Broadcasting of Quantum correlations using Dynamic State Dependent Cloner

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by

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CERTIFICATE

It is certified that the work contained in this thesis, titled "Broadcasting of Quantum correlations using Dynamic State Dependent Cloner " by Manish Kumar Shukla, has been carried out under my supervision and is not submitted elsewhere for a degree.

Date	Adviser: Dr. Indranil Chakrabarty

Dedicated to my grandfather.

His blessings have helped me pursue this research.

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Abstract

In this thesis, we extensively study the problem of broadcasting of entanglement. We first introduce the elements of quantum information theory and then define various kinds of quantum and classical correlations. We also talk about various measures to quantify correlations, which are important in the context of broadcasting correlations. Next, we re-conceptualize the idea of state dependent quantum cloning machine, and in that process, we introduce different types of state dependent cloners like static and dynamic state dependent cloners. We derive the conditions under which we can make these cloners independent of the input state. Our main motivation for defining various types of cloning machines is that if we have partial information about the state to be cloned beforehand, we can choose the best performing cloner. In the broadcasting part, as our resource initial state, we start with general two qubit state and consider specific examples like, non maximally entangled state (NME), Werner like state (WLS), and Bell diagonal state (BDS). We apply both state dependent and state independent cloners, both locally and non-locally, in each of these cases. Incidentally, we find several instances where state dependent cloners outperform state independent cloners in broadcasting. This work gives us a holistic view on the broadcasting of entanglement in various two qubit states, when we have an almost exhaustive set of cloning machines in our arsenal.

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Chapter 1

Elements of Classical Information Theory

In this chapter, first of all, we talk about Maxwell's thought experiment based on the conjecture that information is physical and relate the information entropy with thermodynamic entropy. In the next part, we give introduction to the subject of information theory. We start with the means to quantify information and then we give definitions like Joint entropy, mutual information and others in these contexts.

1.1 What is Information? A Historical Background

Historically, information has always been associated with data and knowledge. It is always interpreted as an answer to a question of some kind. From an information theoretic point of view, information is something that resolves uncertainty. Lets us assume a system A interacts with system B, before the interaction there is uncertainty in the system A regarding the system B and vice versa. However, with interaction there is a flow of information from either side and there is reduction of uncertainty in each one of them. The amount of information gain is equal to the reduction in uncertainty of an event. To measure the uncertainty associated with an event, the most common way is to relate the uncertainty with the probability of occurrence of that event.

1.2 Maxwell's demon and second law of thermodynamics

The second law of thermodynamics says that in an isolated system, the entropy of system never decreases. To take an example, if we take two metal rods at different temperatures and bring them in contact, then the universe evolves in such a way that the final equilibrium temperature of the two rods becomes same. To get a better understanding of this, Maxwell conceived a thought experiment. To directly quote Maxwell, the description of the experiment is, "if we conceive of a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are as essentially finite as our own, would be able to do what is impossible to us. For we have seen that molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now, let us suppose that such a vessel is divided into two portions, A and B, by a division in which there is a small hole, and a being, who can see the individual molecules. The being opens and closes this hole, so as to allow only the swifter molecules to pass from A to B, and only the slower molecules to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics."

In simpler words, suppose we have a box which is divided into two parts A and B, and there is a trapdoor connecting the two parts. This trapdoor is guarded by an intelligent demon. If we fill both parts A and B with a gas at same temperature, the demon operates in such a way that it opens the door only when a gas molecule with a speed higher than average speed is trying to go from side A to B and if a molecule slower than average is trying to go from B to A. If the system is allowed to evolve for some time, we will find that the part A has low speed molecules and part B has high speed molecules. This means that part A is cooler than part B, which is a violation of second law of thermodynamics. One more important condition is that the demon must allow molecules to pass in both directions otherwise a pressure of one of the parts would also be higher.

The resolution comes from the fact that for the demon to gain information about the speed of the molecules, there would be some expenditure of energy which would cause increase in entropy of the demon. This increase would be larger than the lowering of the entropy of the gas. Although, there have been several arguments against this explanation, notably by Rolf Landaur, where he argued that some of the measurement process may not increase the entropy. The solution for this comes from the fact that the demon has a finite memory and after some time would require to delete some information. But deletion of information is irreversible and hence would surely require energy [6].

1.3 Measures of Information

1.3.1 Entropy

Let us start with three statements:

- Sun rises in east.
- The phone will ring in half an hour. There will be snowfall in Hyderabad.

It is interesting to ask of these three statements, which one is most random. Since, the first and the last are certain and impossible to occur respectively, the obvious choice is the second one. Secondly, if we also ask that out of these three, which one is most informative. The intuitive answer is the second one.

This example manifests that there is a non trivial way, by which the randomness of a system is connected with the information of the system. Probability being a good measure of randomness, it is also assumed that information(I) will be some function of the probability, $I = f(p)$. The concept of information is too broad to be captured completely by a single definition. However, for any probability distribution, we define a quantity called the entropy, which has many properties that agree with the intuitive notion of what a measure of information should be. Shannon was the first one to come up with a measure of information. It is referred as Shannon's entropy and is represented by the symbol H. He defined information as the amount of uncertainty in predicting the outcome of a set of events, if only their probabilities are given. So, in one sense, we can say that entropy is a kind of measure of uncertainty of a random variable[21]. But not any function of these probabilities can be considered a measure of information. Any measure of information should obey the following conditions:

- It should be a positive value.
- It should be a continuous function of probabilities.
- If all probabilities are same, i.e. $p_i = \frac{1}{n}$, then H should be a monotonic increasing function of n.
- If a choice be broken down into two successive choices, the original H should be the weighted sum of the individual values of H.

Based on the above stated conditions, entropy $H(X)$ of a discrete random variable is defined as,

$$H(X) = -\sum_{x \in X} p(x) log_2 p(x). \tag{1.1}$$

The unit of entropy depends on the base of the log function. If the base is 2, then the unit is $bits$; if base is e, then unit is $nats$. In general, if base of log is b, then the symbol used is $H_b(X)$. It is important to note that the entropy is a functional of X, i.e. it does not depend on what value X takes, it only depends on its probability distribition. Let us consider an example of a fair coin toss. $p(H) = \frac{1}{2}$ and $p(T) = \frac{1}{2}$, the entropy is $H(X) = -\frac{1}{2} log \frac{1}{2} - \frac{1}{2} log \frac{1}{2} = 1$. So we can see that in this case, we have one bit of uncertainty.

Another way to look at entropy could be in terms of expectation values. Expectation value of random variable $g(X)$ is given by,

$$E_p g(X) = \sum_{x \in X} g(x) p(x). \tag{1.2}$$

So entropy of X can also be interpreted as the expectation value of $log \frac{1}{p(x)}$.

1.3.2 Joint entropy and conditional entropy

In the previous section, we talked about a single random variable and the entropy associated with it. When we are given a pair of random variables and the joint probability distribution $p(x, y)$ associated with it, the joint entropy $H(X, Y)$ is defined as,

$$H(x, y) = -\sum_{x \in X} \sum_{y \in Y} p(x, y) logp(x, y)[21]. \tag{1.3}$$

This can also be seen as $H(X, Y) = -Elogp(x, y)$. This extension is very intuitive as we can consider (X, Y) to be single vector valued random variable.

Next, we define conditional entropy, $H(Y|X)$, of a random variable Y, given another random variable X. It is defined as the expected value of the entropy of the conditional distribution, averaged over conditioning random variables.

$$\begin{aligned} H(Y|X) &= \sum_{x \in X} p(x) H(Y|X=x) \\ &= -\sum_{x \in X} p(x) \sum_{y \in Y} p(y|x) log p(y|x) \\ &= -\sum_{x \in X} \sum_{y \in Y} p(x,y) log p(y|x) \\ &= -E_{p(x,y)} log p(Y|X) \end{aligned} \tag{1.4}$$

The naturalness of the definition of joint entropy and conditional entropy is exhibited by the fact that the entropy of a pair of a random variables is entropy of one plus the conditional entropy of the other,

$$H(X,Y) = H(X) + H(Y|X) \tag{1.5}$$

which can be further extended to,

$$H(X,Y|Z) = H(X|Z) + H(Y|X,Z). \tag{1.6}$$

This is called the chain rule of entropy. Here, it is important to note that $H(Y|X) \neq H(X|Y)$, but $H(X) - H(X|Y) = H(Y) - H(Y|X)$.

1.3.3 Relative entropy and mutual information

As we have discussed in previous sections that entropy of a random variable is a measure of uncertainty associated with the random variable, it can also be seen as a measure of the amount of information required on average to describe the random variable. In this section, we introduce the concept of relative entropy and mutual information.

The relative entropy, also referred to as $KullbackLeibler$ distance, is a kind of distance measure between two distributions, which arises as an expected logarithm of the likelihood ratio. The relative entropy $D(p \parallel q)$ is a measure of the inefficiency of assuming that the distribution is q when the true distribution is p.

$$\begin{aligned} D(p \parallel q) &= \sum_{x \in X} p(x) log \frac{p(x)}{q(x)} \\ &= E_p log \frac{p(x)}{q(x)}. \end{aligned} \tag{1.7}$$

Here, based on the continuity arguments we use $0 log \frac{0}{q} = 0$ and $p log \frac{p}{0} = \infty$ as a convention. It is important to note that $D(p \parallel q) \neq D(q \parallel p)$.

The next quantity which we define here is called mutual information. It is a measure which tells us how much information one random variable contains about the other random variable. It can also be seen as the reduction in uncertainty of one random variable, if we gain complete information about the other random variable. To define mutual information more formally, let us consider two random variables X and Y, whose joint probability function is given by $p(x, y)$, and $p(x)$ and $p(y)$ are marginal probability mass functions. The mutual information, which is denoted as $I(X; Y)$, is described as the relative entropy between the joint probability distribution $p(x, y)$ and the product of marginal probabilities, $p(x)p(y)$, i.e.,

$$I(X;Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) log \frac{p(x,y)}{p(x)p(y)}. \tag{1.8}$$

It is easy to see that if X and Y are independent probability distributions, which means $p(x, y) = p(x)p(y)$, we get $I(X; Y) = 0$. The relation between entropy and mutual information is also very intuitive and can be easily shown with the help of Baye's rule of probability theory. The following relation holds between entropy and mutual information,

$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X). \tag{1.9}$$

These relations are evident in the venn diagram shown in figure 1.1.

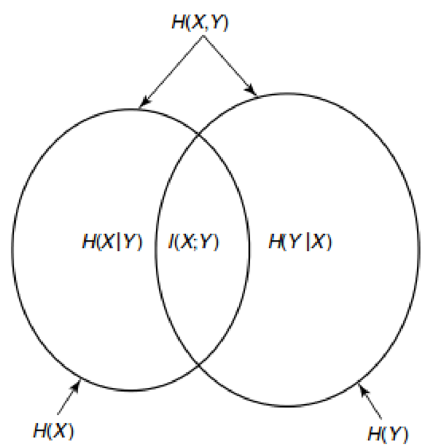

Figure 1.1 The figure shows the relation between mutual information and entropy.

1.3.4 Chain rules

In this section, we will state without proving how to apply the chain rules associated with quantities entropy, relative entropy and mutual information. First is the the chain rule for entropy, which is defined as, let $X_1, X_2, ..X_n$ be drawn according to $p(x_1, x_2, ...x_n)$. Then,

$$H(X_1, X_2...X_n) = \sum_{i=1}^{n} H(X_i|X_{i-1,..X_1}). \tag{1.10}$$

To define the chain rule for mutual information, we need to first define a quantity called conditional mutual information. It is defined for three random variables X, Y and Z as,

$$I(X;Y|Z) = H(X|Z) - H(X|Y,Z). \tag{1.11}$$

Given this relation, we can now define the chain rule for mutual information as,

$$I(X_1, X_2, ...X_n; Y) = \sum_{i=1}^{n} I(X_i; Y|X_{i-1}..X_1). \tag{1.12}$$

Lastly, we define the chain rule for relative entropy. It is expressed as,

$$D(p(x,y) \parallel q(x,y)) = D(p(x) \parallel q(x)) + D(p(y|x) \parallel q(y|x)). \tag{1.13}$$

Here, $D(p(y|x) \parallel (y|x))$ is called conditional relative entropy, and it is defined as,

$$D(p(y|x) \parallel q(y|x)) = \sum_x \sum_y p(y|x) log \frac{p(y|x)}{q(y|x)} [21]. \tag{1.14}$$

1.3.5 Properties and constraints

In this subsection, we state some important properties and inequalities associated with the quantities defined in previous subsections.

1. Jensen's inequality, which states that, if f is a convex function and X is a random variable, then, $Ef(x) \geq f(EX)$
2. Information inequality: Let $p(x)$, $q(x), x \in X$, be two probability mass functions then, $D(p \parallel q) \geq 0$, with equality only when $\forall x p(x) = q(x)$. An important consequence of this is non negativity of mutual information.
3. Conditioning reduces entropy, i.e., $H(X|Y) \leq H(X)$, with equality only when X and Y are independent.
4. Independent bound on entropy: Let $X_1, X_2, ...X_n$ be drawn according to $p(x_1, x_2, ..., x_n)$. Then $H(X_1, X_2, ...X_n) \leq \sum_{i-1}^{n} H(X_i)$. The equality only holds when X_i are independent.
5. Data processing inequality: If three random variables X, Y and Z form a markov chain ($X \rightarrow Y \rightarrow Z$), then $I(X;Y) \geq I(X;Z)$.

1.3.6 Tsallis and Renyi entropy

The generalizations of Shannon's entropy are obtained by relaxing the condition of additivity, i.e., given two independent systems A and B, for which the joint probability density satisfies,

$$p(A,B) = p(A)p(B). \tag{1.15}$$

If we calculate the joint entropy of the system, we find that it obeys,

$$S(A,B) = S(A) + S(B), \tag{1.16}$$

The Tsalli's entropy for a discrete case is defined as,

$$S_q(p_i) = \frac{k}{q-1}(1 - \sum_i p_i^q) \tag{1.17}$$

where q is a real number, referred as entropic index. If we calculate the joint Tsalli's entropy of the two independent systems we find that,

$$S_q(A,B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B). \tag{1.18}$$

In the limit $q \to 1$, the additivity holds and Tsalli's entropy reduces to Boltzmann-Gibbs entropy.

Another generalization of entropy, the Renyi entropy is defined as,

$$h_r(X) = \frac{1}{1-r} log\left[\int f^r(x)dx\right], \tag{1.19}$$

where $0 < r < \infty$. If we take the limit $r \to 1$, we find that it reduces to Shannon entropy.

$$h(X) = h_1(X) = -\int f(x) log f(x) dx. \tag{1.20}$$

Chapter 2

Elements of Quantum Information

2.1 Bit and a Qubit

In classical information theory, we label the unit of information as a bit. In principle, a classical bit can take two values (0 or 1) which can represent different things like yes or no; switch on or off. Any information which is stored on a classical computer is in the form of combination of bits. In quantum information theory, physical systems at quantum scales are used to store information. At quantum scales, the principle of superposition plays an important role, so a quantum bit can take values 0 or 1, as well as all the possible states obtained by superposition of these two states. Quantum bits are referred as qubit. Although, the concept of quantum bits was first developed by Stephen Wiesner [58], the term qubit was coined by Benjamin Schumacher [53]. One way to physically make a qubit is to use electron spin, where spin up is corresponding to bit value 0 and spin down is corresponding to 1. As electron spin exists in a superposition of up and down, so the qubit is in a state of superposition of 0 and 1. Another very popular way to implement a qubit in laboratory is to use photon polarization.

2.2 Stern Gerlach Experiment

One of the most important experiments which demonstrate that the spatial orientation of the angular momentum is quantized is the Stern Gerlach experiment. It also demonstrates the effect of measurement on quantum systems. The experiment was first conducted by the German physicists Otto Stern and Walther Gerlach in 1922. In the original experiment, silver atoms were made to pass through a non uniform magnetic field , due to which they got deflected before they hit a detector screen[29]. To avoid large deflections in the path, eclectically neutral particles were selected.

The first conclusion which we can draw from this experiment is that the spin angular momentum is quantized and can take only two possible values. This is because when the experiment was conducted, it was observed that on the detector there are two regions in which most of the atoms strike. On the contrary, if spin angular momentum had a continuous value, one should have observed a continuous spectrum of deflection[28, 30].

Second inference that can be drawn is that the atomic spin in nature exists as genuine quantum superposition of up spin and down spin, rather than a statistical mixture of up spin and down spin. To demonstrate this, the first part of the setup is regular Stern Gerlach setup to observe the deflection along z axis. Let us consider that after passing through the magnetic field, 50% of the atoms move in $+z$ direction and 50% of them move in $-z$ direction. The stream of atoms which is moving in $-z$ direction is blocked and the stream which is moving in $+z$ direction is now made to pass through another Stern Gerlach setup where the magnetic field is such that the splitting is observed in x direction. We observe that the remaining atoms again split in two directions and yet again in a quantized manner. In another scenario, if we pass the up deviated stream of atoms after the first measurement in z direction , and again make one of the deviated streams pass through exactly the same apparatus, to once again observe the deflection along z direction, we observe that this time the stream does not diverge in two directions. Rather all the atoms deviate in the same direction. This tells us that the spin exists in a superposition state and when we measure it along any one axis, the atoms deflect in one of the possible paths according to the probability of following that path. The atoms remain in the measured spin state unless they are disturbed by measurement along any other axis.

