The Role of Uncertainty in Adaptive Control of Switched Euler-Lagrange Systems

by

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Report No: IIIT/TR/2019/-1

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November 2019

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Abstract— **This work presents a Lyapunov-based approach to adaptive control of uncertain Euler-Lagrange (EL) systems in a slow switching scenario. Fundamental trade-offs arising from considering uncertain dynamics with unknown uncertainty bounds are presented and discussed. Contrary to the non-switched scenario, the use of acceleration feedback seems to be unavoidable in the switched scenario: this is due to the fact that an acceleration feedback and an appropriate Lyapunov function must be adopted to make the switching law independent from the unknown uncertainty bounds. In the absence of such feedback or using different Lyapunov functions, a stabilizing switching law would exist but could not be determined as it would depend on an unknown uncertainty bound.**

I. Introduction

Switched systems are an important class of hybrid systems that consists of subsystems with continuous dynamics, where at any given time instant one subsystem remains active while others being inactive. Switching between active and inactive subsystems is governed by a switching law [1]. With applications in various domains ranging from networked control systems, intelligent transportation systems, power electronics to robotics, the switched systems framework has been very attractive to the control systems community over the past decades [2]–[7] (see also references therein).

In the quest to operate switched systems under various unknown scenarios and to adapt the control law so as to cope with various sources of parametric uncertainty, some notable adaptive control works are reported for linear ([8]–[11]) as well as nonlinear switched systems ([12]–[17]). However, only a few of the aforementioned control designs can tackle the unavoidable unmodelled dynamics and external disturbances (i.e., provide a robust adaptation in the sense of [18]). These works include [10] for time-driven slow switching (e.g., dwell time or average dwell-time switching via multiple Lyapunov functions) and [17] for arbitrary switching (by assuming a common Lyapunov function). Slow switching is typically more relevant because, as observed by many researchers (e.g., [13], [15]), obtaining a common Lyapunov function and a common robust adaptive law to attain arbitrary switching can be difficult and often not possible.

Switched Euler-Lagrange (EL) systems is an extension to the standard EL systems that arises naturally in many electromechanical systems [1], [19]. Therefore, a relevant question arises: to which extent is the state-of-the-art adaptive control for switched systems applicable to switched EL systems? In this regard, the following observations are made:

- Most state-of-the-art adaptive control designs, irrespective of considering switched or non-switched dynamics, rely upon the system dynamics to be linear in parameters (LIP) [12]–[14], [20]–[25]. Also, LIP-based designs built upon the state-space model of the system [12]–[14] cannot be applied to a state-space model of EL system, which is always nonlinear in parameters (NLIP) due to inversion of the mass matrix.
- The designs capable to handle NLIP dynamics as in [15], [16] demand detailed structural knowledge of the system in order to appropriately select the regressor terms in the adaptive law.
- Stabilizing slow switching laws are designed based on multiple Lyapunov functions, exploiting the condition of exponential decrease in between switching instants and bounded jumps at switching instants. It turns out that, in the presence of uncertain dynamics, not all Lyapunov-based arguments proposed for non-switched EL systems (cf. [20]–[26] and references therein) can be directly extended to the slowly switching scenario (cf. Remark 4). Fundamental trade-offs arise from considering uncertain dynamics with unknown uncertainty bounds.

In light of the above discussions and to the best of the authors' knowledge, an adaptive strategy for switched EL systems is to a large extent is missing. Toward this direction, we propose an adaptive solution with the following major contributions:

- NLIP structure can be handled without requiring structural knowledge of the system in order to appropriately select the regressor terms in the adaptive law;
- The trade-offs when designing the stabilizing slow switching law in the presence of uncertainty are highlighted. Contrary to the non-switched scenario, the use of acceleration feedback seems to be unavoidable in the switched scenario: this is due to the fact that an acceleration feedback and an appropriate Lyapunov function must be adopted to make the dwell-time switching law independent from unknown uncertainty bounds. In the

This work has been partially supported by the Fundamental Research Funds for the Central Universities grant no. 4007019109 (RECONSTRUCT) and by the special guiding funds for "double first-class" grant no. 4007019201 (corresponding author: S. Baldi).

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absence of such feedback or using different Lyapunov functions, a stabilizing switching law would exist but could not be determined as it would depend on an unknown uncertainty bound (cf. Corollary 1).

The rest of the paper is organized as follows: Section II describes the class of EL systems and highlights various issues in the state of the art; Section III details the proposed control framework, while the corresponding stability analysis is carried out in Section IV; a simulation study is provided in Section V, while Section VI presents the concluding remarks.

The following notations are used throughout the paper: $\lambda_{\min}(\bullet)$ and $||\bullet||$ represent minimum eigenvalue and Euclidean norm of $(\bullet)$ respectively; $\mathbf{I}$ denotes identity matrix with appropriate dimension.

II. System Dynamics and Problem Formulation

Consider the following switched EL dynamics

$$\mathbf{M}_\sigma(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_\sigma(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}_\sigma(\mathbf{q}) + \mathbf{F}_\sigma(\dot{\mathbf{q}}) + \mathbf{d}_\sigma = \boldsymbol{\tau}_\sigma, \quad (1)$$

where $\mathbf{q},\dot{\mathbf{q}} \in \mathbb{R}^n$ are the system states (usually termed as position and velocity); for each subsystem σ, $\mathbf{M}_\sigma(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is the mass/inertia matrix; $\mathbf{C}_\sigma(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^{n\times n}$ denotes the Coriolis, centripetal terms; $\mathbf{G}_\sigma(\mathbf{q}) \in \mathbb{R}^n$ denotes the gravity vector; $\mathbf{F}_\sigma(\dot{\mathbf{q}}) \in \mathbb{R}^n$ represents the vector of damping and friction forces; $\mathbf{d}_\sigma(t) \in \mathbb{R}^n$ denotes bounded external disturbance and $\boldsymbol{\tau}_\sigma \in \mathbb{R}^n$ is the generalized control input.

