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Sum-Rate Maximization in Non-Orthogonal Multiple Access Relay Networks

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***Abstract*—Non-orthogonal multiple access (NOMA) has been conceived to be promising technology for the fifth generation (5G) wireless communication systems. In this paper, we investigate the downlink sum rate maximization problem in a relay based network which employs NOMA principles for encoding and decoding. The problem to be optimized is non-convex. In order to get around the difficulty to deal with non-convexity, we propose a minorization-maximization technique based algorithm to jointly optimize source power allocation and relay precoder design. We then solve the problem to optimally allocate power at source and design the precoder at relay. For the case of two destination nodes, we propose a grid search based approach. Simulation results are provided to illustrate the performance of the proposed algorithm and also the comparison of the proposed approach with existing approaches.**

***Index Terms*—Non-orthogonal multiple access, optimization, relay, sum rate.**

I. Introduction

Multiple access techniques in wireless communication systems have always been a crucial factor in determining the spectral efficiency, quality of service (QoS) and reliability. Hence, the design of multiple access technique forms the crucial part of physical layer design. Contemporary orthogonal multiple access techniques (OMA) such as time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA) suffer from spectral inefficiency. In this regard, non-orthogonal multiple access (NOMA) has been envisaged to be the promising technology for fifth generation (5G) wireless systems. NOMA utilizes power domain to multiplex the users as opposed to OMA such as TDMA, FDMA and CDMA, which multiplex in time, frequency and code, respectively [1]–[3]. It has been shown in the earlier works that NOMA outperforms conventional OMA techniques in terms of spectral efficiency and achievable sum rate [4], [5].

In NOMA, the source transmits the signal by multiplexing the messages intended for all the users using the superposition coding technique. At the destination node, users employ successive interference cancellation (SIC) technique to decode their signal [6]. For this to happen successfully the users are arranged in increasing order of their effective channel gains. The power allocation to the users can be done according to their effective channel gains i.e., weak user gets more power share than a strong user, or can be done based on their required QoS [5], [7]. In this way, effective sum rate can be improved and also weak users will be served with optimal resources.

When distance between source and destination node is larger it causes severe path loss of the signal. To circumvent this problem relay systems play a crucial role. Broadly there are two types of relay systems one being amplify and forward (AF) relays and other being decode and forward (DF) relays. Relay based NOMA systems have been studied in [8], [9].

There has been increase in interest in designing NOMA-based systems with various optimization criterion. Optimal source precoder design to maximize sum rate in downlink NOMA based multiple output single input system (MISO) system using minorization-maximization (MM) technique is studied in [10]. In [11], maximizing the achievable rate of the destination node with best channel conditions is exploited subject to minimum target rates to other destination nodes. In this paper, we investigate sum rate maximization in NOMA based system with a source node, AF relay node and multiple destination nodes. The problem being non convex, we employ MM technique to approximate the problem to convex when one of the optimization variables is fixed. Then we employ coordinate ascent based algorithm to maximize the objective.

$Notations$: Bold lowercase and uppercase letters are used to represent vectors and matrices, respectively. The transpose, hermitian transpose, and trace of a matrix A are denoted as $\mathbf{A}^T$, $\mathbf{A}^H$, and $tr\left(\mathbf{A}\right)$, respectively. The $vec\left(\mathbf{A}\right)$ denote the vectorization of matrix and the Kronecker product are denoted by $\mathbf{A}$ and $\otimes$, respectively. The notations $\mathbf{A} \succ 0$ and $\mathbf{A} \succeq 0$ indicate positive definite and positive semi-definite matrices, respectively. The symbols $\mathbb{C}^{N\times N}$, $\mathbb{R}^{N\times N}$ and $\mathbb{R}_+^{N\times N}$ are used for $N \times N$-dimensional complex, real and nonnegative real spaces, respectively. The $\mathbf{l_2}$ norm of a vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is denoted as $\|\mathbf{x}\|$ which is defined as $\|\mathbf{x}\| = \sum_{n=1}^{N} |x_n|^2$ where $|x_n|$ is the absolute value of n^{th} coordinate of vector $\mathbf{x}$. $\mathcal{CN}\left(\boldsymbol{\mu}, \mathbf{C}\right)$ indicates circularly symmetric complex Gaussian vector with mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$. The minimum, maximum and statistical expectation, argument functions are denoted by min(·), max(·), $\mathbb{E}[\cdot]$ and $\arg_{\{\}}(\cdot)$, respectively.