The last inference which we can draw from the experiment is that measurement disturbs the quantum state. If we try to measure two non commuting measurement operators one after the other, we find that measurement of one changes the quantum state and hence the measurement outcome after second measurement is also changed[38]. This becomes evident when we further extend the experiment to three measurements. We first make a measurement along z axis and block all the atoms which went in negative z direction and passed the atoms which went in $+z$ direction through a Stern Gerlach experiment setup to measure along x axis. We again observe the splitting in x direction. Now, we block the stream which went in $-x$ direction and pass the stream which deviated along $+x$ directions through a Stern Gerlach setup to again measure deflection along z axis. To our surprise, we see that the stream again deviates in two directions. This is because the measurement along x axis disturbs the homogeneous quantum state ($+z$) obtained after first measurement along z axis.

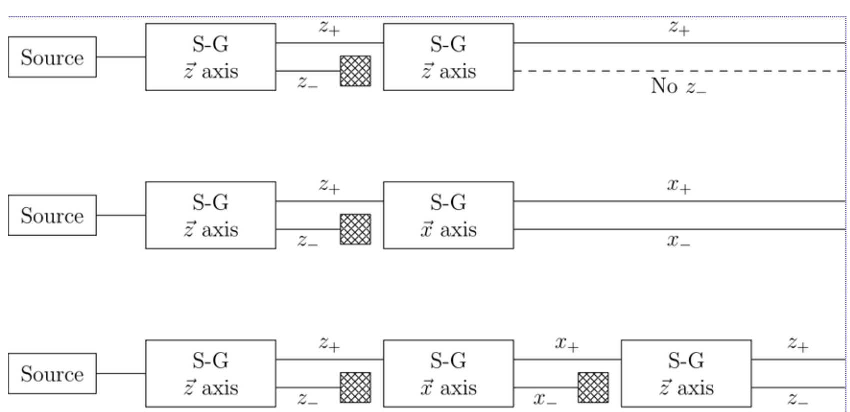

Figure 2.1 The figure shows the experimental setup of Stern Garlach experiment performed multiple times along various axis.

The figure 2.1 shows the experimental setup of Stern Garlach experiment used to demonstrate the inferences stated above.

2.3 Representation of qubit

2.3.1 Ket-Bra representation

The first postulate of quantum mechanics tells us that associated with any isolated physical system is a complex vector space with inner product (that is, a Hilbert space), known as the state space of the system. The system is completely described by its state vector, which is a unit vector in systems state space. We use the standard bra-ket (ket$\rightarrow |\psi\rangle$, bra $\rightarrow \langle\psi|$) notations to represent these state vectors[41].

Quantum mechanics does not give information about the state space and state vector of the system. To figure these out, physicists have developed many beautiful rules and models. Here, we would be dealing with very simple quantum system, i.e., the qubit. It has a two dimensional state space with basis vectors $|0\rangle$ and $|1\rangle$. An arbitrary state in this space is given by,

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle , \tag{2.1}$$

where α and β can be complex. Since $|\psi\rangle$ is a unit vector, $< \psi|\psi >= 1$. An important terminology that would be very frequently used is "superposition". These states are linear combination of other states with some amplitude, i.e.,

$$|\psi\rangle = \sum_i \alpha_i |\phi\rangle . \tag{2.2}$$

A very good representation of these general pure qubit states is obtained by using Bloch sphere which is shown in figure 2.2.

2.3.2 Density matrix representation

We have till now talked about quantum systems in terms of state vectors. There is an alternate formulation which is equivalent to the state vector formulation but is more mathematically convenient in certain cases. This formulation uses density operator or matrix to describe the state of the system. Density matrix formulation is more useful in cases where the state is not completely known. Suppose we have a quantum system, which can be in one of the number of states $|\psi_i\rangle$, where i is an index, with respective probability p_i. We call such a system to be a mixed state or $\{p_i, |\psi_i\rangle\}$, which is an ensemble of pure states $|\psi_i\rangle$. There is nothing quantum about these mixtures, these are just like class with two types of students. In density operator terms, this is represented as,

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i| . \tag{2.3}$$

Not all matrices can be considered a valid quantum state. For a density matrix to represent a valid quantum state, it should obey the following rules,

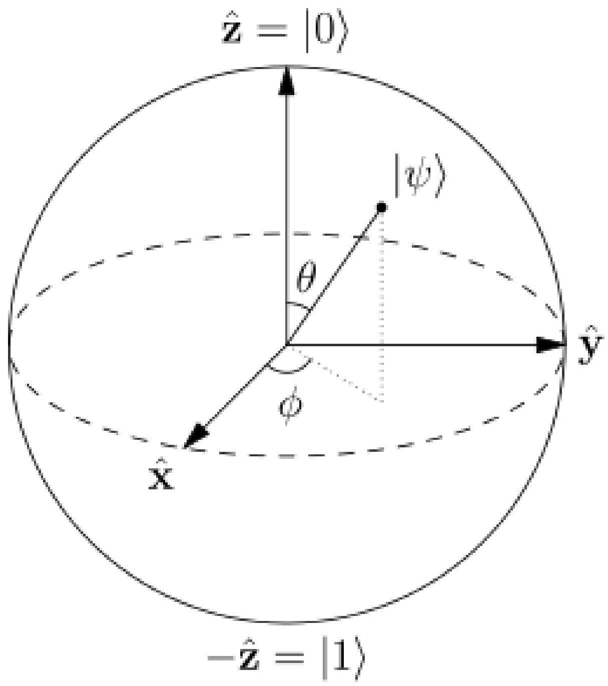

Figure 2.2 The figure shows the sphere which describes a general pure qubit.

1. Trace condition: $Tr[\rho] = 1$.
2. Positivity condition $\rho \geq 0$.

Given a density matrix, we can tell whether it describes a pure state or a mixed state by calculating $Tr[\rho^2]$. If the trace is equal to 1, then the system is corresponding to a pure state else it describes a mixed state. Also, it is important to note that the density matrix does not tell us about the preparation of the state as there can be multiple preparations which can lead to the same density matrix.

2.4 Evolution: Open quantum systems

In this section, we will see how quantum systems evolve in time. The postulates of quantum mechanics tell us that the evolution of a closed system is governed by a unitary transformation. At some instant of time t_1, our quantum system is in the state $|\psi_1\rangle$ and at a later instant of time t_2, it becomes $|\psi'\rangle$. Then the relation between $|\psi\rangle$ and $|\psi'\rangle$ is given by,

$$|\psi'\rangle = U\,|\psi\rangle\,. \tag{2.4}$$

For evolution of a single qubit system, any unitary operator can be realized in realistic systems. Also, the unitary operator U depends only on the time instances t_1 and t_2.

This unitary evolution model requires that the quantum system must be closed, but in most of the cases this is not true. The quantum system which we want to study interacts with its surroundings so it is not closed and hence cannot be described by unitary evolution. Nevertheless, there are quantum systems which can be approximated to be closed systems and their evolution is described by unitary evolution to some good degree of approximation[42].

In principle we can describe every open system as a part of a larger closed system and can study the unitary evolution of the larger system. We can trace out the environment from the larger state description to obtain the final post measurement state of the system we are interested in. This kind of evolution can be described as,

$$\rho_U = \rho_S \otimes \rho_E, \rho_U' = U\rho_U U^T, \rho_S^{'} = Tr[\rho_U']. \tag{2.5}$$

2.5 Measurement

Measurement plays an important role in the theory of quantum mechanics. There are several interpretations of quantum mechanics and currently there is no consensus between them. The issue of measurement lies at the heart of these interpretations. Measurement as described by Dirac is,

”A measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured, the eigenvalue this eigenstate belongs to being equal to the result of the measurement.”

We will introduce quantum measurements in two parts. First, we will talk about projective measurement which is a simple model of measurement on quantum systems and then later work towards a generalized model of measurement. In generalized measurement model, we perform an orthogonal measurement on a larger system which contains the system A which we need to measure[43]. So, to understand generalized measurement, we first need to understand projective model of measurement.

2.5.1 Projective Measurement

In an experimental setup to measure an observable M, one needs to establish a correlation between the observable and the pointer variable associated with the physical device. To achieve this, one needs to modify the Hamiltonian of the world to take into account the coupling between the pointer and the quantum system. This coupling establishes entanglement between eigenstates of the observable and the distinguishable pointer state[48]. The hamiltonian of the coupled quantum system with the pointer state has the form,

$$H = H_o + \frac{1}{2m}P^2 + \lambda MP, \tag{2.6}$$

where $\frac{1}{2m}P^2$ is the Hamiltonian of the pointer calculated independently, which would henceforth be ignored on the grounds that the mass of pointer is large. H_o is the unperturbed Hamiltonian of system and λ is the coupling constant. Here, M is the observable which we need to measure. It is coupled with the momentum P of the pointer.

We assume that the measurement is performed quickly, so that we can neglect the free evolution of the system during the measurement procedure. The approximate hamiltonian then becomes $H = \lambda MP$

(where $[M, P] = 0$) and the time evolution operator is given by,

$$U(t) = exp[-i\lambda tMP]. \tag{2.7}$$

Now, M can be written in diagonal basis as,

$$M = \sum_a |a\rangle M_a \langle a|, \tag{2.8}$$

and the evolution is described as,

$$U(t) \simeq \sum_a |a\rangle exp[-i\lambda tMP] \langle a|. \tag{2.9}$$

As mentioned earlier, P is the momentum operator, so in position representation, $P = -i\frac{d}{dx}$. Using Taylor expansion, we find that ,

$$e^{-ix_oP}\psi(x) = \psi(x - x_o). \tag{2.10}$$

Our initial system is not entangled with the pointer state ($|\psi(x)\rangle$) and is in the superposition of eigen state of M. In time t, the system evolves to,

$$U(t)(\sum_a \alpha_a |a\rangle \otimes |\psi(x)\rangle) = \sum_a |a\rangle \otimes |\psi(x - \lambda tM_a)\rangle. \tag{2.11}$$

We can see that the position of the pointer is correlated with the observable. Now, if we observe the state of the pointer, we will prepare an eigen state of the observable with probability $|\langle a| |\psi\rangle|^2$. This model of measurement is called Von-Neumann model of orthogonal measurement. This model can be used to measure any observable associated with the system, if we are able to couple it efficiently with the pointer state.

Given a set of observables such that they follow the properties given below,

$$E_a = E_a^T, E_aE_b = \delta_{ab}E_a, \sum_a E_a = 1. \tag{2.12}$$

If we carry out the measurement procedure on pure state $|\psi\rangle\langle\psi|$, the final post measurement is given by,

$$\frac{E_a |\psi\rangle \langle\psi| E_a}{\langle\psi| E_a |\psi\rangle}, \tag{2.13}$$

with probability, $prob(a) = \langle\psi| E_a |\psi\rangle$. If we do not choose any particular outcome and rather find the density matrix, which is found by summing over all possible outcomes according to their probability of occurrence, we find that,

$$|\psi\rangle \langle\psi| \to \sum_a E_a |\psi\rangle \langle\psi| E_a. \tag{2.14}$$

This is an ensemble of pure states describing the measurement outcomes. If we measure a quantum state ρ which is mixed initially, we find that it transforms in the following way,

$$\rho \to \sum_a E_a\rho E_a. \tag{2.15}$$

2.5.2 Generalized measurement and POVM

The projective measurement described above does not give a complete picture about the quantum measurement and turns out to be too restrictive. There are measurements that can be performed on the system but cannot be described by the Von-Neumann model of measurement. This motivates us to introduce a more complete theory of quantum measurements, which is called generalized measurement or positive operator valued measurement (POVM).

A generalized measurement is described by relaxing the orthogonality condition of the projective measurement, that is, $E_a E_b \neq \delta_{ab}$. We still need the operator to be positive ($E_1 \geq 0$) and completeness relation ($\sum_i E_i = I$) should hold. Such a measurement is described as $M = \{E_i\}$. Each E_i is associated with an outcome of measurement, and since these operators are positive, we can rewrite it as $E_i = M_i^T M_i$. The post measurement state in this case becomes,

$$\rho' = \frac{M_i \rho M_i^T}{tr(E_i \rho)}, \tag{2.16}$$

and the probability of obtaining the i^{th} output is $Pr\{i\} = tr(E_i \rho)$.

The simplest example which demonstrates the need for having such measurements is when we consider a measurement apparatus for a single qubit measurement, but it has a certain probability of failure p. In case of a failure, the measurement apparatus does not interact with the qubit and returns no measurement outcome. For describing this situation, there is no projective description, rather we can describe it using POVM. It is described as,

$$E_1 = pI \tag{2.17}$$

$$E_2 = (1-p)\left|0\right\rangle\left\langle 0\right| \tag{2.18}$$

$$E_3 = (1-p)\left|1\right\rangle\left\langle 1\right|. \tag{2.19}$$

Another example of POVM which is used to discriminate between the states $|\psi_1\rangle = |0\rangle$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is given by,

$$E_1 = \frac{\sqrt{2}}{1+\sqrt{2}}\left|1\right\rangle\left\langle 1\right| \tag{2.20}$$

$$E_2 = \frac{\sqrt{2}}{1+\sqrt{2}}(|0\rangle - |1\rangle)(\langle 0| - \langle 1|) \tag{2.21}$$

$$E_3 = I - E_1 - E_2. \tag{2.22}$$

It is important to note that a POVM can always be realized by extending the Hilbert space to a larger space, and performing measurement in that larger space. This was given by Neumark as a theorem.

2.6 Entanglement as a Resource

There are various ways to define entanglement mathematically. State of a bipartite system is called entangled, if it cannot be written as a direct product of two states from the two subsystem Hilbert spaces.

The other way to look at it is through the Schmidt number. With any bipartite pure state $|\psi\rangle_{AB}$, we can associate a positive integer, the Schmidt number, which is the number of non zero Eigen values in ρ_A (or ρ_B) and hence the number of terms in the decomposition of $|\psi\rangle_{AB}$. We say that the state is entangled, if the Schmidt number is more than one, else separable.

Quantum entanglement is a physical phenomenon that occurs when a group of particles interact in ways such that the quantum state of each particle cannot be completely described independently. Instead, a quantum state may be given for the system as a whole.

An example of maximally entangled state is the bell state

$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB}) \tag{2.23}$$

The term "Maximally entangled" means that when we trace over qubit B to find the density operator $\rho_A$of qubit A, we obtain a multiple of identity operator, and similarly for qubit B. This means that if we measure spin of qubit A along any axis, the result is completely random, and we find spin up with probability $\frac{1}{2}$ and spin down with probability $\frac{1}{2}$. Therefore, if we perform any local measurement of A or B, we acquire no information about the preparation of the state, instead we merely generate a random bit. Of course, we can recover the information by performing an orthogonal measurement that projects onto the bell basis. But if the qubits are distantly separated, we cannot acquire this information locally, that is, by measuring A or measuring B.

2.6.1 Super Dense Coding

Quantum entanglement acts as a resource in numerous information processing tasks. The first example we consider here is of super dense coding. Alice wants to send some information to Bob. She has access to a classical channel as well as a quantum channel. She can send her information through classical channel using Morse codes. But, she decides to send her information through the quantum channel, hoping that she might get some advantage. For using the quantum channel, she prepares a qubit using photons, and sends them to Bob. Did she get any advantage by using a quantum channel instead of a classical one?

In principle, if the quantum state preparation, measurement and the quantum channel is perfect, then she is not worse of using quantum channel. Alice can prepare $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$, and Bob can measure along z axis. In this way, she is able to send one classical bit with each qubit.

But suddenly, Alice remembers that when she met Bob last year, they shared a pair of entangled states $|\phi^+\rangle_{AB}$. Alice carries out a protocol which she and Bob had arranged for emergency situations. She can perform one of the four possible unitary transformations on her part of the shared entangled state:

- 1 (Do nothing),
- σ_1 (rotate by 180 degrees along x-axis),

- σ_2 (rotate by 180 degrees along y-axis),
- σ_2 (rotate by 180 degrees along z-axis).

By doing this she transforms $|\phi^+\rangle_{AB}$ to one of 4 mutually orthogonal states:

- $|\phi^+\rangle_{AB}$,
- $|\psi^+\rangle_{AB}$,
- $|\psi^-\rangle_{AB}$,
- $|\phi^-\rangle_{AB}$.