Here $\sigma(t) : [0\ \infty) \mapsto \Omega$ is a piecewise constant function of time, called the switching signal, taking values in $\Omega = \{1, 2, \cdots, N\}$. The following class of slowly-switching signals is considered:

Definition 1: Average Dwell Time (ADT) [27]: For a switching signal $\sigma(t)$ and each $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_1,t_2)$ denote the number of discontinuities in the interval $[t_1,t_2)$. Then $\sigma(t)$ has an average dwell time ϑ if

$$N_\sigma(t_1,t_2) \leq N_0 + (t_2 - t_1)/\vartheta, \quad \forall t_2 \geq t_1 \geq 0$$

where $N_0 > 0$ is a given scalar termed as chatter bound.

For most EL systems of practical interest, each subsystem in (1) presents a few interesting properties (cf. [19]), which are later exploited for control design and stability analysis:

Property 1: $\exists \overline{c}_\sigma, \overline{g}_\sigma, \overline{f}_\sigma, \overline{d}_\sigma \in \mathbb{R}^+$ such that $||\mathbf{C}_\sigma(\mathbf{q},\dot{\mathbf{q}})|| \leq \overline{c}_\sigma||\dot{\mathbf{q}}||$, $||\mathbf{G}_\sigma(\mathbf{q})|| \leq \overline{g}_\sigma$, $||\mathbf{F}_\sigma(\dot{\mathbf{q}})|| \leq \overline{f}_\sigma||\dot{\mathbf{q}}||$ and $||\mathbf{d}_\sigma(t)|| \leq \overline{d}_\sigma$.

Property 2: The matrix $\mathbf{M}_\sigma(\mathbf{q})$ is symmetric and uniformly positive definite $\forall \mathbf{q}$, implying that $\exists \underline{m}_\sigma, \overline{m}_\sigma \in \mathbb{R}^+$ such that

$$0 < \underline{m}_\sigma\mathbf{I} \leq \mathbf{M}_\sigma(\mathbf{q}) \leq \overline{m}_\sigma\mathbf{I}. \quad (2)$$

The following remarks define the available knowledge regarding system (1) to design the proposed switching control.

***Remark** 1 (Uncertainty):* As a design challenge, the switched system (1) is considered to be unknown in the sense that $\mathbf{M}_\sigma, \mathbf{C}_\sigma, \mathbf{F}_\sigma, \mathbf{G}_\sigma, \mathbf{d}_\sigma$ and their corresponding bounds, i.e., $\underline{m}_\sigma, \overline{m}_\sigma, \overline{c}_\sigma, \overline{g}_\sigma, \overline{f}_\sigma, \overline{d}_\sigma$ are *completely unknown*.

***Remark** 2 (No structural knowledge):* This work, in contrast to the conventional adaptive designs [12]–[16], [20]–[25], utilizes Properties 1 and 2 which do not require LIP structure and furthermore, do not impose structural knowledge of the system in order to appropriately select the regressor terms in the adaptive law.

Problem: Under Properties 1 and 2, the control problem is to design a switched adaptive control framework for (1): (i) without any knowledge of system parameters (in line with Remark 1); (ii) avoiding any need of structural knowledge of system (as outlined in Remark 2).

The following section gives a positive answer to this problem.

III. Switched Controller Design

Let us consider the tracking problem such that [24], [28]:

***Assumption** 1:* The desired trajectories satisfy $\mathbf{q}^d, \dot{\mathbf{q}}^d, \ddot{\mathbf{q}}^d \in \mathcal{L}_\infty$. Furthermore, $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are available for feedback.

For control design purposes, the dynamics (1) is re-arranged as

$$\mathbf{D}_\sigma\ddot{\mathbf{q}} + \mathbf{E}_\sigma(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}},t) = \boldsymbol{\tau}_\sigma, \quad (3)$$

which has been obtained by adding and subtracting $\mathbf{D}_\sigma\ddot{\mathbf{q}}$ to (1), where $\mathbf{D}_\sigma$ is a user-defined constant positive definite matrix and $\mathbf{E}_\sigma \triangleq (\mathbf{M}_\sigma - \mathbf{D}_\sigma)\ddot{\mathbf{q}} + \mathbf{C}_\sigma\dot{\mathbf{q}} + \mathbf{G}_\sigma + \mathbf{F}_\sigma + \mathbf{d}_\sigma$. The selection of $\mathbf{D}_\sigma$ will be discussed later (cf. Remark 7).

Let $\mathbf{e}(t) \triangleq \mathbf{q}(t) - \mathbf{q}^d(t)$ be the tracking error, $\boldsymbol{\xi}(t) \triangleq [\mathbf{e}(t),\ \dot{\mathbf{e}}(t)]$ and $\mathbf{r}_\sigma$ be the filtered tracking error variable defined as

$$\mathbf{r}_\sigma \triangleq \mathbf{B}^T\mathbf{P}_\sigma\boldsymbol{\xi}, \qquad \sigma \in \Omega \quad (4)$$

where $\mathbf{P}_\sigma > \mathbf{0}$ is the solution to the Lyapunov equation $\mathbf{A}_\sigma^T\mathbf{P}_\sigma + \mathbf{P}_\sigma\mathbf{A}_\sigma = -\mathbf{Q}_\sigma$ for some $\mathbf{Q}_\sigma > \mathbf{0}$, $\mathbf{A}_\sigma \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}_{1\sigma} & -\mathbf{K}_{2\sigma} \end{bmatrix}$ and $\mathbf{B} \triangleq \begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}^T$. Here, $\mathbf{K}_{1\sigma}$ and $\mathbf{K}_{2\sigma}$ are two user-defined positive definite gain matrices and their positive definiteness guarantees that $\mathbf{A}_\sigma$ is Hurwitz.