II. System Model

We consider the downlink scenario for the NOMA principle based network which consists of a source node with single antenna, an AF relay node with N antennas and K destination nodes with single antenna each. Direct links between source and K destinations nodes are ignored as they undergo large path loss when compared with the links via relay. The network functions in half-duplex mode i.e., communication between the cognitive source and K cognitive destination nodes takes place in two time slots. In the first time slot, the source encodes all K destination symbols $s_k \in \mathbb{C}^{1\times 1}$ and $\mathbb{E}\big[\,|s_k|^2\big] = 1\ \forall\ k \in \mathbb{K} \triangleq \{1, 2, \ldots, K\}$ using superposition coding and transmits the signal

$$x_s = \sum_{k=1}^{K} \sqrt{P_k}\, s_k, \tag{1}$$

where P_k is the transmit power allocated to the symbol intended for k^{th} destination node. The signal received at the relay is given by

$$\mathbf{r} = \mathbf{h}x_s + \mathbf{w}, \tag{2}$$

where $\mathbf{h} \in \mathbb{C}^{N\times 1}$ is the channel gain vector from source to relay and $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2\mathbf{I})$ is the additive white Gaussian noise vector at relay. In the second time slot, the relay node amplifies the received signal with precoding matrix $\mathbf{F} \in \mathbb{C}^{N\times N}$ and transmits the following signal

$$\mathbf{x_r} = \mathbf{Fr}. \tag{3}$$

The signal received at the k^{th} destination node is given by,

$$y_k = \mathbf{g}_k^H\mathbf{Fh}x_s + \mathbf{g}_k^H\mathbf{Fw} + n_k, \tag{4}$$

where $\mathbf{g}_k \in \mathbb{C}^{N\times 1}$ is the channel gain vector from relay to k^{th} destination node, and noise term $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the additive white Gaussian noise at k^{th} destination node. The destination nodes use SIC for retrieving the message signal. We assume $\|\mathbf{g}_1\| \leq \|\mathbf{g}_2\| \leq \ldots \leq \|\mathbf{g}_K\|$, which is essential for SIC decoding. The achievable signal-to-interference-plus-noise ratio (SINR) $\Gamma_{k,j}$ at j^{th} destination node after it removes k^{th} destination message using SIC, $j \in \mathbb{J} \triangleq \{k, k+1, \ldots, K\}$, based on (4) is

$$\Gamma_{k,j} = \frac{P_k|\mathbf{g}_j^H\mathbf{Fh}|^2}{\left(\sum_{i=k+1}^{K} P_i\right)|\mathbf{g}_j^H\mathbf{Fh}|^2 + \sigma_w^2\mathbf{g}_k^H\mathbf{FF}^H\mathbf{g}_k + \sigma_k^2}, \tag{5}$$

The achievable rate after SIC operation at the j^{th} destination node to decode signal of k^{th} destination node is

$$R_{k,j} = \frac{1}{2}log_2(1 + \Gamma_{k,j}). \tag{6}$$

For k^{th} destination node signal to be decoded successfully at j^{th} destination node it is required that $\tilde{R}_k \triangleq min(R_{k,j})\ \forall\ j \in \mathbb{J}$ has to be at least equal to achievable rate of k^{th} user i.e., $R_{k,k}$. This is the rate with minimum SINR $\Gamma_{k,j}\ \forall\ j \in \mathbb{J}$. Based on this criterion, the sum rate $\tilde{R}_{sum}$ is given as

$$\tilde{R}_{\text{sum}} = \frac{1}{2}\sum_{k=1}^{K} \log_2\big(1 + min\big(\Gamma_{k,j}\big)\big)\ \forall\ j \in \mathbb{J}. \tag{7}$$