After doing this, she sends her qubit to Bob, who receives it and measures along the bell basis, and he can unambiguously distinguish the four possible actions that Alice could have performed. Therefore, a single qubit transferred through quantum channel by Alice, could deliver two bits of classical information to Bob. This process is called dense coding. Another nice fact about this procedure is that if the information is confidential, she need not worry about eavesdropper. The transmitted density matrix is $\rho_a = \frac{1}{2}I_A$, so it carries no information at all. All the information is in the correlations between A and B.

2.6.2 Teleportation

In dense coding, the goal was to send more classical information using quantum information. Teleportation is a related concept where we try to send quantum information using classical information. Alice has an unknown qubit which Bob wants, but the problem is that, they do not share a quantum channel and only have a classical channel. Alice recalls that she had shared an entangled pair with Bob sometime back. They carry out the following protocol: Alice combines the unknown qubit, which she wants to send to Bob with her part of the entangled qubit. On the combined state, she performs Bell measurement, i.e., projection onto one of the four states $|\phi^+\rangle_{CA}$, $|\phi^-\rangle_{CA}$, $|\psi^+\rangle_{CA}$ and $|\psi^-\rangle_{CA}$. She can now communicate the measurement outcome (2 bits of classical information) to Bob using classical channel. Bob on his side, based on the result communicated to him, performs one of the four actions,

- $|\phi^+\rangle_{CA} \to I_B$,
- $|\psi^+\rangle_{CA} \to \sigma_1^B$,
- $|\psi^-\rangle_{CA} \to \sigma_2^B$,
- $|\phi^-\rangle_{CA} \to \sigma_3^B$.

This transforms his part of the shared entangled state to the state of unknown qubit which Alice had. This process is called teleportation. Mathematics of the process is very simple and can be easily replicated.

2.6.3 Broadcasting of entanglement

Entanglement, which lies at the heart of quantum mechanics, acts as a resource in most of the information processing tasks like quantum cloning[50]. In general, these resources are mixed entangled states [37, 56] in comparison to pure entangled states [57, 26]. However, there is always a requirement of pure entangled states as resource for better achievement of these tasks. The process of distilling pure entanglement from the available mixed entangled states has been studied extensively [7, 39]. Interestingly, the requirement is not unidirectional. In a network, we frequently require more number of states with less entanglement than a single state of greater entanglement. This process of creating more number of lesser entangled states from an initial entangled state can be achieved in various ways. In general, when we achieve this with the aid of local and non local cloning transformations, we call it broadcasting of entanglement[14, 2, 18]

Buzek et. al. showed that though perfect broadcasting of entanglement is forbidden as a consequence of no cloning theorem, partial decompression of initial quantum entanglement is possible, i.e., for a pair of entangled particles, two less entangled pairs can be obtained[14]. In this context, it was shown by Bandyopadhyay et. al. that only universal quantum cloning machines, which have fidelity greater than $\frac{1}{2}(1+\sqrt{\frac{1}{3}})$ can broadcast entanglement[3]. There is also a bound on the number of copies that can be created, which is two, if we use local cloning and six, if we use non-local cloning[3]. Recently, we have shown that it is impossible to even partially broadcast quantum correlation that goes beyond entanglement, by using both local and non local quantum copying machines[19]. In the same work, we extensively study the broadcasting of quantum entanglement for various two qubits mixed states.

In this work, in chapter 4, we first discuss about various types of cloning, i.e. state dependent, state independent, local and non local. While discussing about state dependent cloning, we define two kinds of state dependence, namely, static and dynamic state dependent cloning. We also compare the performance of the above stated cloners. We start the next chapter with the definition of broadcasting using cloning transformations and develop a deeper insight into broadcasting using local and non local cloning. In the same chapter, we consider the general two qubit state and study the broadcasting of entanglement with three examples namely, non maximally entangled (NME)[57], Werner like state (WLS)[56, 4], Bell diagonal state (BDS).

2.7 Quantum information theory

The mathematical foundation developed while discussing classical information largely extends to quantum information theory as well, but the Central issue in the former is the issue of non distinguishability of pure non orthogonal states. This property of quantum systems has no classical analog. In the subsections below, we develop the foundation of quantum information theory.

2.7.1 Von-Neumann entropy

Von Neumann entropy of a density operator is expressed as,

$$S(\rho) = -Tr\rho log_2(\rho) = -\sum_{i=1}^{n} \lambda_i log_2 \lambda_1, \tag{2.24}$$

where λ_i are eigen values of density matrix. We can see that Von Neumann entropy vanishes for a pure state, whereas it is maximum for the completely mixed state. The Von Neumann entropy of a completely mixed state can be expressed as,

$$S(\frac{1}{N}1) = -\frac{1}{N}Trlog_2(\frac{1}{N}1) = log_2 N \tag{2.25}$$

where N is the dimension of the Hilbert space. The Von Neumann entropy is related to Shannons measure of information, which is important in the context of information capacity, and to Gibbs entropy from statistical mechanics.

2.7.1.1 Mathematical properties of Von Neumann entropy

There are several important properties associated with Von Neumann entropy[48]. We list these properties in this section.

1. Since $S(\rho)$ only depends on the eigen values of ρ, if we perform a unitary change of basis, the entropy remains unchanged, i.e., $S(U\rho U^{-1}) = S(\rho)$.

2. It is very intuitive to see that the entropy should be maximum when the quantum state is chosen randomly. The upper bound on the Von Neumann entropy is dependent on the number of non-vanishing eigen values, Let that be D. Then $S(\rho) \leq logD$, with the equality holding only when all the non zero eigen values are equal.

3. As log function is convex, as a consequence of which the Von Neumann entropy is concave. To express it mathematically, $S(\lambda_1\rho_1, \lambda_2\rho_2, ..., \lambda_n\rho_n) \geq \lambda_1 S(\rho_1) + \lambda_2 S(\rho_2), ..., +\lambda_n S(\rho_n)$. In simpler words, this means that Von Neumann entropy is larger if we are more ignorant about the preparation of the state.

4. If we measure non commuting observables then there would be more uncertainty. This means that randomness in measurement is least if we choose to measure an observable which commutes with the density matrix. Given a density matrix ρ and the observable $A = a_y |a_y\rangle \langle a_y|$ so that $|a_y\rangle$ is obtained with probability $\langle a_y| \rho |a_y\rangle$, then the Shannon entropy of the ensemble of measurement outcomes $Y = a_y, p(a_y)$ satisfies $H(Y) \geq S(\rho)$, with equality only when they commute.

5. If we have an ensemble of quantum states such that the density matrix representing the quantum ensemble is $\rho = \sum_x p_x |\phi_x\rangle \langle\phi_x|$, the $H(X) \geq S(\rho)$. The equality holds when $|\phi_x\rangle$ are orthogonal. It means that the distinguishably is lost when we mix non-orthogonal pure states.

6. Given a density matrix ρ_{AB} which represents combined bipartite system AB, then $S(\rho_{AB} \leq S(\rho_A) + S(\rho_B))$. This property is called subadditivity of Von Neumann entropy. The equality holds when the density matrix is separable, i.e., $\rho_{AB} = \rho_A \otimes \rho_B$. This is because some of the information about the combined system is contained in the correlations between A and B. Extending the same argument to tripartite system ρ_{ABC}, then $S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC})$. This is called strong subadditivity.

7. For a bipartite system $S(\rho_{AB}) \geq |\S(\rho_A) - S(\rho_B)|$. This property is called triangle inequality or Arai-Lied inequality. This is contrary to the behaviour we see in Shannon's entropy as in that case $H(X,Y) \geq H(X), H(Y)$ which is to say that there is more information in the whole system than in a part of it. But this is not true with Von Neumann entropy where if, $S(\rho_A) = S(\rho_B)$ the $S(\rho_{AB})$. A classic example of this behaviour are the bell states.

Chapter 3

Quantifying Quantum correlations: Entanglement and Beyond

Entanglement is the most important resource in quantum information theory. Consequently, it is very important to quantify how much of this resource is available. The main problem in this aspect is that the phenomena of quantum entanglement is still not very well understood i.e. we only understand its mathematical definition and its manifestations like bell inequality, teleportation. There are, in principle, two approaches to quantify entanglement, the first one being operational and the other being abstract. In operational measures, entanglement is related to some operational task. We say that a system is more entangled, if it allows for better performance of some quantum mechanical tasks which are not possible without entanglement. One such task is teleportation. In the abstract approach, one says that a state function can be used to quantify entanglement, if it satisfies some natural properties. Some of the requirements which a measure of entanglement should satisfy are,

- $C = 0$ for product states $\rho = \rho_A \otimes \rho_B$.
- C is invariant under local unitary transforms.
- The measure should not depend on the choice of basis.

3.1 Measures of Entanglement

3.1.1 Entropy of entanglement

A measure that fulfills these requirements for pure states is the entropy of entanglement. It is one of the simplest methods to measure entanglement. It uses the Von Neumann entropy of reduced density operator to describe the amount of entanglement associated with the combined system. A useful interpretation of the Von Neumann entropy is that it represents the minimum number of bits required to store the result of a random variable. A pure state $\rho_1 = |\psi\rangle \langle\psi|$ can always be written in its Eigen base as $\rho_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and it is evident that its Von Neumann entropy vanishes to 0,

$$S(\rho_1) = 1log_2 1 + 0log_2(0) = 0 \tag{3.1}$$

A suitable measurement of the observable σ_z, therefore, always produces the result $+1$, and the information gain from such a measurement vanishes. For the maximally mixed state, $\rho_1 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. However, the entropy reaches its maximum value,

$$S(\rho_2) = 1. \tag{3.2}$$

Here, every binary observable generates completely random state. Every result must therefore be represented by one bit, hence no compression is possible in this case. The entropy of entanglement is defined for bipartite pure states as the Von Neumann entropy of one of the reduced states, which is expressed as,

$$E(\rho_{AB}) = S(\rho_A) = S(\rho_B), \tag{3.3}$$

where $\rho_A = Tr_B(\rho_{AB})$ and vice versa. If ρ_{AB} is a product state, ρ_A and ρ_B are pure states and hence the entropy of the reduced state vanishes. If state is maximally entangled, e.g. $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, the reduced states are completely mixed states, $\rho_A = \rho_B = \frac{1}{2}I$. The corresponding entropy of entanglement is then $E(|\psi\rangle\langle\psi|) = 1$.

3.1.2 Concurrence

Let us consider a pure state $|\phi\rangle$ of a pair of qubits, i.e., $|\phi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$. We can easily show that $|\phi\rangle$ is factorisable if and only if $ad = bc$. So, one might take the difference between ad and bc as the measure of entanglement. Indeed, this is what concurrence does $C(\phi) = 2|ad - bc|$. Let us consider a pure state $|\phi\rangle$. The concurrence $C(\phi)$ of this state is defined to be $C(\phi) = |\langle\phi|\,|\breve{\phi}\rangle\,|$, where the $tilde$ represents the "spin-flip" operation, $|\breve{\phi}\rangle = (\sigma_y \otimes \sigma_y)|\phi^*\rangle$, where $|\phi^*\rangle$ is the complex conjugate of $|\phi\rangle$ in the standard basis$(|00\rangle, |01\rangle, |10\rangle, |11\rangle)$ and σ_y is the Pauli operator $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. The spin flip operation when applied to a pure product state, takes the state of each qubit to the orthogonal state, that is, the state diametrically opposite on the Bloch sphere. The concurrence of a pure product state is therefore zero. On the other hand, a completely entangled state, like the singlet state, is left invariant by spin flip (except possibly for a phase factor), so for such states, concurrence takes the value 1, which is its maximum possible value. We can define the concurrence of a mixed state ρ of two qubit system to be average concurrence of ensemble of pure states representing ρ, minimized over all decompositions of ρ. That is,

$$C(\rho) = inf \sum_j p_j C(\phi_j), \tag{3.4}$$

where $\rho = \sum_j |\phi_j\rangle\langle\phi_j|$. Now it happens that the function ε(C) defined above, in addition to being monotonically increasing, is also convex. It follows that,

$$\varepsilon(C(\rho)) = inf\varepsilon(\sum_j p_j C(\phi_j)) \leq inf \sum_j p_j \varepsilon(C(\phi_j)) = E_f(\rho), \tag{3.5}$$

that is, $\varepsilon(C(\rho))$ is a lower bound on $E_f(\rho)$.

At this point I state, but do not prove, two remarkable facts about concurrence. First, there always exist a decomposition of ρ that achieves the minimum in the equation above with a state of pure state having same concurrence. This fact makes the inequality in equation an equality, so that $\varepsilon(C(\rho))$ actually gives the entanglement of formation[60, 61]. Second, one can find an explicit formula for $C(\rho)$ which is,

$$C(\rho) = max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{3.6}$$

where the λ_i's are the square root of the eigen values of $\rho\breve{\rho}$. Here $\breve{\rho}$ is the result of applying spin flip operation to ρ

$$\breve{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y). \tag{3.7}$$

3.1.2.1 Examples

- Pure product state $|00\rangle$, the corresponding density matrix is $\rho = |00\rangle\langle 00|$ =$\rho = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.
  Now we need to find the eigen value of the matrix obtained after applying spin flip operation, i.e., $\sqrt{\rho\breve{\rho}}$. We first calculate matrix $(\sigma_y \otimes \sigma_y)$, which is, $\sigma_y \otimes \sigma_y = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$. We can now calculate that the eigen value of $\sqrt{\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)}$, and then calculate $C(\rho) = max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, which comes out to be 0 and hence the concurrence is zero for a product state.

- Let us now consider the bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. The corresponding density matrix will be =$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$. After applying the spin flip operation we get =$\rho\breve{\rho} = \frac{1}{2}\begin{pmatrix} -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \end{pmatrix}$. The corresponding eigen values are 0 and -1, so the concurrence is 1 for maximally entangled state.

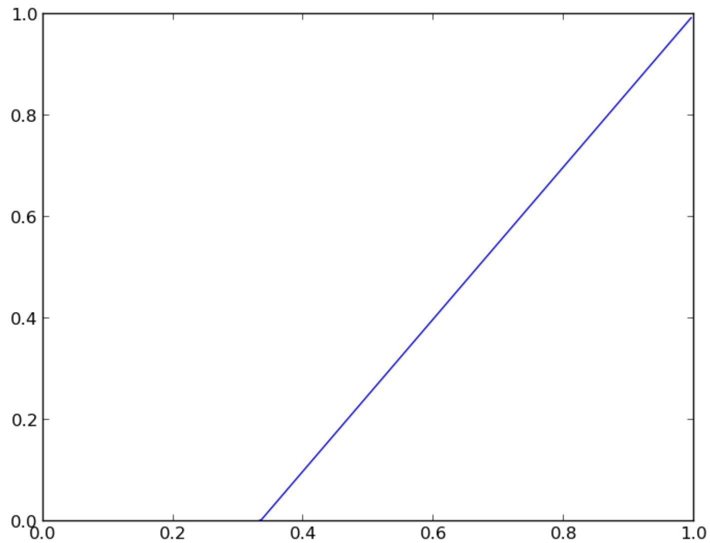

Figure 3.1 The figure shows the value of concurrence as a function of classical mixing parameter p for Werner state.

3.1.3 Negativity

In this subsection, we will discuss another measure of entanglement which is called negativity. Although it does not have a direct physical interpretation, it has certain appealing properties, one of them being ease of computation. It also gives bounds on the singlet distance, on the teleportation distance and distillation rates at certain error levels. The negativity is based directly on the partial transposition criterion for separability and measures, by how much a given state ρ fails to satisfy this criterion[55]. In detail, let ρ^{T_B} denote partial transpose and let $\|\rho^{T_B}\| = tr\sqrt{(\rho^{T_B})^\dagger \rho^{T_B}}$ be its trace norm. If ρ^{T_B} has eigenvalues λ_i, then the norm can be written as $\sum_i |\lambda_i|$. Suppose $\sum_i |\mu_i|$ are negative eigen values of ρ^{T_B}. This set is non-empty only if ρ is not separable. So for a separable ρ we have $tr\rho = \|\rho^{T_B}\| = 1$. In either case we can always write

$$1 = tr(\rho^{T_B}) = \sum_i \lambda_i = \|\rho^{T_B}\| - 2|\sum_j \mu_j|. \tag{3.8}$$

Hence, the last sum measures the amount by which ρ^{T_B} fails to have only positive eigenvalues. Thus it is now intuitive to define the negativity as,

$$N(\rho) = \frac{\|\rho^{T_B}\| - 1}{2} = |\sum_j \mu_j| \tag{3.9}$$

3.1.4 GGM

In quantum systems where we have two subsystems, we observe that highly entangled subsystems gives higher capacity of transmitting classical and quantum systems. When we try to extend the same argument to design a measure of entanglement for multiparty system, to our surprise we find that the quantum capacities of multi-access channels do not have any relation with genuine multiparty entanglement measures[23, 22]. One of the measures used to define the genuine multiparty entanglement is generalized geometric measure (GGM), which is a generalization of geometric measure of entanglement. Here, we give our definitions and propositions assuming we have four subsystems, but these can be easily generalized to more.

