Non Uniform Sampling of Time Limited Signal Using Integer Linear Programming

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Nonuniform Sampling of Time-limited Signal using Integer Linear Programming

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***Abstract*—In this paper, nonuniform sampling of a time-limited signal is proposed. Based on dynamic range, the signal is divided uniformly into equal intervals which constitute coarse partition. Then, total number of samples is nonuniformly distributed among the intervals of coarse partition leading to nonuniform sampling. The problem of nonuniform allocation of samples within coarse partition has been formulated and solutions based on integer linear programming have been proposed. Using intelligent allocation of samples in nonuniform manner, fewer samples are required to represent the signal retaining maximum information of the original signal. Approximation error between uniform sampling and proposed nonuniform sampling is compared. Original time-limited signal is reconstructed using B-Spline interpolation function.**

***Index Terms*—Nonuniform Sampling, Integer Linear Programming, Linear Diophantine Equation, B-Spline.**

I. Introduction

In digital signal processing (DSP), uniform sampling is the most common way of sampling, where samples of continuous-time signals are obtained at regular intervals of time. The exact reconstruction of an original bandlimited signal from uniform samples is possible, only if sampling rate is at least twice the highest frequency to which the signal is bandlimited. This is given by Whittaker-Nyquist-Kotel'nikov-Shannon (WNKS) sampling theorem [1]. If sampling rate is below the Nyquist rate, it results into aliasing effect in the recovered signal.

Although the uniform sampling is more convenient to implement in practice, it may not be applicable when fluctuations in sampling instants can't be ignored or when signal samples are obtained at irregular time intervals. In communication theory, when data from a uniformly sampled signal are lost, it results into a sequence of nonuniform samples . Computerized tomography (CT) and magnetic resonance imaging (MRI) frequently use nonuniform frequency domain samples [2], [3]. Other applications using nonuniform sampling occur in geophysics [4], spectroscopy [5], general signal/image processing and biomedical imaging [6].

One major application of nonuniform sampling is data compression [7], where primary goal is to accurately represent original data (a speech signal, an image or a video etc.) in fewer numbers of bits. With an increased popularity of the internet, our daily digital interaction using email, social media or online shopping generate huge chunk of data. It is required to extract useful information from large volume of data for operations like searching, mining etc. However, the storage and transfer of the large volume of data is a major challenge, which leads to the need for data compression. Using nonuniform sampling, significant data compression can be achieved, where more number of bits are allocated to the region containing more useful data and vice versa. For example, to store speech signal, more bits are allocated to represent voiced sounds and no or very few bits are required to represent silence period of the speech signal.

A bandlimited signal can be uniquely recovered from its nonuniform samples, provided that average sampling rate exceeds Nyquist rate. It is given by generalization of Shannon's theorem, also known as Paley-Wiener-Levinson sampling theorem [8]. However, it is impossible to generate a truly bandlimited signal in any real-world situation, because it requires infinite time to transmit. All real-world signals are, by necessity, time limited and therefore, sampling theorem cannot be applied directly. An anti-aliasing filter is used before sampling to band limit the time limited signal. However, it is practically difficult to realize the anti-aliasing filter with sharp cutoff to maximize the use of the available bandwidth without exceeding the Nyquist limit. In order to relax the requirements of the anti-aliasing filter, a technique known as oversampling is commonly used. The disadvantage of oversampling is an increment in the number of bits per second (A/D converter word-length times the sampling rate), the need for a faster A/D converter and increased cost of processing, storage or transmission.

In the literature [9], [10], nonuniform sampling of bandlimited signal is studied extensively. However, the real world signals are time limited and it is required to bandlimit the signals using anti-aliasing filter before nonuniform sampling. During this operation, valuable information of the signal is lost. In our paper, we propose novel nonuniform sampling method (based on statistical information) for time limited signal. This sampling technique allocates more number of samples in the region containing more information of the signal, while assigning few samples to less important region of the signal. Thus, by intelligently allocating samples in a nonuniform manner, maximum signal information can be retained with less number of samples as compared to uniform sampling. The proposed nonuniform sampling is performed by coarse grain periodic sampling, followed by fine grain nonuniform sampling of a continuous time limited signal. We conceptualize coarse grain periodic sampling based on dynamic range of the time-limited signal. It is done by dividing the signal into uniformly spaced intervals, which constitutes *coarse partition*. Later, total number of samples are distributed nonuniformly in these intervals leading to fine grain nonuniform sampling called *fine partition*.