In addition to the above requirement, in order to boost SINR of the destination nodes with weak channel gains and to ensure non-zero rate allocation for such users we consider

$$P_1 \geq P_2 \ldots \geq P_K. \tag{8}$$

The total power allocation at source and the relay is constrained as

$$\sum_{k=1}^{K} P_k \leq P_s\ , \tag{9a}$$

$$tr\big(\mathbf{F}\mathbb{E}\big[\mathbf{rr}^H\big]\mathbf{F}\big) \leq P_r \tag{9b}$$

where P_s and P_r are power budgets at source and relay nodes respectively. We can reformulate relay power constraint using (2) as

$$\|\mathbf{Fh}\|_2^2 P_s + \sigma_w^2 tr\big(\mathbf{FF}^H\big) \leq P_r. \tag{10}$$

III. Source Power Allocation and Relay Precoder Design

We study the problem of maximizing the sum rate (7) by optimally designing source power allocation and relay precoder design. Thus, this optimization problem can be formulated as follows:

$$\max_{\mathbf{F},\{P_k\}} \quad \frac{1}{2}\sum_{k=1}^{K} \log_2\left(1 + \min_j\left(\Gamma_{k,j}\right)\right), \forall\ j \in \mathbb{J}, \tag{11a}$$

$$\text{s.t.} \quad P_k \leq min\left(P_1 \ldots P_{k-1}\right), \forall\ k \in \mathbb{K}, \tag{11b}$$

$$\sum_{k=1}^{K} P_k \leq P_s, \tag{11c}$$

$$\|\mathbf{Fh}\|_2^2 P_s + \sigma^2 tr\big(\mathbf{FF}^H\big) \leq P_r\ . \tag{11d}$$

The problem in (11) is non-convex as it involves SINR terms and globally optimal solution is intractable to obtain. So, we employ several steps to approximate it to convex which ensures mathematical tractability.

The objective in problem (11) in its present form is difficult to deal with as it involves SINR terms, which are non-convex. So, we can equivalently re-write it as

$$\max_{\mathbf{F},\{P_k\},\{r_k\}} \quad \left(\prod_{k=1}^{K} r_k\right)^{\frac{1}{2K}}, \tag{12a}$$

$$s.t. \quad r_k - 1 \leq \min\left(\Gamma_{k,j}\right), \tag{12b}$$

$$\text{(11b), (11c) \& (11d)}, \tag{12c}$$

$$\forall\ k \in\ \mathbb{K},\ j\ \in\ \mathbb{J},$$

where $r_k \in \mathbb{R}_+^{1\times 1}\ \forall\ k \in \mathbb{K}$ and objective (12a) is obtained by considering the fact that $\log_2(\cdot)$ is a monotonically increasing function and the geometric mean of r_k for $\forall\ k \in \mathbb{K}$ is concave and increasing. Thus, geometric mean can be expressed as system of second-order cone (SOC) constraints [12]. Hence, this conversion doesn't affect the optimal solution of the objective function (11d).

The problem (12) is still non-convex due to the constraint

(12b). We can write the constraint (12b) explicitly using (5) as

$$r_k - 1 \leq \frac{P_k|\mathbf{g}_j^H\mathbf{F}\mathbf{h}|^2}{\left(\sum_{i=k+1}^{K} P_i\right)|\mathbf{g}_j^H\mathbf{F}\mathbf{h}|^2 + \sigma_w^2\mathbf{g}_j^H\mathbf{F}\mathbf{F}^H\mathbf{g}_j + \sigma_k^2}, \mathbb{J}. \tag{13}$$

Nevertheless, it can be approximated to convex when either of the optimization variables $\mathbf{F}$ *or* $\{P_k\}$ is fixed. For this purpose we handle each case individually as follows:

