Energy Efficient Design of Mobile Wireless Sensor Networks: Constrained Clustering

by

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Abstract

Mobility has long been recognized as an efficient method of improving system performance in wireless sensor networks (WSNs), e.g. relieving traffic burden from a specific set of nodes. Though tremendous research efforts have been devoted to this topic during the last decades, yet little attention has been paid for the cluster head selection. In this research paper we formulate the problem of choosing the cluster head (taking into account distance between sensors along with other factors) as a quadratic optimization problem and solve the problem. To the best of our knowledge the derivation of centroid of patterns based on interesting optimization problem is novel and is not known in the literature. We derive the position of centroid by formulating and solving an optimization problem. By allowing mobility to sensors and/or base station/sink.

Wireless Sensor Networks, Mobility, Energy Efficiency, Clustering.

1 Introduction

In wireless sensor networks (WSNs), tiny battery powered sensors are coordinated to gather information about a sensed variable and communicate the information to a Base station/Sink. In many applications, replacement of battery in the sensor is difficult or even not possible. Thus, the design of protocols in such networks must be energy efficient. Various researchers proposed hierarchical energy efficient routing/fusion algorithms such as LEACH, HEED etc. In such algorithms, a group of wireless sensor nodes forming a CLUSTER have a representative leader called CLUSTER HEAD. Periodically the cluster head is rotated among the members of the cluster. The cluster heads essentially participate in data fusion and routing the sensed packets to the Base station/sink. In many interesting applications, the sensors are geographically distributed with certain distance between them. In most existing energy efficient algorithms, distance between sensors is not taken into account for deciding the cluster head. In this research paper we formulate the problem of choosing the cluster head (taking into account distance between sensors along with other factors) as a quadratic optimization problem and solve the problem. The cluster head is chosen to be CENTROID of sensor position coordinates. To the best of our knowledge the derivation of centroid of patterns based on interesting optimization problem is novel and is not known in the literature. We consider the wireless sensor network (WSN) where the sensors know their position (i.e 3 spatial coordinated are known through say GPS).

Cluster head election in LEACH and other energy efficient algorithms involves local exchange of Hello messages among the members of a cluster. Ensuring that cluster head is located at the centriod (that is through minimization of squared distances from centroid/cluster head to sensors) ensures that the signal strength is as good as possible for wireless messages. Also, with selection of cluster heads as centriods ensures that the relaying of messages from sensors to base station /sink is of good quality.

Since remaining battery energy is a quantity utilized to select a cluster head, the sensor node with relatively large residual energy and also closer to cluster head is chosen as the CLUSTER HEAD. Also in case sensors are mobile, the mobile sensor chosen as the cluster head is moved to the CENTROID position. The idea of cluster head selection as being close to centriod applies to all hierarchical protocols where clustering of sensors is carried out. If the placement of sensors (mobile or static) is under the control of user, cluster head is placed at the centroid position. Specifically in the initial deployment of sensors over the sensors field, cluster heads are placed at the centroid.(i.e cluster reconfiguration)
We consider two cases.
i)planned deployment
ii)random deployment
In both cases, we derive the position of centroid by formulating and solving an optimization problem. By allowing mobility to sensors and/or base station/sink we are led to the following four paradigms of wireless sensor networks.
i)Both sensors and base station is static
ii)Mobile sensors and base station static
iii)Static sensors and base station mobile
iv)Both sensors and base station is mobile
In this research paper, we consider the paradigm where Base station/Sink is mobile and sensors are static. The results of this paper are potentially applicable to other mobile paradigms.

2 Random Sensor Deployment : Clustering

We now consider a WSN in which the sensors are random deployed (i.e the spatial position coordinates such as (X_1,X_2,X_3) are random variables). For generality in derivation,

consider N data points in M-dimensional pattern space (i.e M-dimensional random vectors) that are totally N of them (i.e components of these vectors are random variables). Where whose mean needs to be determined.

$X_1, X_2, \ldots, X_N$ with $X_j = [x_{j1}, x_{j2}, \ldots, x_{jm}]$ for

$$1 \leq j \leq N \tag{1}$$

Let X_0 be the desired centroid i.e $X_0 = [x_{01}, x_{02}, \ldots, x_{0m}]$ where random variables to be determined problem: compute X_0 (i.e centroid in such a way that mean value of squared of euclidean distances from centroid to pattern vectors is minimized).

$$J(X_0) = J[\tilde{x}_{01}, \tilde{x}_{02}, \ldots, \tilde{x}_{0m}] = E[\sum_{j=1}^{N} \sum_{i=1}^{m} [x_{ji} - x_{0i}]^2] \tag{2}$$

We would like to solve this unconstrained optimization problem. We exchange differentiation and summation.

$$\frac{\delta J(X_0)}{\delta x_{0i}} = \sum_{j=1}^{N} 2E[x_{ji} - \tilde{x}_{0i}](-1) \tag{3}$$

setting it to zero, we have

$$\sum_{i=1}^{N} Ex_{ji} = (N)\tilde{(}x_{0i}) for 1 \leq i \leq M \tag{4}$$

Thus, we have

$$\tilde{x}_{0i} = \frac{1}{(N)} \sum_{j=1}^{N} (Ex_{ji}) for 1 \leq i \leq M \tag{5}$$

That is

$$\tilde{x}_{01} = \frac{1}{(N)} \sum_{j=1}^{N} (Ex_{j1}) \tag{6}$$

and so on

We will now prove that such a centroid $X_0 = [Ex_{01}, Ex_{02}, \ldots, Ex_{0m},]$ is indeed a global minimum point (i.e we compute second partial derivatives and reason that hessian matrix is positive i.e eigen values are all positive and we have a diagonal hessian matrix).

$$\frac{\delta^2 J(x_0)}{\delta \tilde{x}_{0k}^2} = 2N forki \tag{7}$$

$$\frac{\delta^2 J(X_0)}{\delta \tilde{x}_{0k} \delta \tilde{x}_{0i}} = 0 fork \neq i \tag{8}$$

Thus, Hessian matrix is positive definite diagonal matrix as N > 0

constrained centroid computation.