The control law is designed as

$$\boldsymbol{\tau}_\sigma = \mathbf{D}_\sigma(-\boldsymbol{\Lambda}_\sigma\boldsymbol{\xi} - \Delta\boldsymbol{\tau}_\sigma + \ddot{\mathbf{q}}^d), \quad (5a)$$

$$\Delta\boldsymbol{\tau}_\sigma = \begin{cases} \rho_\sigma \frac{\mathbf{r}_\sigma}{||\mathbf{r}_\sigma||} & \text{if } ||\mathbf{r}_\sigma|| \geq \varpi \\ \rho_\sigma \frac{\mathbf{r}_\sigma}{\varpi} & \text{if } ||\mathbf{r}_\sigma|| < \varpi \end{cases}, \quad (5b)$$

where $\boldsymbol{\Lambda}_\sigma \triangleq [\mathbf{K}_{1\sigma}\ \mathbf{K}_{2\sigma}]$ and the design of ρ_σ will be discussed later. Substituting (5a) in (3) yields

$$\ddot{\mathbf{e}} = -\boldsymbol{\Lambda}_\sigma\boldsymbol{\xi} - \Delta\boldsymbol{\tau}_\sigma + \boldsymbol{\Psi}_\sigma, \quad (6)$$

where $\boldsymbol{\Psi}_\sigma \triangleq -\mathbf{D}_\sigma^{-1}\mathbf{E}_\sigma$ is defined as the *overall uncertainty*. Using Properties 1, 2 and Assumption 1, one can verify that $\exists \theta_{i\sigma}^* \in \mathbb{R}^+\ i = 0, \cdots, 3\ \forall \sigma \in \Omega$ (cf. [29] for details)

$$||\boldsymbol{\Psi}_\sigma|| \leq \theta_{0\sigma}^* + \theta_{1\sigma}^*||\boldsymbol{\xi}|| + \theta_{2\sigma}^*||\boldsymbol{\xi}||^2 + \theta_{3\sigma}^*||\ddot{\mathbf{q}}|| \triangleq \mathbf{Y}_\sigma^T\boldsymbol{\Theta}_\sigma^*, \quad (7)$$

where $\theta_{i\sigma}^*$'s are unknown scalars, $\mathbf{Y}_\sigma = [1\ ||\boldsymbol{\xi}||\ ||\boldsymbol{\xi}||^2\ ||\ddot{\mathbf{q}}||]^T$ and $\boldsymbol{\Theta}_\sigma^* = [\theta_{0\sigma}^*\ \theta_{1\sigma}^*\ \theta_{2\sigma}^*\ \theta_{3\sigma}^*]^T$. Based on the upper bound structure in (7), the gain ρ_σ in (5a) is designed as

$$\begin{aligned}\rho_\sigma &= \hat{\theta}_{0\sigma} + \hat{\theta}_{1\sigma}||\boldsymbol{\xi}|| + \hat{\theta}_{2\sigma}||\boldsymbol{\xi}||^2 + \hat{\theta}_{3\sigma}||\ddot{\mathbf{q}}|| + \zeta_\sigma + \gamma_\sigma \\ &\triangleq \mathbf{Y}_\sigma^T\hat{\boldsymbol{\Theta}}_\sigma + \zeta_\sigma + \gamma_\sigma, \end{aligned} \quad (8)$$

where $\hat{\boldsymbol{\Theta}}_\sigma \triangleq [\hat{\theta}_{0\sigma}\ \hat{\theta}_{1\sigma}\ \hat{\theta}_{2\sigma}\ \hat{\theta}_{3\sigma}]^T$ and $\zeta_\sigma, \gamma_\sigma$ are auxiliary gains needed for closed-loop stabilization (cf. Remark 6). Defining $\varrho_\sigma \triangleq (\lambda_{\min}(\mathbf{Q}_\sigma)/\lambda_{\max}(\mathbf{P}_\sigma))$, the gains $\hat{\theta}_{i\sigma}, \zeta_\sigma$ and γ_σ are adapted using the following laws:

$$\dot{\hat{\theta}}_{jp} = ||\mathbf{r}_p||||\boldsymbol{\xi}||^j - \alpha_{jp}\hat{\theta}_{jp},\ \dot{\hat{\theta}}_{j\overline{p}} = 0,\ j = 0, 1, 2 \tag{9a}$$

$$\dot{\hat{\theta}}_{3p} = ||\mathbf{r}_p||||\ddot{\mathbf{q}}|| - \alpha_{3p}\hat{\theta}_{3p},\ \dot{\hat{\theta}}_{3\overline{p}} = 0 \tag{9b}$$

$$\dot{\zeta}_p = -\left(1 + \hat{\theta}_{3p}||\ddot{\mathbf{q}}||||\mathbf{r}_p||\right)\zeta_p + \bar{\epsilon}_p,\ \dot{\zeta}_{\overline{p}} = 0, \tag{9c}$$

$$\dot{\gamma}_p = 0,\ \dot{\gamma}_{\overline{p}} = -\left(1 + \frac{\varrho_{\overline{p}}}{2}\sum_{i=0}^{3}\hat{\theta}_{i\overline{p}}^2\right)\gamma_{\overline{p}} + \epsilon_{\overline{p}}, \tag{9d}$$

$$\text{with } \alpha_{i\sigma} > \varrho_\sigma/2,\ i = 0, 1, 2, 3, \tag{9e}$$

$$\hat{\theta}_{i\sigma}(t_0) > 0,\ \zeta_\sigma(t_0) = \bar{\zeta}_\sigma > \bar{\epsilon}_\sigma,\ \gamma_\sigma(t_0) = \bar{\gamma}_\sigma > \epsilon_\sigma, \tag{9f}$$

where p and $\overline{p} \in \Omega\backslash\{p\}$ denote the active and inactive subsystems respectively; $\alpha_{ip}, \bar{\epsilon}_p, \epsilon_{\overline{p}} \in \mathbb{R}^+$ are static design scalars and t_0 is the initial time. From (9a)-(9d) and the initial conditions (9f), it can be verified that $\exists \underline{\zeta}_\sigma, \underline{\gamma}_\sigma \in \mathbb{R}^+$ such that

$$\hat{\theta}_{i\sigma}(t) \geq 0,\ 0 < \underline{\zeta}_\sigma \leq \zeta_\sigma(t) \leq \bar{\zeta}_\sigma,$$
$$\text{and } 0 < \underline{\gamma}_\sigma \leq \gamma_\sigma(t) \leq \bar{\gamma}_\sigma \quad \forall t \geq t_0. \tag{10}$$