For given Relay Precoding Matrix $\mathbf{F}$

The contraint (13) can be written as

$$r_k - 1 \leq \frac{P_k\vartheta_j}{\left(\sum_{i=k+1}^{K} P_i\right)\vartheta_j + \omega_j}, \ \forall\ k \in \mathbb{K},\ j \in \mathbb{J}, \tag{14}$$

$$\text{where,}\ \ \vartheta_k = |\mathbf{g}_k^H\mathbf{F}\mathbf{h}|^2, \omega_k = \sigma_w^2\mathbf{g}_k^H\mathbf{F}\mathbf{F}^H\mathbf{g}_k + \sigma_k^2. \tag{15}$$

For given $m_k \in \mathbb{R}_+^{1\times 1}\ \forall\ k \in \mathbb{K}$, it holds that

$$r_k m_k - m_k \leq P_k\vartheta_j, \tag{16a}$$

$$\left(\sum_{i=k+1}^{K} P_i\right)\vartheta_j + \omega_k \leq m_k,\ \forall\ k\ \in \mathbb{K},\ j\ \in \mathbb{J}. \tag{16b}$$

But, the bilinear term in (16a) is non-convex because Hessian matrix for bilinear term is not positive semidefinite. So, we approximate it to convex by employing the following steps

$$r_k m_k = 0.25\left(r_k + m_k\right)^2 - 0.25\left(r_k - m_k\right)^2. \tag{17}$$

As both terms right side of equality are individually convex we can approximate the term $\left(r_k - m_k\right)^2$ by employing MM technique.

Consider a real valued convex function $u(\mathbf{z})$, it follows from first order condition for convexity [13] that

$$u\left(\mathbf{z}\right) \geq u\big(\mathbf{z}^t\big) + \left(\nabla_{\mathbf{z}} u\big(\mathbf{z}^t\big)\right)^T\left(\mathbf{z} - \mathbf{z}^t\right) \triangleq v\big(\mathbf{z}, \mathbf{z}^t\big), \tag{18}$$

where, $v\big(\mathbf{z}, \mathbf{z}^t\big)$ is the Taylor's first order approximation of the function $u\left(\mathbf{z}\right)$ around $\mathbf{z}^t$. The function $v\big(\mathbf{z}, \mathbf{z}^t\big)$ is minorized version of $u\left(\mathbf{z}\right)$ and is called as surrogate function. Following properties hold true from (18):

$$u\big(\mathbf{z}\big) \geq v\big(\mathbf{z}, \mathbf{z}^t\big), \forall\ \mathbf{z}, \tag{19a}$$

$$u\big(\mathbf{z}^t\big) = v\big(\mathbf{z}^t, \mathbf{z}^t\big), \tag{19b}$$

$$\nabla_{\mathbf{z}} u\left(\mathbf{z}\right) = \nabla_{\mathbf{z}} v\big(\mathbf{z}, \mathbf{z}^t\big),\ \ for\ any\ \mathbf{z} = \mathbf{z}^t \tag{19c}$$

The basic idea of MM technique is to maximize the surrogate function $v\big(\mathbf{z}, \mathbf{z}^t\big)$ over $\mathbf{z}$, in order to obtain the next iteration point to linearize the function $u\left(\mathbf{z}\right)$, i.e.,

$$\mathbf{z}^{t+1} = \max_{\mathbf{z}} v\big(\mathbf{z}, \mathbf{z}^t\big). \tag{20}$$

Maximization of the surrogate function drives $u\left(\mathbf{z}\right)$ upwards until it reaches local maximum as follows

$$u\big(\mathbf{z}^{t+1}\big) = u\big(\mathbf{z}^{t+1}\big) - v\big(\mathbf{z}^{t+1}, \mathbf{z}^t\big) + v\big(\mathbf{z}^{t+1}, \mathbf{z}^t\big), \tag{21a}$$

$$\geq v\big(\mathbf{z}^{t+1}, \mathbf{z}^t\big), \tag{21b}$$

$$\geq v\big(\mathbf{z}^t, \mathbf{z}^t\big), \tag{21c}$$

$$= u\big(\mathbf{z}^t\big), \tag{21d}$$

where, (21b) follows from (19a), (21c) is obtained from of (20) and (21d) is due to (19b).

