1} \mathbf{H}_{r1}^H \mathbf{W}_1 \mathbf{W}_1^H \mathbf{H}_{r1} \bar{\mathbf{F}}_r \mathbf{B}_1 \\ & \quad + \alpha_{r2} \mathbf{H}_{r2}^H \mathbf{W}_2 \mathbf{W}_2^H \mathbf{H}_{r2} \bar{\mathbf{F}}_r \mathbf{B}_2. \end{aligned} \quad (31)$$

$$\Rightarrow \text{tr}(\bar{\mathbf{F}}_r \mathbf{B}_r \bar{\mathbf{F}}_r^H) = \beta^{-2} (P_r^{max} - q^2). \quad (32)$$

$$\Rightarrow \lambda [\beta^2 \text{tr}(\bar{\mathbf{F}}_r \mathbf{B}_r \bar{\mathbf{F}}_r^H) - P_r^{max}] = 0. \quad (33)$$

where  $c_i = (\sigma_{ni}^2 + \sigma_{ei}^2 P_i) \text{tr}(\mathbf{W}_i \mathbf{W}_i^H)$ .

From (30), (33), we observe that at the optimal point,  $\lambda \neq 0$ . So, the constraint in (19) becomes equality, i.e., the SR transmits at full power  $P_r^{max}$ . So, (32) becomes:

$$\text{tr}(\bar{\mathbf{F}}_r \mathbf{B}_r \bar{\mathbf{F}}_r^H) = \beta^{-2} (P_r^{max}). \quad (34)$$

From (30), (34), we get:  $\lambda \beta^2 = \frac{(c_1 + c_2)}{P_r^{max}}$ . Substituting this in (31) and following Theorem 2<sup>r</sup> of [4], we get

$$\begin{aligned} \bar{\mathbf{F}}_r^* &= \text{mat} \left( \left[ \sum_{k=1}^2 (\mathbf{B}_k^T \otimes \alpha_{rk} \mathbf{H}_{rk}^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{H}_{rk}) \right] + (\mathbf{B}_r^T \otimes \right. \\ & \left. \frac{(c_1 + c_2)}{P_r^{max}} \mathbf{I}_{N_r}) \right]^{-1} \sqrt{\alpha_{r1} \alpha_{r2}} \text{vec} \left[ \sum_{k=1}^2 \mu_k^{(t-1)} \mathbf{H}_{rk}^H \mathbf{W}_k \right], \\ \beta^* &= \sqrt{\frac{P_r^{max}}{\text{tr}(\bar{\mathbf{F}}_r^* \mathbf{B}_r \bar{\mathbf{F}}_r^{*H})}}. \end{aligned}$$

Therefore, optimal relay precoder is given by  $\mathbf{F}_r^* = \beta^* \bar{\mathbf{F}}_r^*$ . Now, using the optimal  $\mathbf{F}_r^*$ , we compute optimal receive filters by applying the KKT condition:  $\frac{\partial \mathcal{L}}{\partial \mathbf{W}_i^c} = \mathbf{0}_{N_s}, i \in \{1, 2\}$ , which gives the optimal receive filter as

$$\begin{aligned} \mathbf{W}_i^* &= [c_i \mathbf{I}_{N_s} + \alpha_{ri} \mathbf{H}_{ri} \mathbf{F}_r^* \mathbf{B}_i \mathbf{F}_r^{*H} \mathbf{H}_{ri}^H]^{-1} \times \\ & \quad \beta^* \sqrt{\alpha_{r1} \alpha_{r2}} \mu_i^{(t-1)} \mathbf{H}_{ri} \mathbf{F}_r^*. \end{aligned}$$

It must be noted that the optimal matrices  $\mathbf{F}_r^*, \mathbf{W}_i^*, i \in \{1, 2\}$  are obtained by iteratively computing each other's filter until the SMSE converges. The initial value of  $\mathbf{W}_i$  for computing  $\mathbf{F}_r$  can be any random  $N_s \times N_s$  matrix.