We define $\varrho_{M\sigma} \triangleq \lambda_{\max}(\mathbf{P}_\sigma)$, $\varrho_{m\sigma} \triangleq \lambda_{\min}(\mathbf{P}_\sigma)$, $\bar{\varrho}_M \triangleq \max_{\sigma\in\Omega}(\varrho_{M\sigma})$ and $\underline{\varrho}_m \triangleq \min_{\sigma\in\Omega}(\varrho_{m\sigma})$. Following Definition 1 of ADT [27], the switching law is proposed as

$$\vartheta > \vartheta^* = \ln\mu/\kappa, \tag{11}$$

where $\mu \triangleq \bar{\varrho}_M/\underline{\varrho}_m$; κ is a scalar defined as $0 < \kappa < \varrho$ where $\varrho \triangleq \min_{\sigma\in\Omega}(\lambda_{\min}(\mathbf{Q}_\sigma)/\lambda_{\max}(\mathbf{P}_\sigma))$.

While *not using knowledge of the system parameters and structure*, the proposed control and adaptive laws require acceleration measurements (cf. (8), (9b), (9c)). The following remarks highlight that, in the presence of system uncertainty, the use of acceleration measurements seems unavoidable.

***Remark** 3 (On the use of acceleration measurements):* Pioneering adaptive control designs for (non-switched) EL systems made use of acceleration measurements [24]; later, the need for acceleration measurements was removed by introducing in the Lyapunov function a quadratic term depending on the mass matrix (cf. [25, §4]). Therefore, the state-of-the-art adaptive control methods for EL systems rely upon two categories of Lyapunov-based stability analysis: one that explicitly involves the mass matrix $\mathbf{M}_\sigma$ (cf. [20], [21], [26] and references therein) and the one that does not (cf. the inverse-dynamics based designs [22]–[25] and references therein). The designs in the first category typically do not require acceleration measurements. However, both categories impose LIP structure on the system.

***Remark** 4 (On the extension to switched systems):* More important than the LIP assumption, the methods explicitly involving the mass matrix in the Lyapunov function cannot be easily extended to switched EL systems without introducing restrictions. Such designs may not need acceleration feedback, but unfortunately, when extended to slow switching, the resulting switching law will require lower and upper bound knowledge of mass matrix, i.e., $\underline{m}_\sigma$ and $\overline{m}_\sigma$ (cf. Corollary 1). This problem does not occur when adopting the control and adaptive laws (5), (8) and (9), together with a multiple Lyapunov method appropriately defined in the next theorem. Consequently, the necessity of using acceleration measurements seems unavoidable in the presence of uncertainties.

***Remark** 5:* A practical argumentation behind the possibility to get acceleration measurements is that nowadays inertial navigation systems with accelerometers are cheap and commonly used. One might wonder why not trying to estimate acceleration instead. Adaptive works have appeared [30]–[32] where velocity and acceleration measurements are estimated for control design of (non-switched) EL systems. However, partial system dynamics knowledge is typically required, e.g., knowledge of $\mathbf{E}_\sigma$ as in [30], [31] or upper bound knowledge of mass matrix as in [32]. These observations further substantiate the necessity of acceleration measurement in the presence of fully uncertain dynamics.

IV. Stability Analysis of The Proposed Controller

***Theorem** 1:* Under Assumption 1 and Properties 1-2, the closed-loop trajectories of system (6) employing the control laws (5) and (8) with adaptive law (9) and switching law (11) are Uniformly Ultimately Bounded (UUB). An ultimate bound b on the tracking error $\boldsymbol{\xi}$ can be found as

$$b \in \left[0, \sqrt{\frac{2\bar{\varrho}_M^{(N_0+1)}(\delta + \varpi\delta_1)}{\underline{\varrho}_m^{(N_0+2)}(\varrho - \kappa)}}\right], \tag{12}$$

where the scalars δ and δ_1 are defined during the proof.

Proof: Stability relies on the multiple Lyapunov candidate:

$$V = \frac{1}{2}\boldsymbol{\xi}^T\mathbf{P}_\sigma\boldsymbol{\xi} + \sum_{s=1}^{N}\sum_{i=0}^{3}\left\{\frac{(\hat{\theta}_{is} - \theta_{is}^*)^2}{2} + \frac{\gamma_s}{\underline{\gamma}} + \frac{\zeta_s}{\underline{\zeta}}\right\}, \tag{13}$$

where $\underline{\gamma} = \min_{s\in\Omega}(\underline{\gamma}_s)$ and $\underline{\zeta} = \min_{s\in\Omega}(\underline{\zeta}_s)$. Observing that $\boldsymbol{\Lambda}_\sigma\boldsymbol{\xi} = \mathbf{K}_{1\sigma}\mathbf{e} + \mathbf{K}_{2\sigma}\dot{\mathbf{e}}$, the error dynamics in (6) becomes

$$\dot{\boldsymbol{\xi}} = \mathbf{A}_\sigma\boldsymbol{\xi} + \mathbf{B}\left(\boldsymbol{\Psi}_\sigma - \Delta\boldsymbol{\tau}_\sigma\right). \tag{14}$$