Consider a one dimensional time limited signal. It is passed through an anti-aliasing filter i.e. we roughly know the important highest frequency. This bandlimited signal is sampled using uniform sampling and the reconstruction error is measured. Now, original time-limited signal is sampled using proposed nonuniform sampling (hybrid of periodic and nonperiodic sampling) and again reconstruction error is measured. The objective of this paper is to compare reconstruction error between uniform sampling and proposed nonuniform sampling. Assuming that bandlimited (after passing through anti-aliasing filter) signal requires, say L samples for perfect reconstruction using periodic sampling. Therefore, second objective of this paper is, to optimally allocate L samples using proposed nonuniform sampling technique.

This paper is divided into four sections. In section II, an information theoretic approach for nonuniform sampling based on probability distribution is discussed. In section III, we have formulated the problem for allocating samples in nonuniform manner and solved using integer linear programming. Section IV presents reconstruction methods to recover original signal. Simulation results are provided in Section V. Finally, the paper is concluded in section VI.

II. Nonuniform Sampling Approach

From a Real Analysis [11], a function $f(x)$ belongs to L^p space if

$$\int_{-\infty}^{+\infty} f^p(x) < \infty \tag{1}$$

The class of functions in L^2 space belongs to Hilbert space. In practical applications, many functions belong to Hilbert space. Consider $f(t)$, a one dimensional continuous time signal. Using Lebesgues decomposition theorem from Real Analysis, signal $f(t)$ can be decomposed in the following manner.

$$f(t) = g(t) - h(t) \tag{2}$$

where, $g(t)$ and $h(t)$ are non-negative signals corresponding to positive and negative parts of the signal $f(t)$ respectively. The nonuniform sampling of $g(t)$ is discussed in the following sections and the same discussion holds true for the nonuniform sampling of $h(t)$.

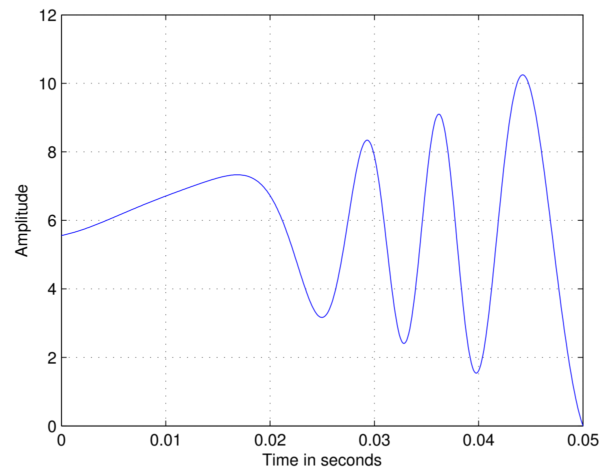

Fig. 1. A signal $g(t)$, positive part of original signal $f(t)$

Consider, $g(t)$ a positive part of signal $f(t)$ as shown in Fig. 1. Now, normalize the signal $g(t)$ in the following manner i.e. define a new signal $r(t)$ such that

$$r(t) = \frac{g(t)}{\int g(t)dt} \tag{3}$$

It is evident that $r(t)$ is a probability density function, which is sampled in nonuniform manner. Our goal is to design a sampling method such that $r(t)$ can be reconstructed from the corresponding samples as accurately as possible. The approximation approach discussed in [12] is used here. The sampling problem boils down in finding a piecewise linear distribution function that approximates to the distribution function corresponding to $r(t)$. In the literature on statistics, there are well developed procedures for approximating a density function (or equivalently the distribution function) with respect to some useful meaningful metric (between the original density and the approximating density). The problem of approximating the probability density function by a set of rectangles is well studied in the literature, which is effectively transferred to arrive at the notion of sampling. Uniform sampling of probability density function approximated using set of rectangles is shown in Fig. 2.