At this stage, we are ready to define two quantities, first is maximal assisted remote singlet production (C_a). It is defined for a single copy of four-party pure quantum state, $|\psi\rangle$, shared between Alice (A), Bob (B), Claire (C) and Danny (D), as the maximal probability with which a single copy of the singlet state, $|\psi_-\rangle = (|01\rangle - |10\rangle)/2$, can be prepared at CD, by using an additional resource of a singlet state shared between A and B, and by using just LOCC operations. In simpler words, we can say that C_a measures the amount of entanglement that can be transferred from AB to CD when AB are assisted by an additional singlet. Also, we would like to note that if the state is not symmetric with respect to interchange of parties, C_a is defined as max of the transfer probabilities corresponding to all possible permutations of the parties.

The second quantity we define here is called maximal unassisted remote singlet production (C_{ua}), and has exactly the same definition as C_a, but without any assistance from any additional singlet. At this stage, I state two propositions without proving them,

- Proposition C1. $C_{ua} \leq C_a \leq P^s_{maxmin}$, where P^s_{maxmin} is defined as follows. Consider the set of four quantities $P^{i:rest}_{max}$, which is the maximum probability of obtaining a singlet state between i and other observers. Choose all six pairs from the set $P^{i:rest}_{max}$, find the minimum for each pair, and then the maximum of these six minima is P^s_{maxmin}.

- Proposition C2. $C_{ua} \leq p^d_{maxmin}$, where p^d_{maxmin} is defined as follows. Consider the set of three quantities $\{P^{AC:BD}_{max}, P^{AD:BC}_{max}, P^{AC:CD}_{max}\}$, choose all three pairs from the set, and the minimum from each set. p^d_{maxmin} is maximum of these minimums.

These capacities, motivated by quantum networks, will be compared with a measure of genuine four-party entanglement measure called GGM, which we have defined below. Consider a four party pure quantum state $|\psi\rangle$, and let $\Lambda(|\psi\rangle) = max|\langle\phi|\,|\psi\rangle\,|$, where the maximum is over all the four party pure quantum states $|\phi\rangle$ that are not genuinely four party entangled. Here, I would like to make it clear that we call an n-party pure quantum state to be genuinely n-party entangled, if it is not a product across any bipartite partition. The GGM of $|\psi\rangle$ is defined as,

$$\varepsilon(|\psi\rangle) = 1 - \Lambda^2_{max}(|\psi\rangle). \tag{3.10}$$

Here, Λ_{max} quantifies the closeness of the state $|\psi\rangle$ to all pure quantum states that are not genuinely multiparty entangled. This definition is a generalization of GM, in which the maximization in Λ_{max} is only over pure product state of bipartite partition. It can be noted that, if the maximization in $\Lambda_{max}(|\psi\rangle_{ABCD})$ is performed over all the states that are the products in the $A : BCD$ split, the result is the maximum Schmidt coefficient, $\lambda_{A:BCD}$, of the state $|\psi\rangle$, when written in the $A : BCD$ split. It is also important to note that a similar expression holds for other splittings as well After a little simplification, we find that the GGM can be computed as,

$$\varepsilon(|\psi\rangle) = 1 - max\{\lambda^2_{i:rest}, \lambda^2_{ij:rest} | i, j = A, B, C, D; i \neq j\} \tag{3.11}$$

It is very interesting to note that the generalized geometric measure is monotonically decreasing under LOCC operations.

Let us take an example of generalized GHZ states to understand the GGM better, $|GHZ\rangle_{ABCD} = \alpha |0000\rangle + \beta |1111\rangle$ (with $|\alpha| > |\beta|$), shared between the four observers. By proposition C1, we find that, $C_{ua} \leq C_a \leq 2|\beta^2|$. Now, if we carry out the measurement in the $|+\rangle, |-\rangle$ basis at A and B, and the resulting state at CD is a pure state, corresponding to each set of measurement at results at A and B, is LOCC-transformed to the singlet state, it is observed that $C_{ua} \geq 2|\beta^2|$. Therefore, $C_{ua} = C_a = 2|\beta^2|$.

3.2 Measures of correlations beyond entanglement

3.2.1 Discord

Two classically same definitions of mutual information are not the same in quantum realm. The definition of quantum discord is motivated from this definition. The interesting thing about quantum discord is that, it is a property of quantum systems which is also exhibited by quantum systems that are not entangled. So, in other words, separability is not the necessary criteria for a system to be classical[44]. In the previous chapter we, have talked about mutual information from a classical information theory point of view, where we saw that we can define mutual information as,

$$J(X : Y) = H(X) - H(X|Y). \tag{3.12}$$

This definition of mutual information shows the correlation between two random variables X and Y. One way to look at mutual information is that it measures the average decrease in uncertainty of first random variable when we find the information associated with other random variable. We also know that,

$$H(X|Y) = H(X, Y) - H(Y). \tag{3.13}$$

This leads us to the other definition of expressing mutual information which is,

$$I(X : Y) = H(X) + H(Y) - H(X, Y). \tag{3.14}$$

The two definitions of mutual definition are equivalent in classical systems. Now, let us try to extend this definition for the quantum systems. To do this, all we need to do is to replace classical probability distribution with density matrices and Shannon entropy with Von Neumann entropy. Now, $H(S) = H(\rho_s)$ which is given by $-Tr\rho_s Log\rho_S$. So, one of the definitions of mutual information becomes,

$$I(S:A) = H(S) + H(A) - H(S,A) \tag{3.15}$$

Here, $H(S,A)$ is the uncertainty present in the combined system described by $\rho_{S,A}$ and to extract all the information about the combined system, we need to perform a measurement or in other words, apply the measurement operator on combined hilbert space of S and A, i.e. $H_S \otimes H_A$.

The other definition of mutual information is not as intuitive, since to specify the conditional entropy $H(S|A)$, we need to specify the state of S after measurement of A. But there is an ambiguity in this statement unless we specify the to-be-measured set of states of A. For this purpose, one dimension projection operators $\{\pi_j^A\}$ are used. Here, j describes various outcomes of measurements. To calculate the information gained about the system S as a result of the measurement $\{\pi_j^A\}$, we calculate, $J(S : A)_{\{\pi_j^A\}} = H(S) - H(S|\{\pi_j^A\})$, where $H(S|\{\pi_j^A\}) = \sum_j p_j H(\rho_S|\pi_j^A)$. Quantum discord is defined as the difference these two definitions as,

$$\delta(S:A)_{\{\pi_j^A\}} = I(S:A) - J(S:A)_{\{\pi_j^A\}}, \tag{3.16}$$

and this definition of quantum discord simplifies to, $H(A) - H(S,A) + H(S|\{\pi_j^A\})$. We can see that it depends both on the combined density operator $\rho_{S,A}$ and on the projectors $\{\pi_j 6A\}$. It is also evident that this definition of quantum discord is asymmetric, if we swap A and S, since it involves measurement on one end. It can also be easily be shown that quantum discord is always greater than or equal to 0.

The figure 3.2 shows how the value of discord changes when we change the value of parameter z which specifies the combined state of the system as $\rho_{S,A} = 1/2(|00\rangle\langle 00| + |11\rangle\langle 11|) + z/2(|00\rangle\langle 11| + |11\rangle\langle 00|)$. The other parameter which is used is θ, which changes the measurement basis. In figure 3.3, the same analysis is shown for Werner state, where z is now classical mixing parameter.

The above definition of quantum discord is dependent on the measurement basis. To make quantum discord purely describe non classical correlations independent of basis, J is first maximized over all measurement basis and hence, basis independent quantum discord becomes,

$$\delta(S:A) = I(S:A) - maxJ(S:A)_{\{\pi_j^A\}} = S(\rho_A) - S(\rho_{S,A}) + min(S(\rho_S|\{\pi_j^A\})). \tag{3.17}$$

3.2.2 Coherence

Coherence describes all properties of the correlation between physical quantities of a single wave, or between several waves or wave packets. If someone were to prepare the state $|\psi\rangle$, the quantum information encoded in the superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$ would immediately be transferred to correlations between the $|\psi\rangle$ and the environment, and become completely inaccessible. In effect, the environment continually measures the $|\psi\rangle$, projecting it onto either the state $|\uparrow\rangle$ or $|\downarrow\rangle$. This process is called

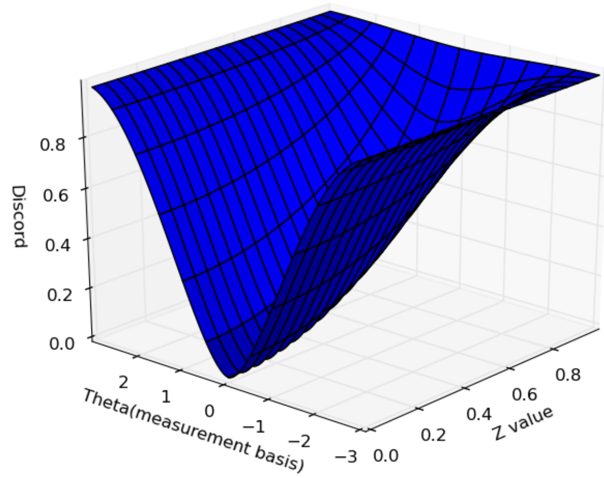

Figure 3.2 The figure shows the value of discord as a function of measurement basis and parameter Z.

decoherence[59]. We will be discussing quantum coherence as a resource and its broadcasting in the last chapter of this work.

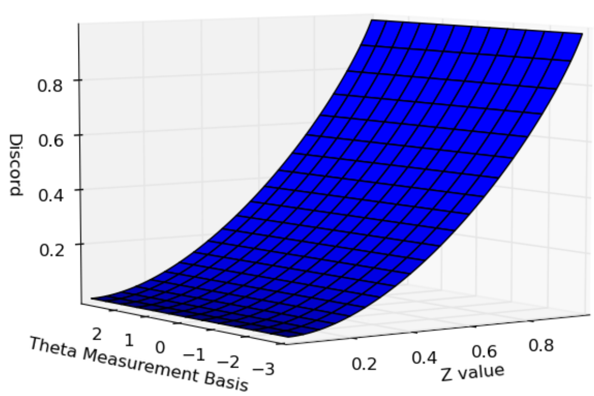

Figure 3.3 The figure shows the value of discord as a function of classical mixing parameter p and measurement basis for Werner state.

Chapter 4

Quantum Cloning

4.1 No cloning theorem

The laws of quantum mechanics, on the one hand gives us a huge technological boost from the classical world in accomplishing various information processing tasks[27, 5, 54, 35, 51, 1, 8, 36, 9, 40, 37, 10, 64, 49, 24], while on the other hand impose certain restrictions on the kind of transformation we can implement on quantum states. These restrictions are given in the form of theorems like no cloning theorem[62], no deletion theorem[46] and many others [47, 20, 17, 63, 45]. No cloning theorem prohibits us from cloning an unknown quantum state perfectly[62]. Nevertheless, given the impossibility of exact cloning, in the year 1996, Buzek *et. al.* went beyond this to introduce the idea of approximate cloning [62, 12, 33, 13, 11, 16, 15, 52] with certain fidelity of success. Approximate quantum cloning machines further can be of two types, a) state dependent [62, 11, 2], b) state independent [12, 13]. In state dependent quantum cloners, the performance of the cloning machine is dependent on the state to be cloned, whereas in state independent quantum cloners, the performance is independent of the input state parameters. The performance of state dependent cloners can be better than state independent cloners for some input states, but on an average, state independent performs better than state dependent cloners. In addition to this, we have probabilistic quantum cloning machines, with which we can clone an unknown quantum state, secretly chosen from a certain set of linearly independent states, with a certain non vanishing probability of success[52, 32, 25, 34].

4.2 Quantum Cloning Beyond No cloning Theorem

The no-cloning theorem states that given an arbitrary quantum state $|\psi\rangle$, there does not exist any completely positive trace preserving map (CPTP) $\mathcal{C}$ which will produce two copies of $|\psi\rangle$ as an output when applied on the state itself, $\mathcal{C} : |\psi\rangle \nrightarrow |\psi\rangle\,|\psi\rangle$. Although perfect cloning is not possible, it never rules out the possibility to clone the quantum states approximately with a fidelity,

$$F = \left\langle \psi | \rho^{out} | \psi \right\rangle , \tag{4.1}$$

where ρ^{out} being the state obtained at the output port after applying the cloning transformation. The most celebrated of all these cloning machines is the B-H cloning machine (U_{bh}), which is an M-dimensional quantum copying transformation acting on a state $|\Psi_i\rangle_{a_0}$ (i = 1, ..., M). This state is to be copied on a blank state $|\Sigma\rangle_{a_1}$. Initially, the copying machine is in state $|X\rangle_x$, which subsequently gets transformed into another set of state vectors $|X_{ii}\rangle_x$ and $|Y_{ij}\rangle_x$. Here, a_0, a_1 and x represent the input, blank and machine qubits respectively. In this case, these transformed state vectors belong to the orthonormal basis set in the M-dimensional space. The transformation scheme U_{bh} is given by [14],

$$\begin{aligned} &U_{bh}\,|\Psi_i\rangle_{a_0}|\Sigma\rangle_{a_1}|X\rangle_x \rightarrow c\,|\Psi_i\rangle_a|\Psi_i\rangle_b|X_{ii}\rangle_x \\ &+d\sum_{j\neq i}^{M}\left(|\Psi_i\rangle_a|\Psi_j\rangle_b+|\Psi_j\rangle_a|\Psi_i\rangle_b\right)|Y_{ij}\rangle_x\,, \end{aligned} \tag{4.2}$$

where i, j = $\{1, ..., M\}$, and the coefficients c and d are real. The coefficients c and d are the probability amplitudes associated with the probability of successfully getting the copied state or not respectively.

Another way to express the general M dimensional Buzek-Hillery quantum cloning machine, from which we can easily derive the state dependent cloner is as follows. Here, M refers to the dimensionality of m qubit system, so $M = 2^m$. The general pure state in M dimensions can be expressed as $|\psi\rangle = \sum_{i=1}^{M}\alpha_i|\psi_i\rangle$, where $|\psi_i\rangle$ are the basis vectors of the m qubit system. The cloning transformation is given by,

$$\begin{aligned} &U_{bh}\,|\psi_i\rangle|\Sigma\rangle|X\rangle \rightarrow |\psi_i\rangle|\psi_i\rangle|X_{ii}\rangle \\ &+\sum_{j=1,j\neq i}^{M}(|\psi_i\rangle|\psi_j\rangle+|\psi_j\rangle|\psi_i\rangle)\,|Y_{ij}\rangle\,. \end{aligned} \tag{4.3}$$

Here we have substituted $c = 1, d = 1$, and have introduced the following unitarity conditions

$$\begin{aligned} \langle X_{ii}\rangle\,X_{ii} &= 1-2(M-1)\lambda, \\ \langle X_{ii}\rangle\,X_{jj} &= 0, i\neq j, \\ \langle Y_{ij}\rangle\,Y_{ij} &= \lambda, \\ \langle X_{ii}\rangle\,Y_{ij} &= 0, \\ \langle Y_{ij}\rangle\,Y_{kl} &= 0, i\neq k, \\ \langle X_{ii}\rangle\,Y_{jk} &= \mu/2, i\neq j. \end{aligned} \tag{4.4}$$

Now onwards, we consider both these quantities λ and μ as machine parameters. The complete density matrix of the combined output state after tracing out machine states is given by,

$$\begin{aligned}\rho_{ab}^{out} = (1-2(M-1)\lambda)\sum_{i=1}^{M}\alpha_i\alpha_i^*(|\psi_i\rangle_a\langle\psi_i|\otimes|\psi_i\rangle_b\langle\psi_i|) \\ +\lambda\sum_{i=1}^{M}\alpha_i\alpha_i^*\sum_{j=1,j\neq i}^{M}|\chi_{ij}\rangle\langle\chi_{ij}| + \frac{\mu}{2}\sum_{i=1}^{M}\alpha_i\sum_{j=1,j\neq i}^{M}\alpha_j^* \\ \sum_{k=1,k\neq i}^{M}(|\psi_i\rangle_a|\psi_i\rangle_b\langle\phi_{jk}| + |\phi_{jk}\rangle\langle\psi_i|_a\langle\psi_i|_b).\end{aligned} \tag{4.5}$$

Here, $|\chi_{ij}\rangle = (|\psi_i\rangle_a|\psi_j\rangle_b + |\psi_j\rangle_a|\psi_i\rangle_b)$ and $|\phi_{jk}\rangle = (|\psi_j\rangle_a|\psi_k\rangle_b + |\psi_k\rangle_a|\psi_j\rangle_b)$. After tracing out the subsystem b we get the new cloned state at the output port a as,

$$\begin{aligned}\rho_a^{out} = \rho_b^{out} = (1-2(M-1)\lambda)\sum_{i=1}^{M}\alpha_i\alpha_i^*|\psi_i\rangle\langle\psi_i| \\ +\frac{\mu}{2}\sum_{i=1}^{M}\alpha_i\sum_{j=1,i\neq j}^{M}\alpha_j^*(|\psi_i\rangle\langle\psi_j| + |\psi_j\rangle\langle\psi_i|) \\ +\lambda\sum_{i=1}^{M}\alpha_i\alpha_i^*\sum_{j=1,j\neq i}^{M}(|\psi_i\rangle\langle\psi_i| + |\psi_j\rangle\langle\psi_j|).\end{aligned} \tag{4.6}$$

We introduce two quantities, $D_{ab} = tr[\rho_{ab}^{(out)} - \rho_a^{(id)}\otimes\rho_b^{(id)}]^2$ and $D_a = tr[\rho_a^{(out)} - \rho_a^{(id)}]^2$ to quantify the amount of distortion in the combined system and the individual system as a result of cloning. Here, $\rho_a^{(id)}$ and $\rho_b^{(id)}$ are the states if cloning was ideal, that is, $\rho_a^{(id)} = \rho_b^{(id)} = \rho_a^{(in)}$. To get the relation between λ and μ, we use the following condition $\frac{\partial D_a}{\partial\alpha_i^2} = 0$, which basically makes D_a independent of the input state parameters. For the M dimensional case, we get

$$\mu = 1 - M\lambda. \tag{4.7}$$

When we substitute μ with $1 - M\lambda$, we find that the distortion D_a reduces to

$$D_a = M(M-1)\lambda^2. \tag{4.8}$$

To calculate the optimal value of the machine parameter λ, we make $\frac{\partial D_{ab}}{\partial\lambda} = 0$ to get λ for which the value of D_{ab} is minimum. This is to ensure that the machine parameter selected should be the one which leads to minimum distortion in the actual complete output obtained and the ideal output that should have been obtained. Now, when we have a cloning transformation for m qubits, we can apply these to study the effect of cloning on teleportation, broadcasting, discord, coherence and compare the results for initial state and the state after cloning. The challenge here is to find a criteria similar to Peres-Horodecki criteria to detect entanglement for m qubit system. In this work, we have used Buzek-Hillery (B-H) QCM as the basic cloning machine and built upon it to get the state dependent cloning machine.