Note that $V(t)$ might be discontinuous at the switching instants and only remains continuous during the time interval between two consecutive switchings. Without loss of generality, the behaviour of the multiple Lyapunov function is studied at the switching instant $t_{l+1},\ l \in \mathbb{N}^+$. Let an active subsystem be $\sigma(t_{l+1}^-)$ when $t \in [t_l\ \ t_{l+1})$ and $\sigma(t_{l+1})$ when $t \in [t_{l+1}\ \ t_{l+2})$. We have before and after switching

$$V(t_{l+1}^-) = (1/2)\boldsymbol{\xi}^T(t_{l+1}^-)\mathbf{P}_{\sigma(t_{l+1}^-)}\boldsymbol{\xi}(t_{l+1}^-)$$
$$+\sum_{s=1}^{N}\sum_{i=0}^{3}\left\{\frac{(\hat{\theta}_{is}(t_{l+1}^-) - \theta_{is}^*)^2}{2} + \gamma_s(t_{l+1}^-)/\underline{\gamma} + \zeta_s(t_{l+1}^-)/\underline{\zeta}\right\},$$

$$V(t_{l+1}) = (1/2)\boldsymbol{\xi}^T(t_{l+1})\mathbf{P}_{\sigma(t_{l+1})}\boldsymbol{\xi}(t_{l+1})$$
$$+\sum_{s=1}^{N}\sum_{i=0}^{3}\left\{\frac{(\hat{\theta}_{is}(t_{l+1}) - \theta_{is}^*)^2}{2} + \gamma_s(t_{l+1})/\underline{\gamma} + \zeta_s(t_{l+1})/\underline{\zeta}\right\},$$

respectively. Thanks to the continuity of the tracking error $\boldsymbol{\xi}$ in (14) and of the gains $\hat{\theta}_{i\sigma}, \zeta_\sigma$ and γ_σ in (9), we have $\boldsymbol{\xi}(t_{l+1}^-) = \boldsymbol{\xi}(t_{l+1})$, $(\hat{\theta}_{is}(t_{l+1}^-) - \theta_{is}^*) = (\hat{\theta}_{is}(t_{l+1}) - \theta_{is}^*)$, $\gamma_s(t_{l+1}^-) = \gamma_s(t_{l+1})$ and $\zeta_s(t_{l+1}^-) = \zeta_s(t_{l+1})$. Further, owing to the facts $\boldsymbol{\xi}^T(t)\mathbf{P}_{\sigma(t)}\boldsymbol{\xi}(t) \leq \bar{\varrho}_M\boldsymbol{\xi}^T(t)\boldsymbol{\xi}(t)$ and $\boldsymbol{\xi}^T(t)\mathbf{P}_{\sigma(t)}\boldsymbol{\xi}(t) \geq \underline{\varrho}_m\boldsymbol{\xi}^T(t)\boldsymbol{\xi}(t)$, one has

$$V(t_{l+1}) - V(t_{l+1}^-) = \frac{1}{2}\boldsymbol{\xi}^T(t_{l+1})(\mathbf{P}_{\sigma(t_{l+1})} - \mathbf{P}_{\sigma(t_{l+1}^-)})\boldsymbol{\xi}(t_{l+1})$$
$$\leq \frac{\bar{\varrho}_M - \underline{\varrho}_m}{2\underline{\varrho}_m}\boldsymbol{\xi}^T(t_{l+1})\mathbf{P}_{\sigma(t_{l+1}^-)}\boldsymbol{\xi}(t_{l+1}) \leq \frac{\bar{\varrho}_M - \underline{\varrho}_m}{\underline{\varrho}_m}V(t_{l+1}^-)$$
$$\Rightarrow V(t_{l+1}) \leq \mu V(t_{l+1}^-), \quad (15)$$

with $\mu = \bar{\varrho}_M/\underline{\varrho}_m \geq 1$. At this point, the behaviour of $V(t)$ between two consecutive switching instants, i.e., when $t \in [t_l \;\; t_{l+1})$ can be studied.

We shall proceed the stability analysis for the two cases (i) $||\mathbf{r}_\sigma|| \geq \varpi$ and (ii) $||\mathbf{r}_\sigma|| < \varpi$ using the Lyapunov function (13). With some abuse of notation, let us denote the active subsystem $\sigma(t_{l+1}^-)$ simply with p for convenience, and any inactive subsystem as $\bar{p}$.

Case (i) $||\mathbf{r}_\sigma|| \geq \varpi$

Using (7), (14), (9) and the Lyapunov equation $\mathbf{A}_p^T\mathbf{P}_p + \mathbf{P}_p\mathbf{A}_p = -\mathbf{Q}_p$, the time derivative of (13) yields

$$\dot{V} \leq -(1/2)\boldsymbol{\xi}^T\mathbf{Q}_p\boldsymbol{\xi} + ||\boldsymbol{\Psi}_p||||\mathbf{r}_p|| - \rho_p||\mathbf{r}_p|| + \sum_{s=1}^{N}\sum_{i=0}^{3}\left\{(\hat{\theta}_{is} - \theta_{is}^*)\dot{\hat{\theta}}_{is} + \dot{\gamma}_s/\underline{\gamma} + \dot{\zeta}_s/\underline{\zeta}\right\} \quad (16)$$
$$\leq -(1/2)\boldsymbol{\xi}^T\mathbf{Q}_p\boldsymbol{\xi} - \mathbf{Y}_p^T(\hat{\boldsymbol{\Theta}}_p - \boldsymbol{\Theta}_p^*)||\mathbf{r}_p|| + \sum_{s=1}^{N}\sum_{i=0}^{3}\left\{(\hat{\theta}_{is} - \theta_{is}^*)\dot{\hat{\theta}}_{is} + \dot{\gamma}_s/\underline{\gamma} + \dot{\zeta}_s/\underline{\zeta}\right\}. \quad (17)$$