In the next subsection, we have discussed proposed information theoretic based nonuniform sampling technique. In this approach, a signal is first divided uniformly in equal intervals called coarse partition. Then, samples are allocated in the nonuniform manner within each interval of coarse partition leading to the nonuniform sampling.

A. Coarse Partition

Assume that amplitude of signal is bounded. Based on dynamic range of the amplitude of the signal, it is divided into M uniformly spaced regions such that dynamic range within each interval of uniformly spaced region is less than δ, where δ is the maximum allowed dynamic range within each interval. This constitutes the coarse partition.

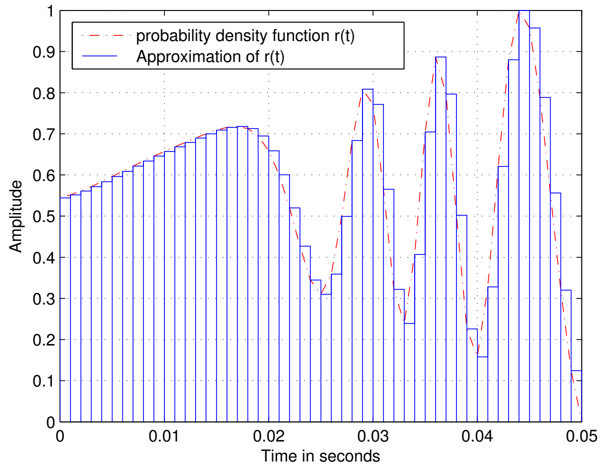

Fig. 2. Uniform sampling of probability density function $r(t)$ approximated by a set of rectangles.

B. Fine Partition

It refers to the nonuniform sampling of the intervals in the coarse partition. We take first order derivative of the signal $g(t)$. Derivative of the signal provides information about variation in the signal amplitude with respect to time. This information is used to allocate different number of samples in each interval of the coarse partition according to signal variation. But, if the signal is corrupted with white Gaussian noise, the derivative signal will have undesired random peaks. Therefore, to remove the effect of noise, appropriate smoothing filter can be used before computing derivative of the signal. Savitzky-Golay is well-known smoothing filter [13], used to compute derivative of noisy signal. Later, normalize the signal ($g'(t) = \frac{dg(t)}{dt}$) in a manner which gives new signal $s(t)$ such that

$$s(t) = \frac{g'(t)}{\int g'(t)dt} \tag{4}$$

It is now clear that $s(t)$ is probability density function. Now, divide the signal $s(t)$ into same M uniformly space regions which was derived earlier for the coarse partition of $g(t)$.

Using one of the numerical integration techniques, compute the area in each of the intervals of the coarse partition of $s(t)$. Here, we have considered absolute values of $s(t)$ to compute the areas in each interval. As function is normalized, these areas constitute probabilities $p_1, p_2, \ldots, p_M$, where

$$p_i = \frac{A_i}{\sum_{i=1}^{M} A_i} \text{ for } i = 1, 2, ..., M \tag{5}$$

These probability values directly reflect the variation in signal amplitude with respect to time. Therefore, more samples are allocated in the intervals with high probability values, whereas fewer samples are allocated in the interval with lesser probability values. Integer linear programming technique is used to determine number of samples in each interval of the coarse partition which is discussed in the next section.

III. Problem Formulation

In this section, the problem formulation is discussed for allocating samples in a nonuniform manner in the intervals of coarse partition. Integer Linear Programming based technique is proposed to determine the number of samples required in each interval of the coarse partition.

Let M be the number of uniformly spaced intervals in the coarse partition of original signal $g(t)$. Assume that q_j, for $j = 1, 2, \ldots, M$ is the probability corresponding to the j^{th} interval of coarse partition of the normalized derivative signal $s(t)$. Let $p_1, p_2, \ldots, p_M$ are the sorted increasing probabilities and $n_1, n_2, \ldots, n_M$ are the corresponding number of samples allocated to the intervals of the coarse partition of $g(t)$. Consider L be the total number of samples required to reconstruct the original signal. The problem of allocating samples in a nonuniform manner in the intervals of coarse partition of $g(t)$ can be formulated as below:

$$\text{Maximize } E[Y] = \sum_{i=1}^{M} n_i \times p_i \tag{6}$$

$$\text{Subject to } \sum_{i=1}^{M} n_i = L$$

Therefore, objective is to find the number of samples in each interval of the coarse partition, which will maximize the expected number of samples subjected to the above constraint on probability masses, p_i.