3 Constrained Clustering: planned sensor placement

We now formulate and solve the constrained clustering problem. Specifically, we consider the case where patterns are deterministic and NOT random vectors.
Problem : The optimization problem is to minimize
Subject to the following constraint (i.e squared Euclidean distance vectors to centroid vector).

$$J(X_0) = E[\sum_{j=1}^{n} \sum_{i=1}^{m} [x_{ji} - x_{0i}]^2] + \lambda [\sum_{k=1}^{m} (x_{Nk} - x_{ok})^2) - S] \tag{9}$$

X_N Extreme point set L=(N-1)

$$\frac{\delta J(X_0)}{\delta x_{0i}} = \sum_{j=1}^{L} 2[x_{ji} - x_{0i}](-1) + \lambda(-2)(x_{Ni} - x_{oi}) for 1 \leq i \leq M \tag{10}$$

setting it to zero,we have

$$\sum_{j=1}^{L} x_{ji} - L(x_{oi}) - \lambda(x_{oi}) + \lambda(x_{Ni}) for 1 \leq i \leq M \tag{11}$$

$$\sum_{j=1}^{L} x_{ji} - x_{oi}(L + \lambda) + \lambda(x_{Ni}) = 0 for 1 \leq i \leq M \tag{12}$$

$$x_{oi} = \frac{\sum_{j=1}^{L} x_{ji} + \lambda(x_{Ni})}{L + \lambda} for 1 \leq i \leq M \tag{13}$$

Equality constraint

$$\sum_{k=1}^{m}(x_{Nk}-x_{ok})^2 = S \tag{14}$$

$$\sum_{k=1}^{m}\left(x_{Nk}-\frac{(\sum_{j=1}^{L}x_{jk}+(\lambda)x_{Nk})}{L+\lambda}\right)^2 = S \tag{15}$$

$$\sum_{k=1}^{m}\left[\frac{(L+\lambda)x_{Nk}-(1+\lambda)x_{Nk}-\sum_{j=1}^{N-1}x_{jk}}{L+\lambda}\right]^2 = S \tag{16}$$

$$\sum_{k=1}^{m}\left[\frac{Lx_{Nk}-\sum_{j=1}^{N-1}x_{jk}}{L+\lambda}\right]^2 = S \tag{17}$$

Let

$$\sigma = \sum_{k=1}^{m}((L)x_{Nk}-\sum_{j=1}^{N-1}x_{jk})^2 = (L+\lambda)^2 = S \tag{18}$$

$$(\lambda^2+L^2+2\lambda L)S = \sigma Let\frac{\sigma}{S} = \beta \tag{19}$$

$$\lambda^2+2\lambda L+L^2 = \beta \tag{20}$$

$$\lambda^2+2\lambda L+L^2-\beta = 0 \tag{21}$$

i.e There are two solution for 'λ'

$$\lambda = \frac{-2L+\sqrt{4L^2-4(L^2-\beta)}}{2} \tag{22}$$

$$= \frac{-2L+\sqrt{4\beta}}{2} \tag{23}$$

$$= -L+\sqrt{\beta)} \tag{24}$$

we now calculate the hessian matrix of second partial derivatives

$$\frac{\delta^2 J(X_0)}{\delta x_{0i}^2} = \sum_{j=1}^{L}\frac{\delta}{\delta x_{oi}}(-2)(x_{ji}-x_{oi})+\lambda(-2)\frac{\delta}{\delta x_{oi}}(x_{Ni}-x_{oi}) \tag{25}$$

$$= \sum_{j=1}^{L}(2)+(-2\lambda)(-1) = 2L+2\lambda \tag{26}$$

$$\frac{\delta^2 J(X_0)}{\delta x_{0k}\delta x_{0i}} = 0fork \neq i \tag{27}$$

Hessian matrix :

$$\left\{ \begin{array}{cccc} 2L+2\lambda & 0 & \ldots & 0 \\ 0 & \ldots & 2L+2\lambda & 0 \\ 0 & 0 & \ldots & 2L+2\lambda \end{array} \right\}$$

Thus, for uniqueness of minimal solution

$$2L + 2\lambda > 0 \tag{28}$$

$$2L + 2(-L + \sqrt{\beta}) = 2\sqrt{\beta} > 0 \tag{29}$$

i.e λ = -L - $\sqrt{\beta}$ is eliminated. Constraint : the squared Euclidean distance from pattern vector x_n to the centroid is equal to value "S".
Note: The derivation in this section can easily be generalized for random sensor pattern deployment (as derived in section 2). Detailed duplication of derivation is avoided for brevity

4 Conclusion

In this research paper, we consider energy efficient, hierarchical routing protocols (such as LEACH,HEED)and proposed cluster head posion as the centriod of sensor positions. We considered random deployment as well as planned deployment of sensors. We formulated and solved the associated optimization problems using Lagrange multiplies method (for N sensors/patterns in M – dimensional space i.e M=3 for WSN application). We thus proposed OPTIMAL CLUSTERING of sensor nodes (with respect to centroid position) for a certain paradigm of mobile wireless sensor networks (WSN). We expect the results to be useful for DYNAMIC CLUSTERING in arbitrary mobile WSN.