Using (9a)-(9b) we have

$$\sum_{i=0}^{3}(\hat{\theta}_{ip} - \theta_{ip}^*)\dot{\hat{\theta}}_{ip} = \sum_{j=0}^{2}(\hat{\theta}_{jp} - \theta_{jp}^*)(||\mathbf{r}_p||||\boldsymbol{\xi}||^j - \alpha_{jp}\hat{\theta}_{jp}) + (\hat{\theta}_{3p} - \theta_{3p}^*)(||\mathbf{r}_p||||\ddot{\mathbf{q}}|| - \alpha_{3p}\hat{\theta}_{3p})$$
$$= \mathbf{Y}_p^T(\hat{\boldsymbol{\Theta}}_p - \boldsymbol{\Theta}_p^*)||\mathbf{r}_p|| + \sum_{i=0}^{3}\{\alpha_{ip}\hat{\theta}_{ip}\theta_{ip}^* - \alpha_{ip}\hat{\theta}_{ip}^2\}. \quad (18)$$

Similarly using the facts $\hat{\theta}_{is} \geq 0$, $0 < \underline{\zeta}_s \leq \zeta_s(t)$, $0 < \underline{\gamma}_s \leq \gamma_s(t)$ from (10) and $\underline{\zeta} = \min_{s\in\Omega}(\underline{\zeta}_s)$, $\underline{\gamma} = \min_{s\in\Omega}(\underline{\gamma}_s)$, (9c) and (9d) leads to

$$\dot{\zeta}_p/\underline{\zeta} = -(1 + \hat{\theta}_{3p}||\ddot{\mathbf{q}}||||\mathbf{r}_p||)(\zeta_p/\underline{\zeta}) + (\bar{\epsilon}_p/\underline{\zeta}) \leq -\hat{\theta}_{3p}||\ddot{\mathbf{q}}||||\mathbf{r}_p|| + (\bar{\epsilon}_p/\underline{\zeta}), \quad (19)$$

$$\dot{\gamma}_{\bar{p}}/\underline{\gamma} = -(1 + (\varrho_{\bar{p}}/2)\sum_{i=0}^{3}\hat{\theta}_{i\bar{p}}^2)(\gamma_{\bar{p}}/\underline{\gamma}) + (\epsilon_{\bar{p}}/\underline{\gamma}) \leq -\frac{\varrho_{\bar{p}}}{2}\sum_{i=0}^{3}\hat{\theta}_{i\bar{p}}^2 + (\epsilon_{\bar{p}}/\underline{\gamma}), \quad (20)$$

Substituting (18)-(20) in (17) yields

$$\dot{V} \leq -(1/2)\lambda_{\min}(\mathbf{Q}_p)||\boldsymbol{\xi}||^2 + \sum_{i=0}^{3}\{\alpha_{ip}\hat{\theta}_{ip}\theta_{ip}^* - \bar{\alpha}_{ip}\hat{\theta}_{ip}^2\} + (\bar{\epsilon}_p/\underline{\zeta}) - \sum_{\forall\bar{p}\in\Omega\setminus\{p\}}\sum_{i=0}^{3}\left\{\varrho_{\bar{p}}\hat{\theta}_{i\bar{p}}^2 - (\epsilon_{\bar{p}}/\underline{\gamma})\right\}. \quad (21)$$

Since $\hat{\theta}_{is} \geq 0$, $\zeta_s(t) \leq \bar{\zeta}_s$ and $\gamma_s(t) \leq \bar{\gamma}_s$ by design (10), one obtains

$$V \leq \frac{1}{2}\lambda_{\max}(\mathbf{P}_p)||\boldsymbol{\xi}||^2 + \sum_{s=1}^{N}\sum_{i=0}^{3}\frac{(\hat{\theta}_{is}^2 + {\theta_{is}^*}^2)}{2} + \bar{\gamma}_s\underline{\gamma} + \bar{\zeta}_s\underline{\zeta}. \quad (22)$$

Hence, using (22), the condition (21) is further simplified to

$$\dot{V} \leq -\varrho V + \bar{\epsilon}_p/\underline{\zeta} + \sum_{i=0}^{3}\{\alpha_{ip}\hat{\theta}_{ip}\theta_{ip}^* - \bar{\alpha}_{ip}\hat{\theta}_{ip}^2\} + \sum_{s=1}^{N}\sum_{i=0}^{3}\frac{\varrho_s{\theta_{is}^*}^2}{2} + \varrho_s\bar{\gamma}_s/\underline{\gamma} + \varrho_s\bar{\zeta}_s/\underline{\zeta} + \epsilon_s/\underline{\gamma}, \quad (23)$$

where $\varrho = \min_{p\in\Omega}\{\varrho_p\}$; $\bar{\alpha}_{ip} = (\alpha_{ip} - (\varrho_p/2)) > 0$ by design from (9e). Further rearrangement yields

$$\alpha_{ip}\hat{\theta}_{ip}\theta_{ip}^* - \bar{\alpha}_{ip}\hat{\theta}_{ip}^2 = -\bar{\alpha}_{ip}\left(\hat{\theta}_{ip} - \frac{\alpha_{ip}\theta_{ip}^*}{2\bar{\alpha}_{ip}}\right)^2 + \frac{\left(\alpha_{ip}\theta_{ip}^*\right)^2}{4\bar{\alpha}_{ip}}. \quad (24)$$

We had defined earlier $0 < \kappa < \varrho$. Then, using (24), $\dot{V}(t)$ from (23) gets simplified to

$$\dot{V}(t) \leq -\kappa V(t) - (\varrho - \kappa)V(t) + \delta, \quad (25)$$

where $\delta \triangleq \max_{p\in\Omega}\left(\sum_{i=0}^{3}\left(\alpha_{ip}\theta_{ip}^*\right)^2/(4\bar{\alpha}_{ip}) + (\bar{\epsilon}_p/\underline{\zeta})\right) + \sum_{s=1}^{N}\sum_{i=0}^{3}(\varrho_s/2){\theta_{is}^*}^2 + \varrho_s\bar{\gamma}_s/\underline{\gamma} + \varrho_s\bar{\zeta}_s/\underline{\zeta} + (\epsilon_s/\underline{\gamma})$.