We reason below that if no constraints are imposed on $\{n_i\}$, then we have a trivial problem.

Case 1: Since, $\{p_1, p_2, \ldots, p_M\}$ are sorted increasing probabilities , i.e., $p_1 \leq p_2 \leq \ldots \leq p_M$, setting $n_1 = 0, n_2 = 0, \ldots, n_M = L$ maximizes $E[Y]$. However, this is a trivial solution.

Case 2: If minimum number of samples in each of the coarse partition is lower bounded by at least N, the allocation of samples would be as following: $n_1 = N$, $n_2 = N, \ldots, n_M = L - (M-1)N$ with $n_M \geq N$

Case 3: If minimum number of samples in each of the coarse partition is lower bounded by at least N and other samples must differ by at least 1, the allocation of samples would be as following: $n_1 = N$, $n_2 = N+1, \ldots, n_{M-1} = N + (M-2)$, $n_M = L - S$ with $(L - S) \geq N$, where $S = N + (N+1) + \cdots + (N + (M-2))$

Case 4: If minimum number of samples in each of the coarse partition is lower bounded by at least N and other samples must differ by at least d, the allocation of samples would be as following: $n_1 = N$, $n_2 = N + d, \ldots, n_{M-1} = N + (M-2)d$, $n_M = L - S$ with $(L - S) \geq N$, where $S = N + (N+d) + \cdots + (N + (M-2)d)$

Thus we are naturally led to imposition of realistic (practical) constraints on the integer values of n_i. Let, n_i's are in Arithmetic Progression (AP) series, i.e., $n_1 = a, n_2 = a+d, n_3 = a+2d, \ldots, n_M = a+(M-1)d$. Here, a and d are unknown parameters, which can be computed using Linear-Diophantine equation.

We know that $a, a+d, a+2d, \ldots, a+(M-1)d$ are number of samples in each interval and add up to L, which is the total number of samples. Therefore,

$$a + (a + d) + (a + 2d) + \cdots + (a + (M - 1)d) = L \quad (7)$$

LHS is in Arithmetic Progression, therefore we can write

$$M \times a + \frac{(M-1)M}{2} \times d = L \quad (8)$$

The equation is in the form of Linear Diophantine equation $px + qy = r$, where p, q and r are integers. By Linear-Diophantine theorem, the equation $px + qy = r$ will have solutions (where x and y are integers) if and only if r is a multiple of the greatest common divisor (GCD) of p and q. Moreover, if (x_0, y_0) is one solution, then the other solutions will have the form:

$$x = x_0 + k\frac{q}{gcd(p,q)}, \; y = y_0 - k\frac{p}{gcd(p,q)} \quad (9)$$

Comparing with (8), we get $p = M, \; q = \frac{(M-1)M}{2}$ and $r = L$. Therefore, by Linear-Diophantine theorem, this equation will have solutions if and only if L is a multiple of the greatest common divisor(GCD) of M and $\frac{(M-1)M}{2}$.

If M is odd, GCD of M and $\frac{(M-1)M}{2}$ is M, then other solutions will be of the form

$$x = x_0 + k\frac{M-1}{2}, y = y_0 - k \quad (10)$$

If M is even, we can find GCD using Extended Euclidean algorithm. From the Extended Euclidean algorithm, given any integers p and q you can find integers m and n such that $pm+qn = gcd(p, q)$; the numbers m and n are not unique, but we only need one pair. Since, we are assuming that $gcd(p, q)$ divides r, there exists an integer l such that $gcd(p, q)l = r$. Multiplying $pm + qn = gcd(p, q)$ through by l, we get

$$p(ml) + q(nl) = gcd(p, q)l = r \quad (11)$$

So this gives one solution, with $x_0 = ml$ and $y_0 = nl$. Thus, if $px_0 + qy_0 = c$ is any solution, then all solutions are of the form

$$x = x_0 + k\frac{q}{gcd(p,q)}, y = y_0 - k\frac{p}{gcd(p,q)} \quad (12)$$

where k is any integer.