Hence, from the above discussion it is clear that we can maximize a convex function by maximizing its surrogate function. As the linear surrogate functions are computationally less complex, we employ this approach in our problem. Now, the term $\left(r_k - m_k\right)^2$ in (17) can be approximated by its first order Taylor series $\mathcal{L}_k$ around $r_k^t,\ m_k^t$ as

$$r_k m_k = 0.25\left(r_k + m_k\right)^2 - 0.25\mathcal{L}_k, \tag{22}$$

where, $\mathcal{L}_k = \left[\left(r_k^t - m_k^t\right)^2 + 2\left(r_k^t - m_k^t\right)\left(r_k - r_k^t - m_k + m_k^t\right)\right]$

Thus, constraints in (16) become

$$0.25\left(r_k + m_k\right)^2 - 0.25\mathcal{L}_k - m_k \leq P_k\vartheta_j, \tag{23a}$$

$$\left(\sum_{i=k+1}^{K} P_i\right)\vartheta_j + \omega_j \leq m_k,\ \forall\ k\ \in \mathbb{K},\ j\ \in \mathbb{J}. \tag{23b}$$

Thus, the constraints in (23) is now convex in optimization variables $\{P_k\}$ and $\{r_k\}$.

For given Source Power Allocation $\{P_k\}$:

The constraints which depend on $\mathbf{F}$ are (13) and (11d). We can reformulate as follows :

Using the equality [14]

$$\text{vec}\left(\mathbf{M}_1\mathbf{M}_2\mathbf{M}_3\right) = \left(\mathbf{M}_3^T \otimes \mathbf{M}_1\right)\text{vec}\left(\mathbf{M}_2\right),$$

where $\mathbf{M}_1, \mathbf{M}_2$ and $\mathbf{M}_3$ are arbitrary matrices, the constraints (13) and (11d) can be written as

$$r_k - 1 \leq \frac{\mathbf{f}^H\mathbf{B}_{k,j}\mathbf{f}}{\mathbf{f}^H\mathbf{C}_{k,j}\mathbf{f} + \sigma_k^2},\ \forall\ k \in\ \mathbb{K},\ j\ \in\ \mathbb{J}, \tag{24a}$$

$$\mathbf{f}^H\mathbf{D}\mathbf{f} \leq P_r, \tag{24b}$$

where, $\mathbf{f} = vec\left(\mathbf{F}\right)$

$$\begin{aligned}\mathbf{B}_{k,j} &= P_k\left(\mathbf{h}^*\mathbf{h}^T \otimes \mathbf{g}_j\mathbf{g}_j^H\right),\\ \mathbf{C}_{k,j} &= \left(\mathbf{h}^*\mathbf{h}^T \otimes \mathbf{g}_j\mathbf{g}_j^H\right)\left(\sum_{i=k+1}^{K} P_i\right) + \sigma_w^2\mathbf{I} \otimes \mathbf{g}_j\mathbf{g}_j^H,\\ \mathbf{D} &= \left(\mathbf{h}^*\mathbf{h}^T \otimes \mathbf{I}\right)P_s + \sigma_w^2\mathbf{I}.\end{aligned} \tag{25}$$

For given $m_k \in \mathbb{R}_+^{1\times 1}$, it holds that

$$r_k m_k - m_k \leq \mathbf{f}^H\mathbf{B}_{k,j}\mathbf{f}, \tag{26a}$$

$$\mathbf{f}^H\mathbf{C}_{k,j}\mathbf{f} + \sigma_k^2 \leq m_k,\ \forall\ k\ \in \mathbb{K},\ j\ \in \mathbb{J}. \tag{26b}$$

We approximate the bilinear term similar to (22)

$$0.25\left(r_k + m_k\right)^2 - 0.25\mathcal{L}_k - m_k \leq \mathbf{f}^H\mathbf{B}_{k,j}\mathbf{f}, \tag{27a}$$

$$\mathbf{f}^{\ H}\mathbf{C}_{k,j}\mathbf{f} + \sigma_k^2 \leq m_k. \tag{27b}$$

The term $\mathbf{f}^H\mathbf{B}_{k,j}\mathbf{f}$ in (27a) can be linearized to its surrogate function using Taylor's first order approximation. As the function above is real valued with complex domain we use Writinger's derivative [15], [16] to obtain linear function.

