Capacity and Link Costs Optimal Hierarchical Ring Networks

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Abstract

Hierarchical Ring Networks are widely used in communication systems, logistics and shared multiprocessors. Efficient design of these networks will help cut costs in the installation and operation of systems that use them. This work looks at the "modified 2-level Hierarchical Ring Netwoks (HRN) design problem" which can be defined as the problem of constructing link costs and capacity costs optimal 2-level HRNs. A mathematical formulation of this problem is presented.

1 Introduction

The modified 2-level HRN design problem is defined as one in which, given the locations of a set of nodes, V, the aim is to find a partition of this set such that, the sum of the costs of the links connecting all the nodes of each group into rings (local rings), the costs of the links connecting each of these groups at a single point constituting another ring (global ring) and the cost of the capacity assigned to each link in the network thus constructed, such that the network is able to service every traffic demand, is minimum.

The term "modified" is borrowed from [8] and is used to differentiate it from the problem of designing only link costs-optimal HRNs.

An example of a 2-level HRN is shown in Fig. 1. The rings marked LR_i are called as the local rings and the single ring marked GR is called the global ring. This global ring could perhaps be the "backbone network" of a very large public network. The global ring–local ring terms are alternately used with terms "federal ring–metro ring", respectively.

Due to their widespread utility (for example [2],[13],[14],[12]), a lot of work has focussed on trying to efficiently design them, sometimes with a few variations ([1]–[11]). However, a mathematical formulation has been done for this exact problem only in [8], [9], [10]. In [11], a generic version

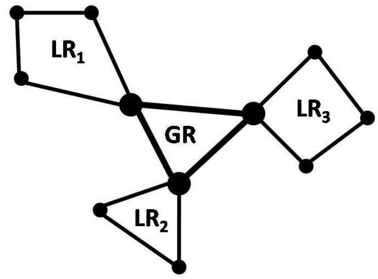

Figure 1: An example of a 2-level HRN. LR_is ares the local rings and GR is the global ring

of the problem – the k-level HRN design problem is considered. However, most of them relate only to link costs optimization. Only [8] considers the capacity and link costs optimal design problem. However, the design of the global ring (federal ring) has been left out of the solution there. Also, the formulation makes use of experimentally determined parameters, which have the problem of being susceptible to scenario changes.

In this work, an non-linear programming formulation of the modified problem is presented that takes into account both, the global ring and the local rings and that does not rely on experimentally determined parameters.

2 Problem Formulation

The formulation is presented under the assumption that the routing in the network follows the model of Unidirectional Self Healing Rings (USHRs). It holds in USHRs that traffic in a ring moves only in one direction and that communication flow in the ring takes up capacity in all the links in the ring. Let $G = (V, E)$ be an undirected complete graph, where V is the set of vertices and E is the set of edges. (The terms "node" and "vertex" are used interchangeably.) Let the number of vertices in G, $|V|$, be equal to n. Let the edge between vertices i and j , be denoted as $\{i, j\}$ and let its corresponding weight (cost) be given by w_{ij} . Also, let the number of desired rings be equal to R. The problem is modeled as dividing the set of vertices into $R + 1$ groups. Here the $(R + 1)$th group refers to the global ring. It is understood that the vertices in this group are also present in the local groups. Let the lower and upper limits for the size of the rings be L_1 and L_2, respectively. Let d_{ij} denote the amount of traffic between vertices i and j. Let c be the unit capacity cost, equal for all the edges in the network. Variables that will be used in the formulation are:

1. N_{ir} is a binary constraint that is equal to 1 if the vertex i is present in the ring r

2. x_{ijr} is once again a binary constraint that is equal to 1 if the edge $\{i,j\}$ is present in ring r

3. λ_{ij} is the capacity assigned to the edge $\{i,j\}$. If the edge does not occur in the graph, it is equal to zero

The design of optimal 2-level HRNs can be considered as the minimization of

$$C = \sum_{r=1}^{R+1} \sum_{j=1}^{n} \sum_{i=1}^{n} x_{ijr} w_{ij} + c \sum_{j=1}^{n} \sum_{i=1}^{n} \lambda_{ij} \quad (1)$$