Out of infinitely many solutions, we are interested in only one solution, which maximizes $E(Y)$. Also, solutions with only real positive integers are selected, i.e. $x > 0$ and $y > 0$.

The expected value $E(Y)$ can be represented as below.

$$\begin{aligned} E[Y] &= \sum_{i=1}^{M} n_i \times p_i \\ &= ap_1 + (a + d)p_2 + \cdots + (a + (M - 1)d)p_M \\ &= a + d\sum_{j=1}^{M}(j-1)p_j \\ &= a + \mu d \end{aligned} \quad (13)$$

where $\mu = \sum_{j=1}^{M}(j-1)p_j$

Since, expected value $E(Y)$ increases, if more number of samples are allocated to the interval which has maximum probability. It is intuitive that a solution of Linear-Diophantine equation which has maximum value of d must be selected. Using the value of d, we can select corresponding value of a, since (a, d) together satisfies Linear Diophantine equation. Thus, we can find number of samples required in each interval of coarse partition.

Theorem: Assume that $\{(a_1, d_1), \ldots, (a_l, d_l), \ldots, (a_K, d_K)\}$ are the set of solutions of Linear Diophantine equation among the infinitely many solutions, which are positive real integers. If $a_1 < \cdots < a_l < \cdots < a_K$, then $d_1 > \cdots > d_l > \cdots > d_K$. In such a case, (a_1, d_1) is the best solution, which maximizes the expected value $E(Y)$.

Proof: To prove that (a_1, d_1) maximizes $E(Y)$, we need to prove the below expression

$$a_l + \mu d_l \leq a_1 + \mu d_1 \text{ for } l = 2, 3, \ldots, K \quad (14)$$

$$i.e. \; a_l - a_1 \leq \mu(d_1 - d_l) \text{ for } l = 2, 3, \ldots, K$$

$$i.e. \; \frac{a_l - a_1}{d_1 - d_l} \leq \mu \text{ for } l = 2, 3, \ldots, K \quad (15)$$

Since, (a_1, d_1) and (a_l, d_l) both satisfy the Linear Diophantine equation (8). Therefore, we can write

$$M \times a_1 + \frac{(M-1)M}{2} \times d_1 = L \quad (16)$$

$$M \times a_1 = L - \frac{(M-1)M}{2} \times d_1$$

$$M \times a_l + \frac{(M-1)M}{2} \times d_l = L \quad (17)$$

$$M \times a_l = L - \frac{(M-1)M}{2} \times d_l$$

Multiplying (15) by M on both sides, we get

$$M \times \frac{a_l - a_1}{d_1 - d_l} \leq M \times \mu \quad (18)$$

$$\frac{M \times a_l - M \times a_1}{d_1 - d_l} \leq M \times \mu \quad (19)$$

$$\frac{\left[L - \frac{(M-1)M}{2} \times d_l\right] - \left[L - \frac{(M-1)M}{2} \times d_1\right]}{(d_1 - d_l)} \leq M \times \mu \quad (20)$$

$$\frac{\frac{(M-1)M}{2} \times (d_1 - d_l)}{(d_1 - d_l)} \leq M \times \mu \quad (21)$$

$$\frac{(M-1)}{2} \leq \mu \quad (22)$$

Substituting the value of μ, we get

$$\frac{(M-1)}{2} \leq \sum_{j=1}^{M}(j-1)p_j \quad (23)$$

Therefore, it is enough to prove the above expression in order to prove (a_1, d_1) is the best solution, which maximizes $E(Y)$.

By the proof of contradiction, it can be proved that the equality in (23) holds only for uniform distribution, i.e., $p_1 = p_2 = \cdots = p_M = \frac{1}{M}$. Therefore,

$$\sum_{j=1}^{M}(j-1)p_j = \frac{1}{M}\sum_{j=1}^{M}(j-1) = \frac{(M-1)}{2} \tag{24}$$