4.2.1 State dependent cloner

In general, we can copy an unknown quantum state with some imperfection. Cloning machines can be classified into two groups based on the kind of imperfection they create. If the performance of the cloning machine is dependent on the input state i.e., if the cloner performs well for some state and non so well for some other state, the cloning machine is called state dependent(SDC). We can define state dependence by relaxing the condition $\frac{\partial D_{ab}}{\partial \alpha_i^2} = 0$, so that we don't have a value of λ for which the cloning fidelity is independent of the input state parameters. Instead, we calculate $\frac{\partial D_{ab}}{\partial \lambda} = 0$, which gives the condition for optimality i.e. we get values of optimal λ as a function of input state parameters. This leads to two kinds of state dependent cloning machines and hence two definitions for state dependent cloning follows.

4.2.1.1 Static state dependent cloner(SSDC)

A static state dependent cloning machine is one whose value of optimal machine parameter λ is a fixed constant value. When we clone the given input state, the complete output state obtained after tracing out the machine states is a function of input states(α^2) as well as machine state parameters (λ and μ). To obtain the relation between between λ and μ, we impose the condition that D_a is independent of input parameters, i.e. $\frac{\partial D_a}{\partial {\alpha_i}^2} = 0$. The relation obtained is given in equation (4.7). D_a hence obtained is independent of input parameters and is only dependent on machine parameter λ. So, for a cloning machine with fixed machine parameter, the value of D_a remains same irrespective of input state which is being cloned. This cloner is state dependent in the sense that the value of D_{ab} is state dependent.

To give a better understanding of this type of cloning, we first prepare a static cloning machine and use it to study cloning in single qubit case. The general single qubit pure state can be expressed as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle(\alpha^2 + |\beta|^2 = 1)$. We use the following cloning transform,

$$\begin{aligned} |0\rangle|\Sigma\rangle|X\rangle &\rightarrow |0\rangle|0\rangle|X_{00}\rangle + (|0\rangle|1\rangle + |1\rangle|0\rangle)|Y_{01}\rangle \\ |1\rangle|\Sigma\rangle|X\rangle &\rightarrow |1\rangle|1\rangle|X_{11}\rangle + (|1\rangle|0\rangle + |0\rangle|1\rangle)|Y_{10}\rangle \end{aligned} \tag{4.9}$$

where $|\Sigma\rangle$ represents one qubit blank state and $|X\rangle$, $|Y\rangle$ represent machine states. We trace out the machine state to get the complete output state ρ_{ab}^{out} and impose the following conditions,

$$\begin{aligned} \langle X_{ii}\rangle X_{ii} &= 1 - 2\lambda, \\ \langle X_{ii}\rangle X_{jj} &= 0, i \neq j, \\ \langle Y_{ij}\rangle Y_{ij} &= \lambda, \\ \langle X_{ii}\rangle Y_{ij} &= 0, \\ \langle Y_{ij}\rangle Y_{kl} &= 0, i \neq k, \\ \langle X_{ii}\rangle Y_{jk} &= \mu/2, i \neq j. \end{aligned} \tag{4.10}$$

The complete output state (ρ_{ab}^{out}) is given by,

$$\rho_{ab} = \begin{bmatrix} \alpha^2(1-2\lambda) & \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} & \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} & 0 \\ \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} & \lambda & \lambda & \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} \\ \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} & \lambda & \lambda & \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} \\ 0 & \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} & \frac{\alpha\sqrt{1-\alpha^2}\mu}{2} & (1-\alpha^2)(1-2\lambda) \end{bmatrix}$$

and the reduced output state(ρ_a^{out}) is given by,

$$\rho_a^{out} = \rho_b^{out} = \begin{bmatrix} \alpha^2(1-2\lambda)+\lambda & \alpha\sqrt{1-\alpha^2}\mu \\ \alpha\sqrt{1-\alpha^2}\mu & \beta^2(1-2\lambda)+\lambda \end{bmatrix}.$$

The distortion in the reduced output state is given by, $D_a = Tr[\rho_a^{out} - \rho_a^{id}]^2$, where ρ_a^{id} is the same as ρ_a^{in}. The D_a, in this case, is given by,

$$D_a = 2(1-2\alpha^2)^2\lambda^2 - 2(\alpha^2-1)\alpha^2(\mu-1)^2 \tag{4.11}$$

The relation obtained when we apply the condition $\frac{\partial D_a}{\partial \alpha^2}$ is $\mu = 1-2\lambda$ and when we substitute this value in the expression of D_a, we get $D_a = 2\lambda^2$. It is evident that D_a is independent of input parameters and only dependent on machine parameter λ.

Next, we calculate $D_{ab} = Tr[\rho_{ab}^{out} - \rho_a^{id} \otimes \rho_b^{id}]^2$ where $\rho_b^{id} = \rho_a^{in}$. When we substitute $\mu = 1-2\lambda$, we obtain,

$$D_{ab} = 2(1-2\alpha^2)\alpha^2 + 12(\alpha^2-1)\lambda\alpha^2 + 8\lambda^2. \tag{4.12}$$

To obtain the completely state independent version of this cloner we can impose additional condition that D_{ab} is also input state independent, i.e. $\frac{\partial D_{ab}}{\partial \alpha^2} = 0$. When we follow the stated procedure, we observe that when $\lambda = \frac{1}{6}$, D_{ab} is independent of input parameters and is equal to constant value $\frac{2}{9}$. Here, we are interested in the state dependent version of cloner, so instead we obtain the optimal machine state parameter, which is obtained by making $\frac{\partial D_{ab}}{\partial \lambda} = 0$. The equation of the optimal machine state parameter obtained is given by[2],

$$\lambda_{optimal} = \frac{3\alpha^2(1-\alpha^2)}{4} \tag{4.13}$$

We can construct various static state dependent cloning machines by substituting values of input state parameters in equation (4.13). Table 4.1 shows how we can construct different static state dependent cloning machines and their respective distortions.

If we select one of these cloning machines (say M_1 with $\lambda = 0.0675$) and use it to clone the general single qubit pure state, its performance in terms of D_a and D_{ab} is given in Table 4.2.

4.2.1.2 Dynamic state dependent cloner(DSDC)

A dynamic state dependent cloning machine is one where the value of optimal machine parameter is not fixed for a machine, rather, it changes dynamically according to the state being cloned. Unlike the

Machine	α^2	λ_{SSDC}	$D_{a_{SSDC}}$
M_1	0.1	0.0675	0.0091
M_2	0.2	0.12	0.0288
M_3	0.3	0.1575	0.0496
M_4	0.4	0.18	0.0648
M_5	0.5	0.1875	0.0703
M_6	0.6	0.18	0.0648
M_7	0.7	0.1575	0.0496
M_8	0.8	0.12	0.0288
M_9	0.9	0.0675	0.0091

Table 4.1 λ_{SSDC} are machine parameters of various static state dependent cloning machines(M_1 to M_9). $D_{a_{SSDC}}$ is the distortion in reduced state when any input state is cloned using the respective static state dependent cloner.

α^2	λ_{SSDC}	$D_{a_{SSDC}}$	$D_{ab_{SSDC}}$
0.1	0.0675	0.0091	0.1436
0.2	0.0675	0.0091	0.2269
0.3	0.0675	0.0091	0.2864
0.4	0.0675	0.0091	0.3221
0.5	0.0675	0.0091	0.3340
0.6	0.0675	0.0091	0.3221
0.7	0.0675	0.0091	0.2864
0.8	0.0675	0.0091	0.2269
0.9	0.0675	0.0091	0.1436

Table 4.2 Performance of SSDC M_1 for various input states.

case of static state dependent cloning machine, here we do not need to create many cloning machines, rather we have only one cloning machine which can choose the value of optimal machine parameter (λ) according to the state being cloned. To get a better understanding, we use the same example as in the case of static state dependent cloning. We have the same functions for D_a and D_{ab}. Now, when we substitute $\mu = 1 - 2\lambda$ and the value of λ from equation (4.13) we get,

$$\begin{aligned} D_a &= \frac{9}{8}\alpha^4(1-\alpha^2)^2 \\ D_{ab} &= \frac{1}{2}(1-\alpha^2)\alpha^2(4-9(1-\alpha^2)\alpha^2) \end{aligned} \tag{4.14}$$

In Table 4.3 we show the performance of dynamic state dependent cloning machine when it is used to clone various single qubit quantum states. We can see here that both D_a and D_{ab} change when the input state is changed.

α^2	λ_{DSDC}	$D_{a_{DSDC}}$	$D_{ab_{DSDC}}$
0.1	0.0675	0.0091	0.1436
0.2	0.12	0.0288	0.2048
0.3	0.1575	0.0496	0.2216
0.4	0.18	0.0648	0.2208
0.5	0.1875	0.0703	0.2188
0.6	0.18	0.0648	0.2208
0.7	0.1575	0.0496	0.2216
0.8	0.12	0.0288	0.2048
0.9	0.0675	0.0091	0.1436

Table 4.3 λ_{DSDC} are machine parameters of dynamic state dependent cloning machine. $D_{a_{DSDC}}$ is the distortion in reduced state and $D_{ab_{DSDC}}$ is the distortion in complete output state.

In the context of broadcasting, when a specific static state dependent cloning machine with a fixed value of optimal machine parameter is used, we obtain a range of values of input parameters, where the broadcasting of entanglement is possible. When a different static state dependent cloning machine with a different optimal machine parameter is used, we obtain a different range of input parameters where broadcasting is possible. On the other hand, if we use a dynamic state dependent cloning machine, we obtain a single range of values of input parameters where broadcasting is possible. This is because when an input state is supplied to the cloner for broadcasting, the cloner automatically selects the optimal machine parameter according to the input state.

4.2.1.3 Local cloner

Now let us try to clone the combined state of two quantum systems. There can be two ways of doing this, first where we apply the cloning transformation on state of each particle individually, which is called local cloning or we can apply a single cloning transform on combined two particle state of the system which is called non-local cloning. Let the input state be,

$$\rho_{ab} = \begin{bmatrix} \alpha^2 & 0 & 0 & \alpha\beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha\beta & 0 & 0 & \beta^2 \end{bmatrix}$$

then the reduced density matrix representing each of these subsystems will be,

$$\rho_a = \rho_b = \begin{bmatrix} \alpha^2 & 0 \\ 0 & \beta^2 \end{bmatrix}.$$

Now, if parties a and b apply the cloning transformation on their individual systems, which is given by equation (4.9), we refer to it as local cloning. We trace out the machine states using conditions given in equation (4.10) to get the complete output state. The local cloner can be seen as cloning transformation in 2 dimensions, that is M=2 so we get $\mu = 1 - 2\lambda$. In order to calculate the value of optimal λ, we calculate $\frac{\partial D_{ab}}{\partial \lambda} = 0$, where $D_{ab} = tr[\rho_{ab}^{(out)} - \rho_a^{(id)} \otimes \rho_b^{(id)}]^2$.

4.2.1.4 Non-local cloner

The other way to clone the two qubit state would be to apply a global cloning transformation and this is given by,

$$\begin{aligned} |00\rangle |\Sigma\rangle |X\rangle \rightarrow |00\rangle |00\rangle |X_{00}\rangle + (|00\rangle |01\rangle + |01\rangle |00\rangle) |Y_{01}\rangle + \\ (|00\rangle |10\rangle + |10\rangle |00\rangle) |Y_{02}\rangle + (|00\rangle |11\rangle + |11\rangle |00\rangle) |Y_{03}\rangle . \end{aligned} \tag{4.15}$$

Here $|\Sigma\rangle$ represents blank state and $|X\rangle$, $|Y\rangle$ represent machine states. Similarly, we can also write the cloning transformations for rest of the basis vectors $|01\rangle$, $|10\rangle$ and $|11\rangle$. We trace out the machine states to get the complete output state ρ_{abcd} and impose the following conditions

$$\begin{aligned} \langle X_{ii}\rangle X_{ii} &= 1 - 6\lambda, \\ \langle X_{ii}\rangle X_{jj} &= 0, i \neq j, \\ \langle Y_{ij}\rangle Y_{ij} &= \lambda, \\ \langle X_{ii}\rangle Y_{ij} &= 0, \\ \langle Y_{ij}\rangle Y_{kl} &= 0, i \neq k, \\ \langle X_{ii}\rangle Y_{jk} &= \mu/2, i \neq j. \end{aligned} \tag{4.16}$$

The non-local cloner here can be seen as cloning transformation in 4 dimensions, that is M=4, so we get $\mu = 1 - 4\lambda$. In order to calculate the value of optimal λ, we calculate $\frac{\partial D_{abcd}}{\partial \lambda} = 0$, where $D_{abcd} = tr[\rho_{abcd}^{(out)} - \rho_{ab}^{(id)} \otimes \rho_{cd}^{(id)}]^2$.

4.2.2 State independent cloner

If a cloning machine performs the same for any input state, it is called state independent quantum cloning machine. The optimal state independent version of the B-H cloner (U_{bhsi}) can be obtained by imposing following conditions, $\langle X_{ii}\rangle X_{ii} = \langle Y_{ij}\rangle Y_{ij} = \langle X_{ii}\rangle Y_{ji} = 1$, $\langle X_{ii}\rangle Y_{ij} = \langle Y_{ji}\rangle Y_{ij} = \langle X_{ii}\rangle X_{jj} = 0$ and the output state obtained after cloning can be expressed in the form $\rho_a^{out} = s |\psi\rangle \langle\psi| + \frac{1-s}{4}\mathbb{I}$, where $|\psi\rangle$ is the original input state, s is the classical mixing parameter, which tells about the quality of cloning and $\mathbb{I}$ is the identity matrix. After imposing these conditions and selecting the transformation for which D_a is minimum, we get the values of coefficients c and d as follows,

$$c^2 = \frac{2}{M+1}, d^2 = \frac{1}{2(M+1)} \tag{4.17}$$

Another way to derive state-independence for the cloner described in equation (4.3), where c=1, d=1, with the following conditions,

$$\begin{aligned} \langle X_{ii}\rangle X_{ii} &= 1-2(M-1)\lambda, \\ \langle X_{ii}\rangle X_{jj} &= 0, i\neq j, \\ \langle Y_{ij}\rangle Y_{ij} &= \lambda, \\ \langle X_{ii}\rangle Y_{ij} &= 0, \\ \langle Y_{ij}\rangle Y_{kl} &= 0, i\neq k, \\ \langle X_{ii}\rangle Y_{jk} &= \mu/2, i\neq j, \end{aligned} \tag{4.18}$$

is to get the value of λ for which the performance of cloner becomes independent of the input state. We apply the following conditions $\frac{\partial D_{ab}}{\partial \alpha_i^2}=0$ and $\frac{\partial D_a}{\partial \alpha_i^2}=0$, where $D_{ab}=tr[\rho_{ab}^{(out)}-\rho_a^{(id)}\otimes\rho_b^{(id)}]^2$, $D_a=tr[\rho_a^{(out)}-\rho_a^{(id)}]^2$ and α_i^2 are the input state parameters. Solving this, we get the value of machine parameter $\lambda=\frac{1}{2(M+1)}$. For the case of local and non local cloner described in the section above, we get $\lambda=\frac{1}{6}$ and $\lambda=\frac{1}{10}$ respectively.