The sum-rate for the designed system is

$$R = \sum_{m=1}^2 \mathbb{E} \{ \log_2 [\mathbf{I}_{N_s} + \mathbf{SINR}_m] \} \quad (\text{bits/sec/Hz}),$$

where,

$$\begin{aligned} \mathbf{SINR}_m &= [\alpha_{1r} \alpha_{2r} \mu_m^{(t-1)2} \mathbf{W}_m^H \mathbf{H}_{rm} \mathbf{F}_r \mathbf{F}_r^H \mathbf{H}_{rm}^H \mathbf{W}_m] \times \\ & \quad [\mathbf{W}_m^H (\sigma_{nm}^2 \mathbf{I}_{N_s} + \alpha_{mr} \sigma_{nr}^2 \mathbf{H}_{rm} \mathbf{F}_r \mathbf{F}_r^H \mathbf{H}_{rm}^H + \\ & \quad \mu_m^2 \Psi_m \mathbf{Q}_m \mathbf{V}_m \mathbf{V}_m^H \mathbf{Q}_m^H \Psi_m + \alpha_{1p} \rho_p \mathbf{H}_{p1} \times \\ & \quad \mathbf{H}_{p1}^H + \alpha_{1r} \alpha_{pr} \rho_p \mathbf{H}_{rm} \mathbf{F}_r \mathbf{H}_{rp} \mathbf{H}_{rp}^H \mathbf{F}_r^H \mathbf{H}_{rm}^H + \\ & \quad \alpha_{mr} \mathbf{H}_{rm} \mathbf{F}_r \mathbf{A}_z \mathbf{F}_r^H \mathbf{H}_{rm}^H) \mathbf{W}_m]^{-1}, m = 1, 2, \end{aligned}$$

with,

$$\mathbf{A}_z = \Psi_r^{(t-1)} \mathbf{F}_r^{(t-1)} (\Phi^{(t-2)} + \mathbf{A}_z^{(t-1)}) \mathbf{F}_r^{(t-1)H} \Psi_r^{(t-1)H}.$$

Similar to  $\mathbf{A}_f, \mathbf{A}_z$  is  $\mathbf{0}_{N_s}$  for  $t < 3$ .

## V. SIMULATION RESULTS

We consider the following parameters:  $N_s = 2, N_r = 4, M = 3$  relays,  $p_r^{max} = 1, P_k^{min} = 0.6, P_k^{max} = 2, k \in \{1, 2\}$ . We consider the path loss as defined by the 3GPP LTE for outdoor macro cells [11], viz.,

$$\alpha = 15.3 + 37.6 \log_{10}(d_{ij}) \text{ dB}, \quad (35)$$

where  $d_{ij}$  is the distance between nodes  $i$  and  $j$ , in metres, at 2 GHz frequency. Unless otherwise specified, consider the separation, in metres, between the nodes to be as follows:  $d_{1R_1} = 800, d_{1R_2} = 1600, d_{1R_3} = 1000, d_{1p} = 1400, d_{2R_1} = 900, d_{2R_2} = 750, d_{2R_3} = 650, d_{2p} = 1400, d_{pR_1} = 850, d_{pR_2} = 700, d_{pR_3} = 1200$ . The distances were selected randomly. The corresponding path loss is given by (35). We assume all the channel matrices and channel estimation error matrices to follow Rayleigh fading and their elements to be independent and identically distributed complex Gaussian random variables, each with zero mean and unit variance. We averaged the results of around 1000 Monte Carlo simulations to arrive at each of the following results.

Fig. 2(a) shows the result for simulating optimal SR selection process for the aforementioned separation values. It shows the relay selected for varying values of interference threshold  $\theta$ . We observe three distinct range of  $\theta$  values for which a particular relay is selected. For example, relay 1 is selected for  $\theta = -45$  dBm or higher.