Taylor's first order approximation of the function $f(\mathbf{z})$ around point $\mathbf{z}_0$ is given by

$$f(\mathbf{z}, \mathbf{z}_0) = f(\mathbf{z}_0) + 2\Re\{\left(\frac{\partial f}{\partial \mathbf{z}}\right)^T (\mathbf{z} - \mathbf{z}_0)\} \tag{28}$$

The constraint in (27) can be written in SOC constraint form as

$$0.25\,(r_k + m_k)^2 - 0.25\mathcal{L}_k - m_k \leq \tilde{\mathcal{L}}_{k,j}, \tag{29a}$$

$$\left\| \begin{pmatrix} \tilde{\mathbf{C}}_{k,j}\mathbf{f} \\ \sigma_k \\ (m_k - 1)/2 \end{pmatrix} \right\|_2 \leq \ (m_k + 1)/2, \tag{29b}$$

$$\text{where, } \tilde{\mathcal{L}}_{k,j} = 2\Re\left\{ \left(\mathbf{f}^{\ t}\right)^H \mathbf{B}_{k,j}\mathbf{f} \right\} - \left\{ \left(\mathbf{f}^{\ t}\right)^H \mathbf{B}_{k,j}\mathbf{f}^{\ t} \right\},$$
$$\mathbf{C}_{k,j} = \tilde{\mathbf{C}}_{k,j}\tilde{\mathbf{C}}_{k,j}^H, \because \mathbf{C}_{k,j} \succeq 0$$

The final optimization problem which is convex is

$$\max_{\mathbf{F}, \{P_k\}, \{r_k\}, \{m_k\}} \quad \left(\prod_{k=1}^{K} r_k \right)^{\frac{1}{2K}} \tag{30a}$$

$$\text{s.t. (11b), (11c), (23) for given } \mathbf{F} \tag{30b}$$

$$\text{(24b), (29) for given } \{P_k\} \tag{30c}$$

Algorithm 1 MM based NOMA Sum Rate algorithm

initialize $\left[\ r_k^0, m_k^0, P_k^0, \mathbf{F}^{\ 0}\ \right]$ from feasibility set of (11), n=0;
repeat
  Given $\mathbf{F}$ solve (30) iteratively for $\{r_k\}, \{m_k\}, \{P_k\}$
  Given P_k solve (30) iteratively for $\{r_k\}, \{m_k\}, \{\mathbf{F}\}$
  $n = n + 1$;
until Convergence

Remarks : When initial conditions are chosen from feasible set of the original problem (11), $(n+1)^{th}$ iteration in algorithm also produces a solution set from feasibility set. The **Algorithm 1** returns non-decreasing sequence of objective values because surrogate functions used for approximating non-convex functions are non-decreasing with each iteration. As the feasibility set is convex and compact (feasibility set is bounded by power budgets at source and relay), the algorithm do converges to a finite value.

Grid Search based algorithm : In previous section, we proposed MM based problem (30) which achieves local optimal solution for (12). Now, we propose grid search based algorithm over $\{r_k\} \forall\ k \in \mathbb{K}$.

For given relay precoding matrix $\mathbf{F}$, the constraint (14) can be formulated as

$$(r_k - 1)\left[\left(\sum_{i=k+1}^{K} P_k\right)\vartheta_j + \omega_j\right] \leq P_k\vartheta_j, \ \forall\ k \in \mathbb{K}, j \in \mathbb{J}. \tag{31}$$

The problem (12) can be solved using K-D grid search over $\{r_k\}$ and convex feasibility over $\{P_k\}$. This feasibility checking problem is a linear optimization problem.