For any other probability distribution, we get

$$\frac{(M-1)}{2} < \sum_{j=1}^{M}(j-1)p_j \tag{25}$$

Proof: To prove that minimum value of $\sum_{j=1}^{M}(j-1)p_j$ occurs only for uniform distribution, i.e., $p_1 = p_2 = \cdots = p_M = q = \frac{1}{M}$, we use the proof of contradiction. Suppose that the probability masses $p_1 = q + \epsilon, p_2 = p_3 = \cdots = p_{M-1} = q = \frac{1}{M} and p_M = q + \epsilon$ gives the minimum value of $\sum_{j=1}^{M}(j-1)p_j$, where $\epsilon > 0$ is a small positive real number. Using the given probabilities, we can calculate value of the expression $\sum_{j=1}^{M}(j-1)p_j$ as below:

$$\begin{aligned}\sum_{j=1}^{M}(j-1)p_j = 0 \times (q-\epsilon) + \sum_{j=2}^{M-1}(j-1) \times q \\ + (M-1) \times (q+\epsilon)\end{aligned} \tag{26}$$

$$\begin{aligned}\sum_{j=1}^{M}(j-1)p_j &= \sum_{j=1}^{M-1}(j-1) \times q + (M-1) \times (q+\epsilon) \\ &= \sum_{j=1}^{M}(j-1) \times q + (M-1) \times \epsilon \\ &= \frac{1}{M} \times \sum_{j=1}^{M}(j-1) + (M-1) \times \epsilon \\ &= \frac{1}{M} \times \frac{(M-1)M}{2} + (M-1) \times \epsilon \\ &= \frac{(M-1)}{2} + (M-1) \times \epsilon\end{aligned} \tag{27}$$

Since $\epsilon > 0$, it contradicts our supposition. Therefore, we can conclude that the value of $\sum_{j=1}^{M}(j-1)p_j > \frac{(M-1)}{2}$ for any probability distribution and minimum value occurs for uniform distribution.

IV. Reconstruction of Original Signal

In literature, perfect reconstruction of bandlimited continuous time signal from discrete time samples is given by Lagrange interpolation. For uniform sampling, Lagrange interpolation reduces to sinc interpolation and can be realized using a well-designed low pass filter. However, in case of nonuniform sampling, limitations on numerical computation make Lagrange interpolation impractical.

Therefore in practice, approximations for exact interpolation function are obtained for accurate and efficient reconstruction from nonuniform samples. B-splines interpolation is popular approximation technique used for reconstruction of original signal due to their smoothness and finite support, which makes them computationally easy to implement. Splines are piecewise polynomials with pieces that are smoothly connected together. The joining points of the polynomials are called knots. For a spline of degree, each segment is a polynomial of degree , which would suggest that we need coefficients to describe each piece. However, there is an additional smoothness constraint that imposes the continuity of the spline and its derivatives up to order at the knots, so that, effectively, there is only one degree of freedom per segment.

Splines are piecewise polynomials with pieces that are smoothly connected together. The joining points of the polynomials are called knots. For a spline of degree n, each segment is a polynomial of degree n, which would suggest that we need $n+1$ coefficients to describe each piece. However, there is an additional smoothness constraint that imposes the continuity of the spline and its derivatives up to order $n-1$ at the knots, so that, effectively, there is only one degree of freedom per segment. Any spline function of order k on a given set of knots can be uniquely expressed as a linear combination of B-splines as below [16]:

$$s(x) = \sum_{k \in Z} c(k)\beta^n(x-k) \tag{28}$$

where $\beta^n(x)$ is central B-spline function of degree n, $c(k)$ are B-spline coefficients.

We now consider spline interpolation problem where coefficients are determined such that the function goes through the data points exactly. In order to derive these coefficients, discrete B-spline kernel b_m^n is used, which is obtained by sampling the B-spline of degree n expanded by factor m:

$$b_m^n(k) = \beta^n(\frac{x}{m})\big|_{x=k} \overset{z}{\leftrightarrow} B_m^n(z) = \sum_{k \in Z} b_m^n(k)z^{-k} \tag{29}$$

Now, for a given signal samples $s(k)$, we need to determine coefficients $c(k)$ of B-spline model such that we have a perfect fit at integers; i.e. $\forall k \in Z$.

$$\sum_{l \in Z} c(l)\beta^n(l-k)\big|_{x=k} = s(k) \tag{30}$$

Using the discrete B-spline, this constraint can be rewritten in the form of a convolution

$$s(k) = (b_1^n * c)(k) \tag{31}$$

Defining the inverse convolution operator

$$(b_1^n)^{-1}(k) \overset{z}{\leftrightarrow} \frac{1}{B_1^n(z)} \tag{32}$$

The solution is found by inverse filtering

$$c(k) = (b_1^n)^{-1}(k) * s(k) \tag{33}$$

Using the above coefficients, we can reconstruct original signal approximately and accurately from nonuniform samples.