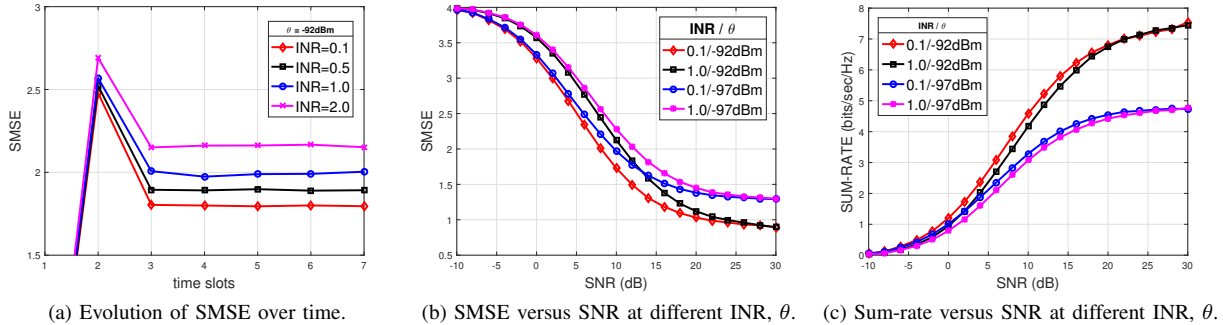


Fig. 3

Fig. 2(a) clearly signifies the effect of  $\theta$  and separation between nodes on optimal SR selection process.

Fig. 2(b) shows the variation in transmit power of the transceiver 1 versus  $\theta$  for two sets of separation values of  $d_{1p}$ ,  $d_{2p}$ , which correspond to path loss given by (35). As seen, the transmit power of transceiver varies from  $P_1^{\min}$  to  $P_1^{\max}$  as  $\theta$  varies from  $-105\text{dBm}$  to  $-80\text{dBm}$ . As expected, the transceiver transmits more power when the interference tolerance limit of the PU increases.

The practical feasibility of the proposed algorithm for  $\mathbf{F}_r$  and  $\mathbf{W}_m$ ,  $m \in \{1, 2\}$ , design is depicted by the result in Fig. 2(c). It shows the number of iterations required to obtain the optimal value of  $\mathbf{F}_r$ ,  $\mathbf{W}_m$ , at  $\text{SNR} = 5\text{dB}$ , for varying INR and  $\theta = -92\text{dBm}$ . The optimal matrices are obtained when the SMSE converges. We observe that beginning with any random  $2 \times 2$  matrix  $\mathbf{W}_m$ , the SMSE converges after 6 iterations for all the INR values.

Fig. 3(a) exhibits the significance of the effect of feedback term  $\mathbf{\Lambda}_f$ , on the performance of the system over time. It shows the variation of SMSE, for different INR values, with  $\theta = -92\text{dBm}$ . As seen, the SMSE begins to stabilize from  $3^{\text{th}}$  time slot due to the effect of feedback term  $\mathbf{\Lambda}_f$ , which starts from  $t = 3$ .

Fig. 3(b) and Fig. 3(c) illustrate the SMSE and sum-rate performance of the designed system, respectively versus SNR for different INR and  $\theta$ . The proposed designs demonstrate good performance at low INR with the performance degrading slightly as INR increases. This signifies the need for precoding with multiple antennas to suppress RSI. Also, it can be observed from these figures that the performance at high SNR is bounded by the interference threshold  $\theta$ . This is because  $\theta$  controls the transmit power of transceivers as seen in Fig. 2(b).

## VI. CONCLUSION

In this paper, we considered a cognitive full-duplex two-way relaying network with multiple relays. We presented an optimal relay selection scheme based on SNR maximization at the relay, while limiting the interference to the primary user. The precoders at the secondary transceivers were designed to nullify the effect of the transceiver-relay channel link. To account for the perpetuating residual self-interference caused by AF operation

of the FD relay, we proposed the design of optimal relay precoder and transceiver receive filters. These matrices were obtained by SMSE minimization of end-to-end communication. An iterative technique having low computational complexity, low memory requirement for computing the feedback term was presented. The simulation results validated the practical feasibility of the proposed algorithms. The results also verified that the SMSE and sum-rate performance of the system are capped by the interference threshold of the primary user.

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