For given power $\{P_k\}$, the constraints (11d) and (14) can be formulated using (24) into SOC constraints as follows

$$\| \begin{pmatrix} \tilde{\mathbf{C}}_{k,j}\mathbf{f} \\ \sigma \end{pmatrix} \|_2 \leq \ |\mathbf{f}^H \frac{\mathbf{b}_{k,j}}{\sqrt{r_k - 1}}|, \ \forall\ k \in\ \mathbb{K},\ j\ \in\ \mathbb{J}, \tag{32a}$$

$$\|\tilde{\mathbf{D}}\mathbf{f}\|_2 = \sqrt{P_r}, \tag{32b}$$

$$\text{where, } \mathbf{B}_{k,j} = \mathbf{b}_{k,j}\mathbf{b}_{k,j}^H,\ \mathbf{C}_{k,j} = \tilde{\mathbf{C}}_{k,j}\tilde{\mathbf{C}}_{k,j}^H,\ \mathbf{D} = \tilde{\mathbf{D}}\tilde{\mathbf{D}}^H. \tag{32c}$$

Decomposition of matrices is possible due to the fact that $\mathbf{B}_{k,j} \succeq 0$ and is also rank one, $\mathbf{C}_{k,j} \succeq 0$, $\mathbf{D} \succ 0\ \forall\ k \in \mathbb{K},\ j \in \mathbb{J}$. The problem (12) can be solved using K-D grid search over $\{r_k\}$ and convex feasibility over $\mathbf{F}$. This feasibility checking problem is a SOC problem. Hence, the problem (12) can be iteratively solved to obtain globally optimal solution for quasi-convex problem over $\mathbf{F}$ *and* $\{P_k\}$. But, solving this problem requires solving sequence of linear and SOC problems which is computationally exhaustive, but achieves better optimum solution than MM based algorithm.

IV. Simulation Results

All the simulations are performed on MATLAB. For simulations, we consider the channel model to be Rayleigh fading. We consider the gain vector to be $\mathbf{h} \sim \mathcal{CN}\,(\mathbf{0}, \mathbf{I})$ and Channel gain vectors from relay node to destination nodes to be $\mathbf{g}_k \sim \mathcal{CN}\left(\mathbf{0}, \sqrt{d_k^{-\alpha}}\right) \ \forall\ k\ \in\ \mathbb{K}$, where d_k is the normalized distance from relay node to the k^{th} destination node and α denotes the path loss exponent. Path loss exponent considered is $\alpha = 2$. We consider $d_1 = 100$, $d_K = 1$ and all other destination nodes are equally spaced between d_K and d_1. This ensures the assumption of channels $\|\mathbf{g}_1\|_2 \leq \ \|\mathbf{g}_2\|_2\ \ldots\ \leq\ \|\mathbf{g}_K\|_2$. We assume relay node has N = 4 antennas. Source SNR is P_s/σ^2 = 25 dB. We consider all the noise components w *and* $n_k\ \forall\ k \in \mathbb{K}$ to be i.i.d and have unit variance. Simulation results are averaged over 500 realizations.

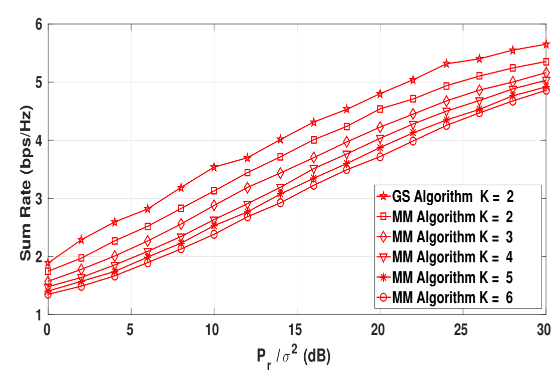

Fig. 1: Achievable sum rate vs P_r/σ^2 of proposed MM based algorithm.