If $\{i,j\}$ belongs to a local ring, then λ_{ij} can be written as the sum of outbound traffic from the ring to which the edge belongs and the inbound traffic from outside vertices to that ring. Otherwise, it is equal to the sum of all traffic in the network minus the traffic that is restricted to the local rings. Hence,

$$\begin{aligned}
\lambda_{ij} \quad = \quad & \sum_{r=1}^{R} x_{ijr} \sum_{i'=1}^{n} \sum_{\substack{j'=1 \\ j' \neq i'}}^{n} d_{i'j'} N_{i'r} \\
& + \sum_{r=1}^{R} x_{ijr} \sum_{i'=1}^{n} \sum_{\substack{j'=1 \\ j' \neq i'}}^{n} \left(1 - N_{i'r}\right) d_{i'j'} N_{j'r} \\
& + x_{ij(R+1)} \left(\sum_{i'=1}^{n} \sum_{\substack{j'=1 \\ j' \neq i'}}^{n} d_{i'j'} - \sum_{r=1}^{R} \sum_{i'=1}^{n} \sum_{\substack{j'=1 \\ j' \neq i'}}^{n} N_{i'r} N_{j'r} d_{i'j'} \right)
\end{aligned}$$

which can be re-written as,

$$\begin{aligned}
\lambda_{ij} \quad = \quad & \sum_{r=1}^{R} x_{ijr} \sum_{i'=1}^{n} \sum_{\substack{j'=1 \\ j' \neq i'}}^{n} d_{i'j'} \left[N_{i'r} + N_{j'r} - N_{i'r} N_{j'r} \right] \\
& + x_{ij(R+1)} \left(\sum_{i'=1}^{n} \sum_{\substack{j'=1 \\ j' \neq i'}}^{n} d_{i'j'} - \sum_{r=1}^{R} \sum_{i'=1}^{n} \sum_{\substack{j'=1 \\ j' \neq i'}}^{n} N_{i'r} N_{j'r} d_{i'j'} \right)
\end{aligned}$$

subject to the following constraints:

$$L_1 \leq \sum_{i=1}^{n} N_{ir} \leq L_2 \quad \forall \; r \neq (R+1) \quad (2)$$

$$\sum_{i=1}^{n} N_{i(R+1)} = R \tag{3}$$

$$\sum_{r=1}^{R} N_{ir} = 1 \quad \forall\ i \tag{4}$$

$$\sum_{i=1}^{n} N_{ir} N_{i(R+1)} = 1 \quad \forall\ r \neq (R+1) \tag{5}$$

$$x_{ijr} \leq N_{ir}, \quad x_{ijr} \leq N_{jr} \quad \forall\ i, j, r \tag{6}$$

$$\sum_{j=1}^{n} x_{ijr} = 2N_{ir} \quad \forall\ i, r \tag{7}$$

$$\sum_{i \in V^*} \sum_{j \in V^*\ j>i} x_{ijr} \quad \leq \quad |V^*| - N_{tr}$$
$$\forall\ V^* \subset V,\ V^* \neq \emptyset, \quad \forall\ t \in V \backslash V^*, \quad \forall\ r \tag{8}$$

$$x_{ijr}, N_{ir} \in \{0, 1\} \quad \forall\ i, j, r \tag{9}$$

Eq. 1 represents the cost function that is to be minimized. Eq. 2 - Eq. 8 are the design constraints. Eq. 2 is the ring cardinality constraint. It ensures that the size of each ring (except the global ring) is between L_1 and L_2. The size of the global ring is controlled by Eq. 3. Eq. 4 ensures that all the vertices are part of only a single local ring. Eq. 5 ensures that only one vertex from each ring is part of the global ring. The constraints in Eq. 6 make sure that an edge is selected in a ring, only if its adjacent vertices are also present in that ring. Eq. 7 is the vertex degree constraint. It makes sure that the degree of each vertex present in a ring (both local and global) is exactly 2. Eq. 8 (from [9]) is the subtour elimination constraint. It ensures that the graph of each ring is connected. Finally Eq. 9 is the binary constraint on the variables.

It is to be noted that this is a non-linear integer programming formulation and hence cannot be solved using linear optimization techniques. The expression for λ_{ij} is cubic in nature and Eq. 5 is quadratic. Conversion of the non-linear constraints into linear constraints comes with the addition of a large number of variables and is thus not considered.

3 Conclusion

In this work, a formulation for the modified 2-level HRN design problem was presented, under the assumption that the routing in the rings was that of Unidirectional Self Healing Rings, where it holds that communication flow in the ring takes up capacity in all links in the ring.

Future work on this problem could have two directions: 1) Efficient reduction of the order of the formulation from cubic to linear and 2) Providing capacity and link costs formulation for k-level HRNs.

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