Case (ii) $||\mathbf{r}_\sigma|| < \varpi$

In this case, the time derivative of (13) yields

$$\dot{V} \leq -(1/2)\boldsymbol{\xi}^T\mathbf{Q}_p\boldsymbol{\xi} + ||\boldsymbol{\Psi}_p||||\mathbf{r}_p|| - \rho_p||\mathbf{r}_p||^2/\varpi + \sum_{s=1}^{N}\sum_{i=0}^{3}\left\{(\hat{\theta}_{is} - \theta_{is}^*)\dot{\hat{\theta}}_{is} + \dot{\gamma}_s/\underline{\gamma} + \dot{\zeta}_s/\underline{\zeta}\right\} \quad (26)$$
$$\leq -(1/2)\boldsymbol{\xi}^T\mathbf{Q}_p\boldsymbol{\xi} + ||\boldsymbol{\Psi}_p||||\mathbf{r}_p|| + \sum_{s=1}^{N}\sum_{i=0}^{3}\left\{(\hat{\theta}_{is} - \theta_{is}^*)\dot{\hat{\theta}}_{is} + \dot{\gamma}_s/\underline{\gamma} + \dot{\zeta}_s/\underline{\zeta}\right\}. \quad (27)$$

Following similar lines of proof as in Case (i) we have

$$\dot{V}(t) \leq -\kappa V(t) - (\varrho - \kappa)V(t) + \delta + (\mathbf{Y}_p^T\hat{\boldsymbol{\Theta}}_p - \hat{\theta}_{3p}||\ddot{\mathbf{q}}||)||\mathbf{r}_p||$$
$$= -\kappa V(t) - (\varrho - \kappa)V(t) + \delta + \sum_{j=0}^{2}\hat{\theta}_{jp}||\boldsymbol{\xi}||^j||\mathbf{r}_p||. \quad (28)$$

From (4) one can verify $||\mathbf{r}|| < \varphi \Rightarrow ||\boldsymbol{\xi}|| \in \mathcal{L}_\infty$ and consequently, the adaptive law (9a) implies $||\mathbf{r}||, ||\boldsymbol{\xi}|| \in \mathcal{L}_\infty \Rightarrow \hat{\theta}_{jp}(t) \in \mathcal{L}_\infty$ $j = 0, 1, 2$. Therefore, $\exists\delta_1 \in \mathbb{R}^+$ such that $\sum_{j=0}^{2}\hat{\theta}_{jp}||\boldsymbol{\xi}||^j \leq \delta_1 \ \forall p \in \Omega$ when $||\mathbf{r}_p|| < \varphi$. Hence, replacing this relation in (28) yields

$$\dot{V}(t) \leq -\kappa V(t) - (\varrho - \kappa)V(t) + \delta + \varpi\delta_1. \quad (29)$$

Therefore, investigating the stability results of Cases (i) and (ii), it can be concluded that $\dot{V}(t) \leq -\kappa V(t)$ when

$$V(t) \geq \mathcal{B} \triangleq (\delta + \varpi\delta_1)/(\varrho - \kappa). \quad (30)$$

In light of this, further analysis is needed to observe the behaviour of $V(t)$ between the two consecutive switching instants, i.e., $t \in [t_l \ t_{l+1})$, for two possible scenarios:

(i) when $V(t) \geq \mathcal{B}$, we have $\dot{V}(t) \leq -\kappa V(t)$ implying exponential decrease of $V(t)$;

(ii) when $V(t) < \mathcal{B}$, no exponential decrease can be derived.

Behaviour of $V(t)$ is discussed below individually for these two scenarios.

Scenario (i): There exists a time, call it T_1, when $V(t)$ enters into the bound $\mathcal{B}$ and $N_\sigma(t)$ denotes the number of all switching intervals for $t \in [t_0 \ \ t_0 + T_1)$. Accordingly, for $t \in [t_0 \ \ t_0 + T_1)$, using (15) and $N_\sigma(t_0, t)$ from Definition 1 we have

$$\begin{aligned}
&V(t) \leq \exp\left(-\kappa(t - t_{N_\sigma(t)-1})\right) V(t_{N_\sigma(t)-1}) \\
&\leq \mu \exp\left(-\kappa(t - t_{N_\sigma(t)-1})\right) V(t^-_{N_\sigma(t)-1}) \\
&\leq \mu \exp\left(-\kappa(t - t_{N_\sigma(t)-1})\right) \\
&\quad \cdot \mu \exp\left(-\kappa(t_{N_\sigma(t)-1} - t_{N_\sigma(t)-2})\right) V(t^-_{N_\sigma(t)-2}) \\
&\qquad \vdots \\
&\leq \mu \exp\left(-\kappa(t - t_{N_\sigma(t)-1})\right) \mu \exp\left(-\kappa(t_{N_\sigma(t)-1} - t_{N_\sigma(t)-2})\right) \\
&\quad \cdots \mu \exp\left(-\kappa(t_1 - t_0)\right) V(t_0) \\
&= \mu^{N_\sigma(t_0,t)} \exp\left(-\kappa(t - t_0)\right) V(t_0) \\
&= c\left(\exp\left(-\kappa + (\ln\mu/\vartheta)\right)(t - t_0)\right)) V(t_0), \quad (31)
\end{aligned}$$

where $c \triangleq \exp(N_0 \ln\mu)$ is a constant. Substituting the ADT condition $\vartheta > \ln\mu/\varrho$ in (31) yields $V(t) < cV(t_0)$ for $t \in [t_0 \ \ t_0 + T_1)$. Moreover, as $V(t_0 + T_1) < \mathcal{B}$, one has $V(t_{N_\sigma(t)+1}) < \mu\mathcal{B}$ from (15) at the next switching instant $t_{N_\sigma(t)+1}$ after $t_0 + T_1$. This implies that $V(t)$ may be larger than $\mathcal{B}$ from the instant $t_{N_\sigma(t)+1}$, necessitating further analysis. Then following similar lines of proof as in [29], [33], we can come to the conclusion that $V(t) < c\mu\mathcal{B}$ for $t \in [t_0 + T_1 \ \ \infty)$. This confirms that once $V(t)$ enters the interval $[0, \mathcal{B}]$, it cannot exceed the bound $c\mu\mathcal{B}$ any time later with the ADT switching law (11).