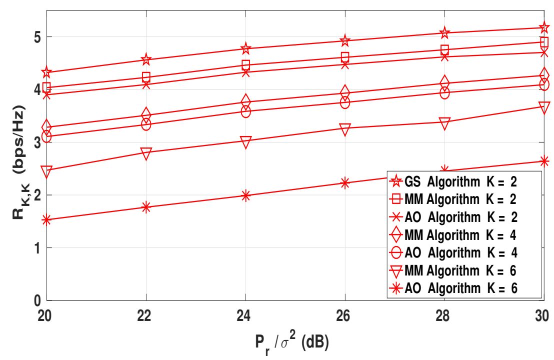

Fig. 2: Comparison of achievable rate of K^{th} user of MM based algorithm with AO based algorithm.

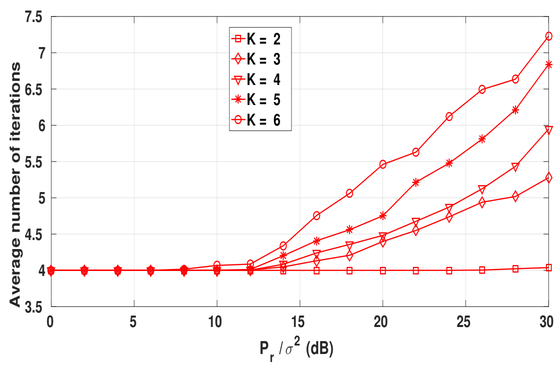

Fig. 3: Average number of iterations required for convergence of the algorithm for different users of MM based algorithm.

Fig. 1 shows variation of sum rate with SNR at relay node P_r/σ^2 with different number of users K = 2, 3, 4, 5, and 6. We simulate grid search (GS) based algorithm for K=2 destination nodes and it can be seen to perform better than MM based algorithm but at the cost of computational complexity.

In [11], alternating optimization (AO) method is considered to maximize rate of the destination node with best channel gain i.e., of K^{th} destination node and their results have shown to perform better than conventional OMA based network. Due to infeasibility of the algorithm as stated in remark 2 of [11], we consider target rates as 0.5, 0.25 and 0.16 for $K = 2$, $K = 4$ and $K = 6$, respectively and source budget $P_s/\sigma^2 = 25\ dB$. Fig. 2 shows results of proposed MM based algorithm outperform AO based algorithm. Hence, this also means proposed algorithm outperforms conventional OMA [11]. For comparison purpose we plot results obtained from grid search algorithm.

Fig. 3 shows the average number of iterations required for algorithm convergence. It can be seen that, average number of iterations for convergence is increasing with increase in number of users for given relay node SNR. It is also increasing with increase in relay node SNR.

Fig. 4 shows the variation of sum rate with varying relay node SNR, $P_r/\sigma^2 = 25\ dB$, and source node SNR, $P_s/\sigma^2 = 25\ dB$, for $K = 5$. It is observed that sum rate increases with increase in either of the source budget or relay budget. This shows that parameters which govern sum rate are source budget and relay budget.

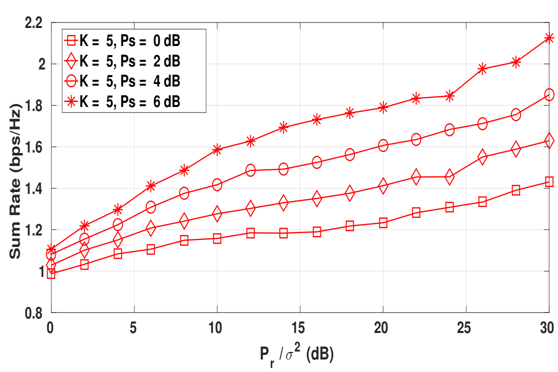

Fig. 4: Achievable sum rate vs P_r/σ^2 with different source power P_s/σ^2 of proposed MM based algorithm.

V. Conclusion

In this paper, we have considered a relay based NOMA system. We proposed MM based algorithm to design the optimal power allocation at source and relay precoder design. This optimal design maximizes the sum rate while maintaining non zero rates to users with small channel gains. The performance of the proposed algorithm was illustrated in the simulation results. It is shown that proposed algorithm performs well when compared to OA algorithm. It is observed that sum rate increases with increase in SNR at source or relay nodes. The average number of iterations taken for convergence also increases with increase in power budget at relay node.

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