4.2.3 Comparative analysis of state dependent and state independent cloner

In this subsection, we give a comparative analysis of both dynamic state dependent(DSDC) and state independent cloners in terms of the distortion of the reduced output state for both local(DSDLC and SILC) and non local case(DSDNLC and SINLC). In Table 4.4 we show the comparison between state dependent and state independent cloner. We have used $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$ as input state when we are performing local cloning [2] and $|\psi\rangle=\alpha|00\rangle+\beta|11\rangle$ while performing non-local cloning. We obtain optimal λ for the local case using equation (4.13) and for the non local case using equation (5.19). We note that when dynamic state dependent local cloner (DSDLC) is used, the distortion in reduced output state (D_a) is less than the case when state independent local cloner (SILC) is used, when α^2 is $0.1, 0.2, 0.3, 0.7, 0.8$ and 0.9, and D_a is greater for $DSDLC$ when α^2 is $0.4, 0.5$ and 0.6. Next, when we use non local cloning then the values of D_a for dynamic state dependent (DSDNLC) cloner are higher than state independent cloner (SINLC), when α^2 is $0.3, 0.4, 0.5, 0.6$ and 0.7, and lower when α^2 is $0.1, 0.2, 0.8$ and 0.9. For the general M-dimensional state-independent non-local case we have,

$$D_a=\frac{M(M-1)}{4(M+1)^2}. \tag{4.19}$$

In the next chapter, we have compared the performance of state dependent cloners, both static and dynamic, when applied locally or non locally to study broadcasting of entanglement.

α^2	λ_{DSDL}	$D_{a_{DSDL}}$	$D_{a_{SIL}}$	λ_{DSDNL}	$D_{a_{DSDNL}}$	$D_{a_{SINL}}$
0.1	0.0675	0.0091	0.0556	0.0480	0.0276	0.12
0.2	0.12	0.0288	0.0556	0.0844	0.0855	0.12
0.3	0.1575	0.0496	0.0556	0.1099	0.1449	0.12
0.4	0.18	0.0648	0.0556	0.125	0.1875	0.12
0.5	0.1875	0.0703	0.0556	0.13	0.2028	0.12
0.6	0.18	0.0648	0.0556	0.125	0.1875	0.12
0.7	0.1575	0.0496	0.0556	0.1099	0.1449	0.12
0.8	0.12	0.0288	0.0556	0.0844	0.0855	0.12
0.9	0.0675	0.0091	0.0556	0.0480	0.0276	0.12

Table 4.4 Comparative analysis of dynamic state dependent local (DSDL), state independent local (SIL), dynamic state dependent non local (DSDNL) and state independent non local (SINL) cloning machines.

Chapter 5

Broadcasting of correlations

Broadcasting of entanglement refers to a situation where, from a given entangled state, we can create multiple entangled states by applying unitary transformations, both locally and non locally. There are several ways to implement this, however, the most predominant way is the one when we use both local and non local cloning machines. It is the reverse process of distilling pure entangled states from mixed entangled states. Though perfect broadcasting of quantum entanglement is prohibited as a consequence of no cloning theorem, however, it is always possible to partially broadcast quantum entanglement with the help of both types of cloning transformations. When we refer to the broadcasting of entanglement, we generally talk about creating more pairs of lesser entangled states from the initial entangled state. Our basic aim is to broadcast the amount of entanglement present in the given input state to many pairs. The procedure we follow to achieve this is to take an entangled state ρ_{12} and a blank state, apply the cloning transformation and trace out the machine state to get the complete output state ρ_{1234}. Ideally, broadcasting of entanglement would be possible if we are able to generate more number of entangled pairs from it. However, for optimal broadcasting of entanglement between non-local outputs, we must not have any entanglement between local output states.

Definition 5.0.1. *Optimal local broadcasting:* An entangled state ρ_{12} is said to be optimally broadcast after the application of local cloning operation $U1 \otimes U2$ (each of the type given by equation 4.3) on the qubits 1 and 2 respectively, if for some values of the input state parameters,

- the non local output states between A and B

$$\rho_{14}^{out} = Tr_{23}[U1 \otimes U2\rho_{12}],$$
$$\rho_{23}^{out} = Tr_{14}[U1 \otimes U2\rho_{12}] \tag{5.1}$$

  are inseparable, and

- the local output states of A and B

$$\begin{aligned} \rho_{13}^{out} &= Tr_{24}[U1 \otimes U2 \rho_{12}], \\ \rho_{24}^{out} &= Tr_{13}[U1 \otimes U2 \rho_{12}] \end{aligned} \tag{5.2}$$

are separable.

Definition 5.0.2. *Optimal non local broadcasting:* An entangled state ρ_{12} is said to be optimally broadcast after the application of non local cloning operation U_{12} (given by equation 4.3) together on the qubits 1 and 2, if for some values of the input state parameters,

- the desired output states,

$$\begin{aligned} \rho_{12}^{out} &= Tr_{34}[U_{12} \rho_{12}] \\ \rho_{34}^{out} &= Tr_{12}[U_{12} \rho_{12}] \end{aligned} \tag{5.3}$$

are inseparable,

- and the remaining output states

$$\begin{aligned} \rho_{13}^{out} &= Tr_{24}[U_{12} \rho_{12}] \\ \rho_{24}^{out} &= Tr_{13}[U_{12} \rho_{12}] \end{aligned} \tag{5.4}$$

are separable.

Without any loss of generality, we have chosen the non local output states to be ρ_{12}^{out} and ρ_{34}^{out}. Alternatively, we could also have chosen ρ_{14} and ρ_{23}.

5.1 Cloning general two qubit state

In this subsection, we make a systematic study of broadcasting of entanglement present in two qubit states, starting with a general representation of two qubit state, written in terms of Bloch vectors and correlation matrix, followed by specific examples like non maximally entangled state (NME), Werner like state (WLS) and Bell diagonal state (BDS). In this context, we have used various cloning machines like state independent local (SILC), static state dependent local (SSDLC), dynamic state dependent local (DSDLC), state independent non local (SINLC), static state dependent non local (SSDNLC) and dynamic state dependent non local (DSDNLC) cloning machines. The general two qubit state, represented in terms of Bloch vector and correlation matrix is expressed as,

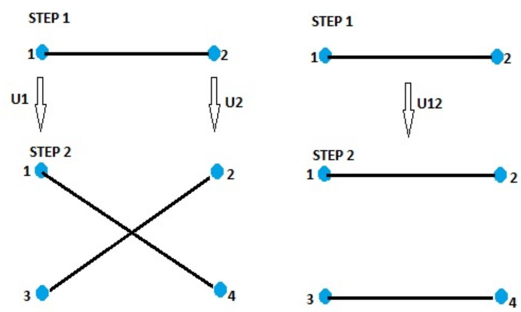

Figure 5.1 The first figure shows, the broadcasting of the state ρ_{12} into ρ_{14} and ρ_{23} by application of local cloning unitary transformations U_1 and U_2 on both sides. The second figure shows, the broadcasting of the state ρ_{12} into ρ_{12} and ρ_{34} by application of non local cloning unitary transformation U_{12} on complete input state.

$$\rho_{12} = \frac{1}{4}\Big[\mathbb{I}_4 + \sum_{i=1}^{3}(x_i\sigma_i \otimes \mathbb{I}_2 + y_i\mathbb{I}_2 \otimes \sigma_i) + \sum_{i,j=1}^{3} t_{ij}\sigma_i \otimes \sigma_j\Big] = \{\vec{x}, \vec{y}, T\}, \tag{5.5}$$

where $x_i = Tr[\rho_{12}(\sigma_i \otimes \mathbb{I}_2)]$, $y_i = Tr[\rho_{12}(\mathbb{I}_2 \otimes \sigma_i)]$, $t_{ij} = Tr[\rho_{12}\sigma_i \otimes \sigma_j]$, σ_i are Pauli matrices. I_n is the identity matrix of order n and i, j can take values $\{1, 2, 3\}$. In the simplified expression, $\vec{x}$, $\vec{y}$ are Bloch vectors and $T = [t_{ij}]$ is the correlation matrix.

Let us consider a situation where qubit 1 is with Alice and qubit 2 is with Bob. We apply two local cloners U_1, U_2 on ρ_{12} with blank state $|\Sigma\rangle_3$ on Alice's side and $|\Sigma\rangle_4$ on Bob's side. On tracing out we get the local output states $\rho_{13}^{out} = Tr_{24}[U_1 \otimes U_2(\rho_{12} \otimes |\Sigma\rangle_3 \otimes |\Sigma\rangle_4)]$ and $\rho_{24}^{out} = Tr_{13}[U_1 \otimes U_2(\rho_{12} \otimes |\Sigma\rangle_3 \otimes |\Sigma\rangle_4)]$. The non local output states are given by $\rho_{14}^{out} = Tr_{23}[U_1 \otimes U_2(\rho_{12} \otimes |\Sigma\rangle_3 \otimes |\Sigma\rangle_4)]$ and $\rho_{23}^{out} = Tr_{14}[U_1 \otimes U_2(\rho_{12} \otimes |\Sigma\rangle_3 \otimes |\Sigma\rangle_4)]$. First of all, we consider the case when we apply state independent local cloners (SILC) independently on each of these qubits 1 and 2. The local and the non local output states are respectively given by,

$$\rho_{13}^{out} = \Big\{\frac{2}{3}\vec{x}, \frac{2}{3}\vec{x}, \frac{1}{3}\mathbb{I}_3\Big\}, \rho_{24}^{out} = \Big\{\frac{2}{3}\vec{y}, \frac{2}{3}\vec{y}, \frac{1}{3}\mathbb{I}_3\Big\}, \tag{5.6}$$

$$\rho_{14}^{out} = \rho_{23}^{out} = \Big\{\frac{2}{3}\vec{x}, \frac{2}{3}\vec{y}, \frac{4}{9}T\Big\}. \tag{5.7}$$

Secondly, we consider the case when we use state dependent local cloners (SDLC). In this situation, the local and the non local outputs obtained after cloning are given by,

$$\rho_{13}^{out} = \Big\{\mu\vec{x}, \mu\vec{x}, T_l^{sd}\Big\}, \rho_{24}^{out} = \Big\{\mu\vec{y}, \mu\vec{y}, T_l^{sd}\Big\}, \tag{5.8}$$

$$\rho_{14}^{out} = \rho_{23}^{out} = \left\{ \mu\vec{x}, \mu\vec{y}, \mu^2 T \right\}, \tag{5.9}$$

where T_l^{sd} =diag$[2\lambda, 2\lambda, 1-4\lambda]$, T is the input state correlation matrix and μ, λ are the machine parameters as usual. For local cloning, these machine parameters are related by $\mu = 1 - 2\lambda$. It is interesting to note that these output states are same whether we use static or dynamic state dependent cloning machine (SSDLC / DSDLC).

Next, we consider the case when we have the non local cloners. In case of non local cloning, we apply single cloner on the combined two qubit state instead of two different cloners locally. The output states obtained in this process are given by,

$$\begin{aligned} \rho_{12}^{out} &= Tr_{34}[U_{12}\rho_{12}], \\ \rho_{34}^{out} &= Tr_{12}[U_{12}\rho_{12}], \\ \rho_{13}^{out} &= Tr_{24}[U_{12}\rho_{12}], \\ \rho_{24}^{out} &= Tr_{13}[U_{12}\rho_{12}]. \end{aligned} \tag{5.10}$$

The local and the non local output states obtained in case of state independent cloning transformations are given by,

$$\rho_{13}^{out} = \left\{ \frac{3}{5}\vec{x}, \frac{3}{5}\vec{x}, \frac{1}{5}\mathbb{I}_3 \right\}, \rho_{24}^{out} = \left\{ \frac{3}{5}\vec{y}, \frac{3}{5}\vec{y}, \frac{1}{5}\mathbb{I}_3 \right\}, \tag{5.11}$$

$$\rho_{12}^{out} = \rho_{34}^{out} = \left\{ \frac{3}{5}\vec{x}, \frac{3}{5}\vec{y}, \frac{3}{5}T \right\}. \tag{5.12}$$

Finally, when we apply state dependent cloning transformations on both the qubits together, the local and the non local output states obtained as a result of this, are given by,

$$\rho_{13}^{out} = \left\{ \mu\vec{x}, \mu\vec{x}, T_{nl}^{sd} \right\}, \rho_{24}^{out} = \left\{ \mu\vec{y}, \mu\vec{y}, T_{nl}^{sd} \right\}, \tag{5.13}$$

$$\rho_{12}^{out} = \rho_{34}^{out} = \left\{ \mu\vec{x}, \mu\vec{y}, \mu T \right\}. \tag{5.14}$$

Here, matrix T_{nl}^{sd} is a $3X3$ diagonal matrix, with the diagonal elements being $2\lambda, 2\lambda, 1-8\lambda$. Here, T is the same input state correlation matrix. However, the machine parameters μ and λ are related differently from the local cloning case as $\mu = 1 - 4\lambda$. It is needless to mention that even in the non local case, the outputs obtained as a result of both static and dynamic cloning operations are identical.

5.2 Broadcasting Non-Maximally entangled state(NME)

Here, we specifically consider the problem of broadcasting non-maximally entangled state (NME), $|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$ where α, β are probability amplitudes ($\alpha^2 + |\beta|^2 = 1$). We use state independent(SIC), static state dependent(SSDC) and dynamic state dependent cloning(DSDC) machines, both locally(SILC. SSDLC, DSDLC respectively) and non locally(SINCL, SSDNLC, DSDLC respectively), for the purpose of broadcasting. We also compare broadcasting ranges of each of the above stated cloners with the help of different tables (5.1, 5.2) and figure (5.2). The same non maximally entangled state (NME) can be expressed in terms of Bloch vector and correlation matrix as $\rho_{12}^{nm} = \left\{\vec{x}^{nm}, \vec{x}^{nm}, T^{nm}\right\}$, where $\vec{x}^{nm} = \{0, 0, (\alpha^2 - \beta^2)\}$ and $T^{nm} = diag[2\alpha\beta, -2\alpha\beta, 1]$.

5.2.1 Local static state dependent and state independent cloners:

The output states obtained as a result of state dependent cloning transformations (static/dynamic) are given by,

$$\begin{aligned} \rho_{13}^{out} = \rho_{24}^{out} = \left\{\mu\vec{x}^{nm}, \mu\vec{x}^{nm}, T_l^{sd}\right\} \\ , \rho_{14}^{out} = \rho_{23}^{out} = \left\{\mu x^{\vec{nm}}, \mu x^{\vec{nm}}, \mu^2 T^{nm}\right\}. \end{aligned} \tag{5.15}$$

We have already seen in section 2.2, that the value of λ obtained in case of local cloning to make the cloning machine state independent is $\frac{1}{6}$. In order to make a static state dependent cloning machine, we first have to calculate optimal value of λ. To calculate optimal λ, we make $\frac{\partial D_{13}}{\partial \lambda} = 0$, where $D_{13} = Tr[\rho_{13}^{out} - \rho_1^{id} \otimes \rho_3^{id}]$ is the distortion in the combined state. There can be two ways in which we can introduce state dependence here. In the first method, we prepare a state dependent cloning machine for a general pure single qubit quantum state and use it locally to clone non-maximally entangled state. If we use this method then the expression for optimal lambda reduces to $\lambda = \frac{3\alpha^2(1-\alpha^2)}{4}$[2]. The other way to use state dependent cloning machine would be to prepare it according to the state which is to be cloned, that is, the single qubit state obtained after tracing out the other qubit from the combined density matrix of non-maximally entangled state, $\rho_1 = Tr_2[\rho_{12}]$, where $\rho_{12} = |\psi\rangle\langle\psi|$. In this work, we restrict our discussion to second definition of state dependence for both static and dynamic cloning. The optimal λ obtained in this case is given by,

$$\lambda_{optimal} = \frac{\alpha^2(1-\alpha^2)}{2(1-\alpha^2+\alpha^4)}. \tag{5.16}$$

We have used various values of α^2 ranging from 0.1 to 0.9 with uniform interval of 0.1, to get the values of optimal machine parameter for various static state dependent cloners. We observe that due to symmetry in the non-maximally entangled state, that is, if we interchange the values of α^2 and β^2 (say value of α^2 is 0.1, then value of β^2 = 0.9 or if value of α^2 is 0.9, then value of β^2 = 0.1), this should essentially give us same optimal machine parameter. We have used the values of optimal machine parameters obtained by the method above to find the broadcasting range of the non maximally entangled states. To find the broadcasting, range we apply PPT criteria to see for what values of α^2 ρ_{13}, ρ_{24} are separable and ρ_{14}, ρ_{23} are inseparable. In table 5.1, we show the comparison between various static state dependent local cloners (SSDLC) and state independent local cloner(SILC) in terms of range of values of α^2, where broadcasting is possible. It is evident that all the static state dependent cloning machines perform better than the state independent version. This is because we obtain $\lambda = \frac{1}{6}$ in case of state independent local cloning when we are cloning pure state, but the actual state on which the local cloner is acting in case of non maximally entangled (NME) state is a mixed state, which is obtained when we trace out the other system $\rho_1 = Tr_2[\rho_{12}]$. In case of static state dependent local cloners, we obtain machine parameters from equation (5.16), which considers the original input states as mixed states.