V. Simulation Results

In order to minimize an approximation error between original signal and reconstructed signal, it is necessary to allocate more number of samples in the region having large variation in the signal amplitude and less number of samples in the region

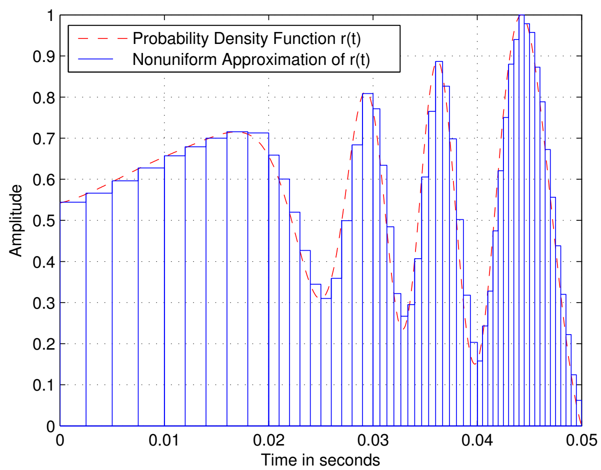

Fig. 3. Nonuniform sampling of probability density function $r(t)$ approximated by a set of rectangles.

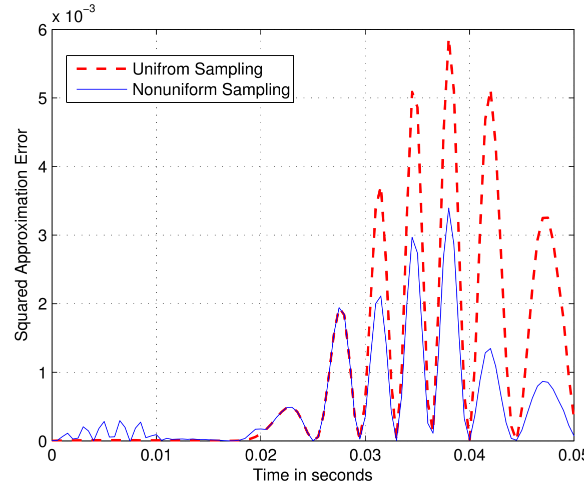

Fig. 4. A comparison of squared approximation error between uniform sampling and nonuniform sampling.

where there is a small variation in the signal. Probability values computed using (5) provide the information about the variation of the signal in the intervals of coarse partition. To illustrate the algorithm, we have considered rectangular approximation of a signal. In order to reduce the approximation error, it is necessary to approximate probability density function by using a set of rectangles in an intelligent manner. In the intervals which have more probability values, will be approximated by more number of rectangles of smaller width, whereas the intervals having less probability values, will be approximated by less number of rectangles of larger width. Nonuniform sampling of probability density function is shown in Fig. 3. Approximation error of uniform sampling and proposed nonuniform sampling is compared in Fig. 4.

Similar approach can be used to sample actual signal $g(t)$. Probability values computed using (5) determine the number of samples to be allocated in the intervals of coarse partition. As probability values depend upon the variation in the signal amplitude, number of samples which are allocated in different intervals of coarse partition will differ significantly, resulting into nonuniform sampling.

VI. Conclusion

In this paper, a nonuniform sampling is discussed for a time limited continuous time signal based on integer linear programming. In nonuniform sampling, more samples are allocated to the highly varying region, whereas few samples are allocated to slow varying region of the signal, thus retaining maximum information of the signal. The problem for nonuniform allocation of samples has been formulated and the solutions based on Integer Linear Programming technique are proposed. The B-spline interpolation function is used for reconstruction of original signal. An squared approximation error is compared between uniform sampling and nonuniform sampling. The work will be extended for online sampling of the signal. Also, it can be extended to the nonuniform sampling of 2D and 3D signals.

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