Scenario (ii): It can be easily verified that the same argument below (31) also holds for Scenario (ii).

Thus, observing the stability arguments of the Scenarios (i) and (ii), it can be concluded that the closed-loop system remains UUB globally with the control laws (5) and (8) with the adaptive law (9) and switching law (11) implying

$$V(t) \leq \max\left(cV(t_0), c\mu\mathcal{B}\right), \quad \forall t \geq t_0. \quad (32)$$

Again, the definition of the Lyapunov function (13) yields

$$V(t) \geq (1/2)\lambda_{\min}(\mathbf{P}_{\sigma(t)})||\boldsymbol{\xi}||^2 \geq (\underline{\varrho}_m/2)||\boldsymbol{\xi}||^2. \quad (33)$$

Using (32) and (33) we have

$$||\boldsymbol{\xi}||^2 \leq (2/\underline{\varrho}_m)\max\left(cV(t_0), c\mu\mathcal{B}\right), \quad \forall t \geq t_0. \quad (34)$$

Therefore, using the expression of $\mathcal{B}$ from (22), an ultimate bound b on the tracking error $\boldsymbol{\xi}$ can be found as (12). ■

The switching law (11) is independent from the unknown uncertainty bounds mentioned in Remark 1 (this is thanks to the uncertainty independent bound derived in (15)). Using alternative multiple Lyapunov functions, a stabilizing switching law would exist but could not be determined as it would depend on an unknown uncertainty bound. This is clarified by the following corollary.

Corollary 1: When the mass matrix $\mathbf{M}_\sigma$ is completely unknown (cf. Remark 1), it is not possible to design a slow switching law using an extended version of the Lyapunov function explicitly involving the mass matrix.

Proof: Similarly to [20], [21], [26], [29], consider a multiple Lyapunov function candidate

$$V_1 = (1/2)\boldsymbol{\xi}^T\mathbf{M}_\sigma\boldsymbol{\xi} + V_2, \quad (35)$$

where V_2 is a positive definite, continously differentiable function (cf. [20], [21], [26], [29] for details). Then, following similar lines as in [29], one would get

$$V_1(t_{l+1}) \leq \mu_1 V_1(t^-_{l+1}), \quad (36)$$

where $\mu_1 = \overline{m}/\underline{m} \geq 1$, $\overline{m} \triangleq \max_{\sigma\in\Omega}(\overline{m}_\sigma)$ and $\underline{m} \triangleq \min_{\sigma\in\Omega}(\underline{m}_\sigma)$. As $\overline{m}$ and $\underline{m}$ are unknown from Remark 1, it can be realized that one cannot design an ADT as in (11) while following the multiple Lyapunov candidate (35) because, after using the argument in (31), a stabilizing ADT would end up depending on unknown parameters. ■

***Remark** 6 (Importance of ζ and γ):* The following two observations clarify the importance of the gains γ and ζ in ensuring closed-loop stability: (i) the term $-\frac{\varrho_{\overline{p}}}{2}$ in (20), contributed by $\dot{\gamma}$, cancels the similar term stemming from (22) leading to (23); (ii) the term $-\hat{\theta}_{3p}||\ddot{\mathbf{q}}||||\mathbf{r}_p||$, contributed by $\dot{\zeta}$ in (19), negates the similar term in the first inequality in (28). Note that boundedness of $||\mathbf{r}_p||$ in Case (ii) implies boundedness of $||\boldsymbol{\xi}||$ but not of $||\ddot{\mathbf{q}}||$: therefore, canceling the term $-\hat{\theta}_{3p}||\ddot{\mathbf{q}}||||\mathbf{r}_p||$ through the auxiliary gain ζ is necessary for guaranteeing closed-loop stability. We are not aware of any similar design in literature.

***Remark** 7 (Selection of gain $\mathbf{D}_\sigma$):* One can see from (6)-(7) that $\theta^*_{3\sigma} \geq ||\mathbf{D}_\sigma^{-1}\mathbf{M}_\sigma - \mathbf{I}||$. As $\theta^*_{3\sigma}$ is considered to be unknown, the proposed design does not put any restriction on the choice of $\mathbf{D}_\sigma$: this is a clear advantage compared to designs proposed in [28], [34], [35] (and references therein) which utilize acceleration feedback and require the knowledge of $\theta^*_{3\sigma}$ (i.e., $\mathbf{D}_\sigma$ is to be designed based on the upper bound knowledge of $\mathbf{M}_\sigma$). However, $\mathbf{D}_\sigma$ should be carefully selected in practice as its high value may lead to unnecessary high control input (cf. (5a)).

V. Simulation Results

Due to lack of space, a simulation verification of the proposed adaptive scheme using switched dynamics of a 2-link manipulator is provided in [].

VI. CONCLUSIONS

A new concept of adaptive control design was proposed for unknown switched EL systems with slow switching. Specifically, it was shown that for such systems, when system structure and bounds of uncertainty are considered to be unknown, use of acceleration feedback seems an unavoidable design trade-off. The performance of the proposed concept was verified using a manipulator system with switched dynamics due to parametric variations in the subsystems.

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