Machine	λ_{SSDLC}	Range (SSDLC)	Range (SILC)
M_1	0.0494	0.0034< α^2 <0.9966	0.1097< α^2 <0.8903
M_2	0.0952	0.0176< α^2 <0.9824	0.1097< α^2 <0.8903
M_3	0.1329	0.0480< α^2 <0.9520	0.1097< α^2 <0.8903
M_4	0.1578	0.0885< α^2 <0.9120	0.1097< α^2 <0.8903
M_5	0.1666	0.1097< α^2 <0.8903	0.1097< α^2 <0.8903
M_6	0.1578	0.0885< α^2 <0.9120	0.1097< α^2 <0.8903
M_7	0.1329	0.0480< α^2 <0.9520	0.1097< α^2 <0.8903
M_8	0.0952	0.0176< α^2 <0.9824	0.1097< α^2 <0.8903
M_9	0.0494	0.0034< α^2 <0.9966	0.1097< α^2 <0.8903

Table 5.1 Comparison of broadcasting ranges of various static state dependent local cloners (SSDLC) and state independent local cloner (SILC).

5.2.2 Non local state independent and static state dependent cloners

Next, we study static state dependent(SSDNLC) and state independent(SINLC) non-local cloners. The local output states obtained by non-local state dependent cloners is given by,

$$\rho_{13}^{out} = \rho_{24}^{out} = \left\{\mu x^{\vec{nm}}, \mu x^{\vec{nm}}, T_{nl}^{sd}\right\} \tag{5.17}$$

and the non-local (inseparable) output states are given by,

$$\rho_{12}^{out} = \rho_{34}^{out} = \left\{\mu x^{\vec{nm}}, \mu x^{\vec{nm}}, \mu T^{nm}\right\}, \tag{5.18}$$

where $\mu = 1 - 4\lambda$. On substituting $\lambda = 1/10$, we get the optimal state independent cloner. In case of state dependent cloner, we calculate value of optimal machine parameter by calculating $\frac{\partial D_{1234}}{\partial \lambda} = 0$, where $D_{1234} = Tr[\rho_{1234}^{out} - \rho_{12}^{id} \otimes \rho_{34}^{id}]^2$,

$$\lambda_{optimal} = \frac{13\alpha^2(1-\alpha^2)}{4(6+\alpha^2-\alpha^4)}. \tag{5.19}$$

Table 5.2 gives the comparison between various static state dependent cloning machines and state the independent cloning machine for the task of broadcasting. We see similar trends as in local case, that is, as the value of λ increases, broadcasting range decreases. Maintaining the symmetry, lambda again decreases and broadcasting range increases. Here, non local static state dependent cloning machines M_1, M_2, M_8, M_9 perform better than state independent cloning machine and M_3, M_4, M_5, M_6, M_7 perform worse than state independent cloning machine.

Machine	λ_{SSDNLC}	Range (SSDNLC)	Range (SINLC)
M_1	0.0480	0.0035< α^2 <0.9965	0.0286< α^2 <0.9714
M_2	0.0844	0.0165< α^2 <0.9835	0.0286< α^2 <0.9714
M_3	0.1099	0.0401< α^2 <0.9599	0.0286< α^2 <0.9714
M_4	0.125	0.0670< α^2 <0.9330	0.0286< α^2 <0.9714
M_5	0.13	0.0797< α^2 <0.9203	0.0286< α^2 < 0.9714
M_6	0.125	0.0670< α^2 <0.9330	0.0286< α^2 <0.9714
M_7	0.1099	0.0401< α^2 <0.9599	0.0286< α^2 <0.9714
M_8	0.0844	0.0165< α^2 <0.9835	0.0286< α^2 <0.9714
M_9	0.0480	0.0035< α^2 <0.9965	0.0286< α^2 <0.9714

Table 5.2 Comparison of broadcasting ranges for various static state dependent non local cloners (SS-DNLC) and state independent non local cloner (SINLC).

5.2.3 Dynamic cloner

The last kind of cloner we discuss is dynamic state dependent cloner. In this case, the value of λ is automatically selected by the machine according to the input state. So a machine, instead of having a fixed machine parameter, has a variable one which takes optimal value for the given input automatically. To do this we first obtain the condition for broadcasting by using PPT criterion, the condition obtained is in terms of λ and α^2 and now we substitute the value of optimal λ, which is a function of α^2, to obtain the condition for broadcasting entirely in terms of α^2. The broadcasting range obtained using these cloners, both locally and non-locally, is same, and that is the entire range of the input parameter α^2. Figure (5.2) shows the broadcasting range(R= upper limit of α^2 - lower limit of α^2) on y axis and k on x-axis. k is the value of α^2 used to create various static state dependent cloners. It is evident that non-local cloners are better than local cloners and state dependent cloners are better than state independent ones for some values of α^2. Dynamic state dependent cloner is the best amongst all, as it gives the largest broadcasting range.

5.3 Broadcasting Werner like state (WLS)

The second example we consider here is Werner like state, $\rho_{12}^{w} = p\,|\psi\rangle\langle\psi| + \frac{1-p}{4}\mathbb{I}_4$. Here $|\psi\rangle = \alpha\,|00\rangle + \beta\,|11\rangle$ is non maximally entangled state, with 'p' being the classical randomness and $\mathbb{I}_4$ being the identity operator. The same state can be expressed in terms of the Bloch vectors and correlation

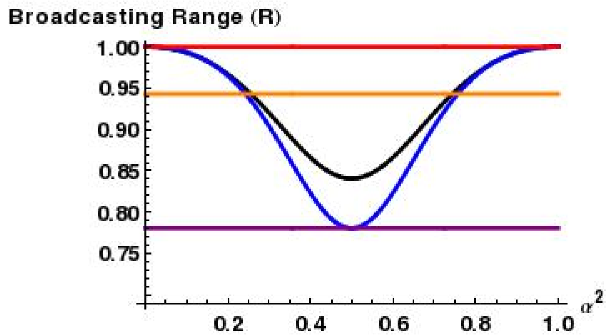

Figure 5.2 The figure shows the broadcasting range of non maximally entangled state(NME) when various cloners are used. Purple $\rightarrow$ state independent local cloner, blue $\rightarrow$ various static state dependent local cloners, black $\rightarrow$ various static state dependent non local cloners, orange $\rightarrow$ state independent non local cloner and red $\rightarrow$ dynamic state dependent cloner for both local and non-local cloning.

matrix as, $\rho_{12}^{w} = \{\vec{x}^{w}, \vec{x}^{w}, T^{w}\}$, where $\vec{x}^{w} = \{0, 0, p(\alpha^2 - \beta^2)\}$ and $T^{w} = \text{diag}[2p\alpha\beta, -2p\alpha\beta, p]$.

5.3.1 Static state dependent and state independent local cloning

Local and non local output states obtained after local state-dependent cloning are given by,

$$\begin{aligned} \rho_{13}^{out} = \rho_{24}^{out} &= \Big\{\mu\vec{x}^{w}, \mu\vec{x}^{w}, T_l^{sd}\Big\}, \\ \rho_{14}^{out} = \rho_{23}^{out} &= \Big\{\mu x^{\vec{w}}, \mu x^{\vec{w}}, \mu^2 T^{w}\Big\}. \end{aligned} \tag{5.20}$$

As before, to get the state independent output, we substitute $\lambda = 1/6$. The optimal value of λ for state dependent cloner is obtained by equating the partial derivative of D_{13} ($D_{13} = Tr[\rho_{13}^{out} - \rho_1^{id} \otimes \rho_3^{id}]$, $\rho_1^{id} = \rho_3^{id} = Tr_2[\rho_{12}^{w}]$) with respect to λ to zero, i.e. $\frac{\partial D_{13}}{\partial \lambda} = 0$.

The optimal machine parameter λ obtained in this way is given by,

$$\lambda_{optimal} = \frac{1 - p^2(4\alpha^4 - 4\alpha^2 + 1)}{2(3 + p^2(4\alpha^4 - 4\alpha^2 + 1))}. \tag{5.21}$$

We apply Peres-Horodecki criteria to find out the broadcasting range i.e. the range of values for which ρ_{13}^{out} (ρ_{24}^{out}) is separable and ρ_{14}^{out} (ρ_{23}^{out}) is inseparable. These ranges, for each of the static state dependent machines (SSDLC), are obtained by substituting the values of machine parameters λ. The parameter k is the value of α^2 we have used to create various static state dependent cloners. The broadcasting ranges (R) obtained for each of these cloners in terms of allowed values of p and α^2 is shown in figure (5.3). We note that broadcasting range (R) in terms of α^2 increases as p increases and it is maximum when $p = 1$. We also note that as the value of k increases from 0.1 to 0.5, the broadcasting range decreases and when k goes from 0.5 to 0.9, the broadcasting range again increases.

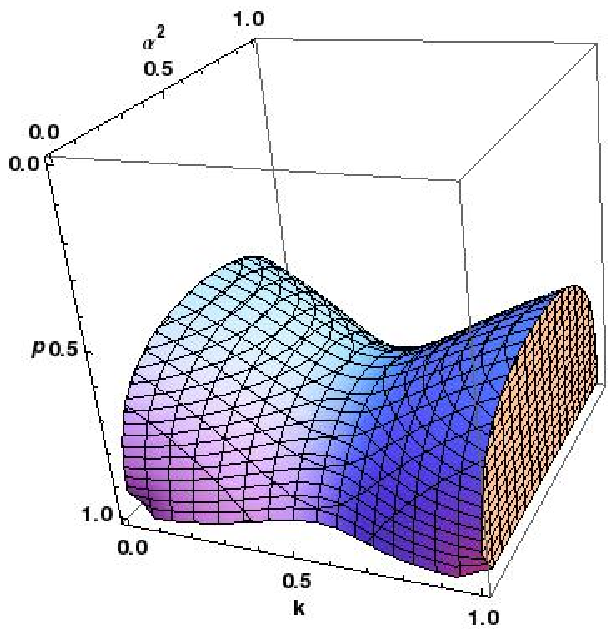

Figure 5.3 The figure shows the broadcastable region of Werner like state in terms of input parameters α^2 and p when different static state dependent local cloners are used for different choices of k.

5.3.2 Static state dependent and state independent non local cloning

Next, we use non-local cloners for the purpose of broadcasting entanglement. The local and the non local output states obtained in this process are given by,

$$\begin{aligned}\rho_{13}^{out} &= \rho_{24}^{out} = \left\{\mu x^{\vec{w}}, \mu x^{\vec{w}}, T_{nl}^{sd}\right\},\\ \rho_{12}^{out} &= \rho_{34}^{out} = \left\{\mu x^{\vec{w}}, \mu x^{\vec{w}}, \mu T^{w}\right\}.\end{aligned} \tag{5.22}$$

For the state independent part, to make the machine free from the input state, we substitute once again $\lambda = \frac{1}{10}$. Coming back to state dependent cloning, we follow the same procedure to get the optimal values of λ as,

$$\lambda_{optimal} = \frac{-3 + p^2(34\alpha^4 - 34\alpha^2 - 3) + 6p^3(3\alpha^4 - 3\alpha^2 + 1)}{2(-15 + p^2(8\alpha^4 - 8\alpha^2 - 33))} \tag{5.23}$$

As before, to construct the different state dependent cloners we take selective values of α^2 and relabel it as the parameter k. Figure (5.4) shows the broadcastable region in terms of the parameters α^2 and p for different values of k. It is interesting to note that static state dependent non local (SSDNLC) cloning machines perform better than the static state dependent local cloning (SSDLC) machines, for any values of the parameter k. From both the figures (5.4, 5.3), it is evident that the state is not broadcastable for all values of p.

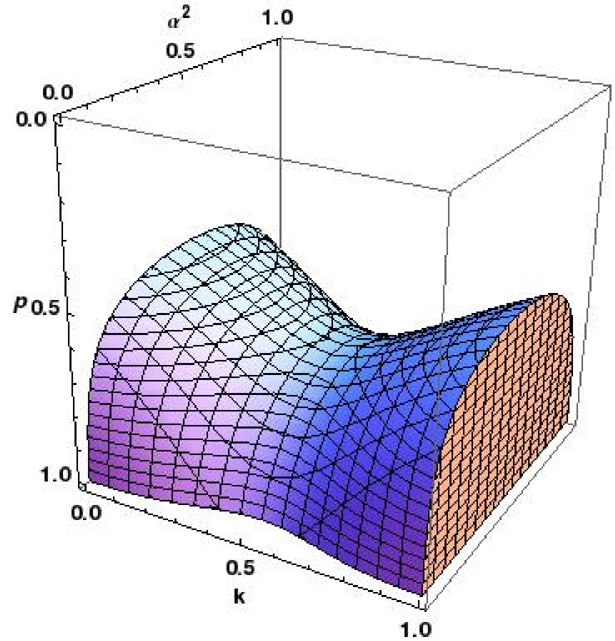

Figure 5.4 The figure shows the broadcastable region of Werner like state in terms of input parameters α^2 and p, when different static state dependent non local cloners are used for different choices of k.

Henceforth, we take three values of p $(0.65, 0.80, 0.95)$ uniformly from the broadcastable region and construct three corresponding tables (5.3, 5.4, 5.5) Where in each of them we compare the broadcastable range for both state dependent local (SSDLC) and non local (SSDNLC) cloning machines, corresponding to the choice of different machines, for different values of k. When we use state independent cloning machines to broadcast entanglement, the ranges obtained in the local case are 0.288< α^2 <0.711 and 0.135< α^2 <0.0.864 for $p = 0.80$ and $p = 0.95$, respectively. We cannot broadcast entanglement using local state independent cloners when $p = 0.65$. When non local state independent cloning machines are used, the ranges obtained are 0.188< α^2 <0.811, 0.07< α^2 <0.92 and 0.03< α^2 <0.96 for $p = 0.65, 0.80, 0.95$ respectively. It is interesting to note that for any value of p, the broadcasting range, in terms of allowed values of α^2 for all static state dependent local cloners is better than state independent local cloner.

5.3.3 Local and non local dynamic cloning

Lastly, we use dynamic state dependent cloner (DSDLC, DSDNLC for local and non local respectively), where the value of optimal machine parameter is automatically selected by the machine. This is based on the input state according to equation(5.21) for the local case and equation (5.23) for the non local case. In table 5.6, we take different values of α^2 and compare the range of values of p where broadcasting is possible in both cases of state independent and dynamic state dependent cloners. In-

k	λ_{SSDLC}	SSDLC	λ_{SSDNLC}	SSDNLC
0.1	0.1115	0.183< α^2 <0.816	0.0744	0.098< α^2 <0.90
0.2	0.1344	0.32< α^2 <0.67	0.0968	0.172< α^2 <0.827
0.3	0.1519	False	0.1125	0.281< α^2 <0.718
0.4	0.1629	False	0.1218	False
0.5	0.1666	False	0.1249	False
0.6	0.1629	False	0.1218	False
0.7	0.1519	False	0.1125	0.281< α^2 <0.718
0.8	0.1344	0.32< α^2 <0.67	0.0968	0.172< α^2 <0.827
0.9	0.1115	0.183< α^2 <0.816	0.0744	0.098< α^2 <0.90

Table 5.3 Comparison of broadcasting ranges for various static state-dependent local and non local cloner when $p = 0.65$.

k	λ_{SSDLC}	SSDLC	λ_{SSDNLC}	SSDNLC
0.1	0.0865	0.044< α^2 <0.955	0.0633	0.029< α^2 <0.97
0.2	0.1191	0.09< α^2 <0.908	0.0920	0.064< α^2 <0.93
0.3	0.1446	0.163< α^2 <0.837	0.1122	0.113< α^2 <0.886
0.4	0.1610	0.244< α^2 <0.755	0.1242	0.165< α^2 <0.83
0.5	0.1666	0.288< α^2 <0.711	0.1282	0.189< α^2 <0.81
0.6	0.1610	0.244< α^2 <0.755	0.1242	0.165< α^2 <0.83
0.7	0.1446	0.163< α^2 <0.837	0.1122	0.113< α^2 <0.886
0.8	0.1191	0.09< α^2 <0.908	0.0920	0.064< α^2 <0.93
0.9	0.0865	0.044< α^2 <0.955	0.0633	0.029< α^2 <0.97

Table 5.4 Comparison of broadcasting ranges for various static state-dependent local and non local cloners when p=0.80.

terestingly, we find that dynamic state dependent local cloner gives a larger range of values of p where broadcasting is possible than state independent local cloner, for all values of α^2. However, in case of non local cloning, the scenario is different. For $\alpha^2 = 0.1, 0.2, 0.8, 0.9$, dynamic state dependent cloners perform better than state independent cloners, whereas for other values of α^2, the findings are the other way round.

In a nutshell, we find that in most of the cases, state dependent cloners perform better at least in terms of the broadcasting range, compared to the state independent cloners when we take Werner like state(WLS), as our initial resource entangled state.

Figure (5.5) shows the broadcastable region when various types of cloners are used. The brown region shows the area where original input state ρ_{12}^{w} (Werner like state) is inseparable. All broadcastable zones obtained as a result of different cloning transformations must be a subset of this. This is because the broadcasting of entanglement only makes sense, in the region where original input state is entangled.

k	λ_{SSDLC}	SSDLC	λ_{SSDNLC}	SSDNLC
0.1	0.0590	0.007< α^2 <0.992	0.0518	0.006< α^2 <0.994
0.2	0.1015	0.027< α^2 <0.972	0.0864	0.023< α^2 <0.97
0.3	0.1360	0.065< α^2 <0.934	0.1106	0.050< α^2 <0.94
0.4	0.1587	0.111< α^2 <0.888	0.1250	0.083< α^2 <0.916
0.5	0.6666	0.135< α^2 <0.864	0.1297	0.097< α^2 <0.090
0.6	0.1587	0.111< α^2 <0.888	0.1250	0.083< α^2 <0.916
0.7	0.1360	0.065< α^2 <0.934	0.1106	0.050< α^2 <0.94
0.8	0.1015	0.027< α^2 <0.972	0.0864	0.023< α^2 <0.97
0.9	0.0590	0.007< α^2 <0.992	0.0518	0.006< α^2 <0.994

Table 5.5 Comparison of broadcasting ranges for various static state-dependent local and non local cloners when $p = 0.95$.

α^2	SILC	DSDLC	SINLC	DSDNLC
0.1	False	0.71< p ≤1	0.75< p ≤1	0.64< p ≤1
0.2	0.865< p ≤1	0.70< p ≤1	0.64< p ≤1	0.62< p ≤1
0.3	0.794< p ≤1	0.71< p ≤1	0.58< p ≤1	0.64< p ≤1
0.4	0.760< p ≤1	0.73< p ≤1	0.56< p ≤1	0.66< p ≤1
0.5	0.75< p ≤1	0.75< p ≤1	0.55< p ≤1	0.67< p ≤1
0.6	0.760< p ≤1	0.73< p ≤1	0.56< p ≤1	0.66< p ≤1
0.7	0.794< p ≤1	0.71< p ≤1	0.58< p ≤1	0.64< p ≤1
0.8	0.865< p ≤1	0.70< p ≤1	0.64< p ≤1	0.62< p ≤1
0.9	False	0.71< p ≤1	0.75< p ≤1	0.64< p ≤1

Table 5.6 Range of values of p where broadcasting of entanglement is possible for various values of α^2, when state independent and dynamic state dependent cloners are used locally and non locally.

the purple and the black semi-circular regions are the broadcastable zones obtained when local and non local state independent cloners are used. The yellow and blue region in between show the broadcastable regions when dynamic state dependent cloners are used locally (DSDLC) and non locally (DSDNLC), respectively.

5.4 Broadcasting Bell Diagonal state

As our last example, we consider the broadcasting of Bell diagonal state, which is given by $\rho_{12}^b = p_1 |\psi_+\rangle \langle\psi_+| + p_2 |\psi_-\rangle \langle\psi_-| + p_3 |\phi_+\rangle \langle\phi_+| + p_4 |\phi_-\rangle \langle\phi_-|$, where p_1, p_2, p_3 and p_4 are classical mixing parameters. Here, $|\psi_\pm\rangle, |\phi_\pm\rangle$ are Bell states. In terms of Bloch vectors and correlation matrix, Bell diagonal states can be expressed as,

$$\rho_{12} = \{\vec{0}, \vec{0}, T^b\}, \tag{5.24}$$

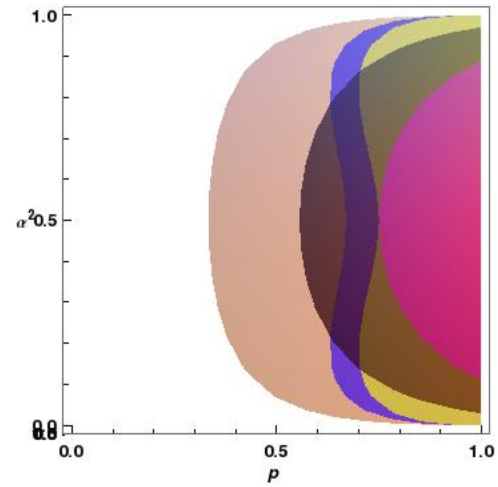

Figure 5.5 The figure shows the broadcastable zone for Werner like state (WLS) when various cloners are used. Brown $\rightarrow$ region where ρ_{12}^{w} is inseparable, purple $\rightarrow$ state independent local cloning, yellow $\rightarrow$ dynamic state dependent local cloning, blue $\rightarrow$ dynamic state dependent non local cloning and black $\rightarrow$ state independent non local cloning.

where $\vec{0}$ is the null matrix and T^b = diag[c_1, c_2, c_3] $(-1 \leq c_i \leq 1)$ is the correlation matrix.

5.4.1 Static state dependent and state independent local cloning

The local and non local output states obtained as a result of state dependent local cloning are given by,

$$\begin{aligned} \rho_{13}^{out} = \rho_{24}^{out} = \{\vec{0}, \vec{0}, T_l^{sd}\}, \\ \rho_{14}^{out} = \rho_{23}^{out} = \{\vec{0}, \vec{0}, \mu^2 T^b\}. \end{aligned} \tag{5.25}$$

It is interesting to note that for local cloning, the optimal value of λ is $\frac{1}{6}$, whether we use state dependent or state independent cloning transformation. We present this result in the form of the following theorem.

Theorem 5.4.1. *In local cloning on bell diagonal state using state dependent cloner, there is only one possible value of optimal machine parameter, which is equal to the machine parameter for state independent case.*

Proof. The local output state obtained by local state dependent cloning is given by,

$$\rho_{13}(\rho_{24}) = \begin{bmatrix} \frac{1}{2}(1-2\lambda) & 0 & 0 & 0 \\ 0 & \lambda & \lambda & 0 \\ 0 & \lambda & \lambda & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\lambda) \end{bmatrix}$$

. The distortion of the combined output state is a function of machine parameter λ only, where in most of the cases, it is a function of machine parameter and input parameter. This distortion of the combined state is given by,

$$D_{13} = \frac{1}{4} - 2\lambda + 6\lambda^2. \tag{5.26}$$

When we take the first derivative of D_{13} with respect to λ, we obtain $\lambda = \frac{1}{6}$. The second derivative test shows that this is the value of λ corresponding to minimum value of D_{13} for both state independent and state dependent cloner. □

In table (5.7), we show the broadcasting range for the state independent and state dependent local cloners, in terms of the parameter c_3, for chosen values of the parameters c_1, c_2. These ranges happen to be the same for both the cloners, as their optimal value of the machine parameter is equal.

c_1	c_2	SILC	SDLC
$-\frac{7}{8}$	$-\frac{7}{8}$	-1$\leq c_3 < -\frac{3}{4}$	-1$\leq c_3 < -\frac{3}{4}$
$-\frac{3}{4}$	$-\frac{3}{4}$	-1$\leq c_3 < -\frac{3}{4}$	-1$\leq c_3 < -\frac{3}{4}$
$-\frac{7}{8}$	$-\frac{3}{4}$	-$\frac{7}{8} \leq c_3 < -\frac{5}{8}$	-$\frac{7}{8} \leq c_3 < -\frac{5}{8}$
$-\frac{3}{4}$	$-\frac{7}{8}$	-$\frac{7}{8} \leq c_3 < -\frac{5}{8}$	-$\frac{7}{8} \leq c_3 < -\frac{5}{8}$

Table 5.7 Comparative analysis of broadcasting ranges of state dependent local (SDLC) and state independent local cloners (SILC) in terms of c_3 for fixed values of c_1 and c_2.

In the figure (5.6), we show the broadcastable regions for all types of cloners used in the local case. The inside octahedron represents the zone where there is no entanglement in original bell diagonal state. It is needless to mention that the broadcastable regions would be a subset of the complement part. The four pyramids obtained at the edge of the tetrahedron are the regions where the broadcasting is possible.

5.4.2 Static state dependent and state independent non local cloning

Next, we use state dependent non-local cloner for broadcasting. The local and the non local output states are given by,

$$\begin{aligned} \rho_{13}^{out} = \rho_{24}^{out} &= \left\{\vec{0}, \vec{0}, T_{nl}^{sd}\right\}, \\ \rho_{12}^{out} = \rho_{34}^{out} &= \left\{\vec{0}, \vec{0}, \mu T^{b}\right\}. \end{aligned} \tag{5.27}$$

It is interesting to note that the local output states are not dependent on input state parameters c_1, c_2, c_3. To calculate the expression for optimal machine parameter in case of state dependent cloner, we calculate $\frac{\partial D_{1234}}{\partial \lambda} = 0$ and obtain optimal λ as a function of input parameters,

$$\lambda_{optimal} = \frac{21{c_1}^2 + 6c_1c_2c_3 + 21{c_2}^2 + 4{c_3}^2 + 12}{8\left(12{c_1}^2 + 12{c_2}^2 + 11{c_3}^2 + 15\right)}. \tag{5.28}$$

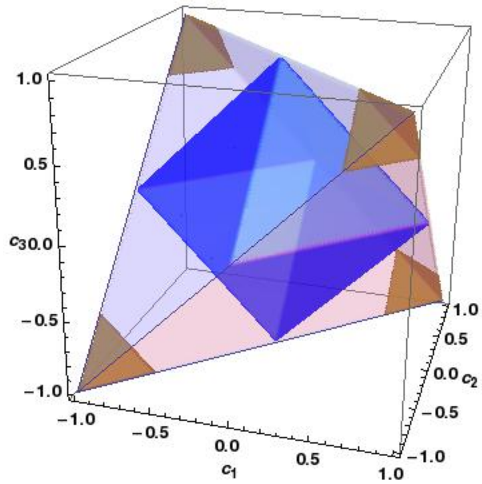

Figure 5.6 The figure shows the broadcastable region for the bell diagonal state (BDS), when various local cloners are used. Blue $\rightarrow$ separable input state, brown $\rightarrow$ broadcastable zone for all cloners and pink $\rightarrow$ inseparable input state.

In table (5.8), we choose values of input parameters c_1, c_2 and c_3 uniformly, to fix a value for optimal machine parameter, so that we can use the machine corresponding to the machine parameter obtained, at the time of broadcasting. We use the static state dependent cloning machines to find the broadcasting range in terms of the parameter c_3, for chosen values of c_1 and c_2. These ranges are shown in table (5.9). It is interesting to note that in table (5.9), as the values of the optimal machine parameter increases, the broadcasting range decreases and vice-versa.

c_1	c_2	c_3	$\lambda_{SSDNLCl}$
$-\frac{7}{9}$	$-\frac{7}{9}$	$-\frac{7}{9}$	0.1278
$-\frac{7}{9}$	$-\frac{7}{9}$	$-\frac{5}{9}$	0.1391
$-\frac{7}{9}$	$-\frac{5}{9}$	$-\frac{7}{9}$	0.1210
$-\frac{7}{9}$	$-\frac{5}{9}$	$-\frac{5}{9}$	0.1319
$-\frac{5}{9}$	$-\frac{7}{9}$	$-\frac{7}{9}$	0.1210
$-\frac{5}{9}$	$-\frac{7}{9}$	$-\frac{5}{9}$	0.1319
$-\frac{5}{9}$	$-\frac{5}{9}$	$-\frac{7}{9}$	0.1116
$-\frac{5}{9}$	$-\frac{5}{9}$	$-\frac{5}{9}$	0.1219

Table 5.8 Values of optimal machine parameters for various values of input parameters.

c_1	c_2	λ_{SSDNLC}	Broadcasting Range
$-\frac{7}{9}$	$-\frac{7}{9}$	0.1278	-1$\leq c_3 < -\frac{5}{9}$
$-\frac{7}{9}$	$-\frac{7}{9}$	0.1391	$-1. \leq c_3 < -0.698728$
$-\frac{7}{9}$	$-\frac{5}{9}$	0.1210	-$\frac{7}{9} \leq c_3 < -0.604651$
$-\frac{7}{9}$	$-\frac{5}{9}$	0.1319	False
$-\frac{5}{9}$	$-\frac{7}{9}$	0.1210	-$\frac{7}{9} \leq c_3 < -0.604651$
$-\frac{5}{9}$	$-\frac{7}{9}$	0.1319	False
$-\frac{5}{9}$	$-\frac{5}{9}$	0.1116	$-1. \leq c_3 < -0.695247$
$-\frac{5}{9}$	$-\frac{5}{9}$	0.1219	$-1. \leq c_3 < -0.840489$

Table 5.9 Broadcasting range in terms of c_3 when values of c_1, c_2 are fixed for various static state dependent non local cloners.

c_1	c_2	SINL	DSDNLC
$-\frac{7}{9}$	$-\frac{7}{9}$	-1$\leq c_3 < -\frac{5}{9}$	-1$\leq c_3 < -0.62809$
$-\frac{5}{9}$	$-\frac{5}{9}$	-1$\leq c_3 < -\frac{5}{9}$	-1$\leq c_3 < -0.72682$
$-\frac{7}{9}$	$-\frac{5}{9}$	-$\frac{7}{9} \leq c_3 < -\frac{1}{3}$	$-0.77777 \leq c_3 < -0.68061$
$-\frac{5}{9}$	$-\frac{7}{9}$	-$\frac{7}{9} \leq c_3 < -\frac{1}{3}$	$-0.77777 \leq c_3 < -0.68061$
$-\frac{2}{5}$	$-\frac{2}{5}$	$-1. \leq$ c3 < -0.86	$-1. \leq$ c3 < -0.839078
$-\frac{1}{3}$	$-\frac{1}{3}$	False	$-1. \leq$ c3 < -0.902749

Table 5.10 Comparison between broadcasting ranges when dynamic state dependent and state independent non local cloners are used.

5.4.3 Local and non local dynamic state dependent cloning

Lastly, we use the dynamic state dependent cloner(DSDNLC) where, when we supply the value of c_1 and c_2, and the optimal machine parameter as a function of c_3, is substituted in the output density matrices, which can be used to calculate the broadcasting ranges. In the Table 5.10 we present a comparative analysis of dynamic state-dependent and state independent cloning. We can see that the broadcasting range of dynamic state dependent cloner is less than the state independent one for most cases. But we also notice that for $c_1 = -2/5$ and $c_2 = -2/5$, the range is greater for dynamic state dependent cloner and for $c_1 = -1/3$ and $c_2 = -1/3$, broadcasting is not possible using state independent cloner. However, it is possible using dynamic state dependent cloner. In figure (5.7) we plot the broadcastable zones when state independent and dynamic state dependent non local cloners are used. The blue octahedron at the center represents the region where the input bell diagonal state is separable. The brown pyramid at the corner represents the broadcastable zone when state independent non local cloner is used. The black region at the corners shows the broadcastable zone when dynamic state dependent cloner is used. It is evident that none of these regions completely overlap the other, indicating that for some values, dynamic state dependent is better and for some other values, state independent one is better.

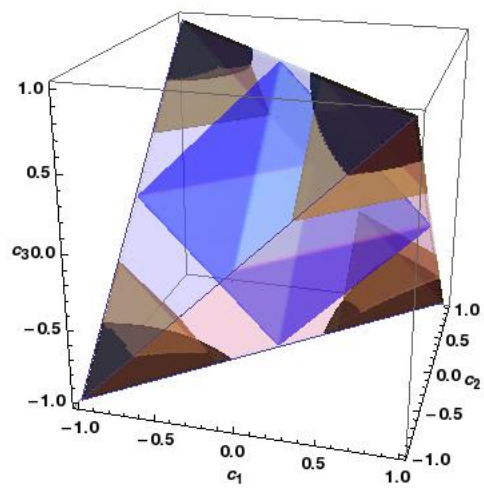

Figure 5.7 The figure shows the broadcastable region for the bell diagonal state (BDS), when various non local cloners are used. Blue → separable input state, pink → inseparable input state, brown → broadcastable zone with state independent non local cloning and black → broadcastable zone when dynamic state dependent non local cloner is used.

Chapter 6

Broadcasting of coherence

In an ongoing work, we have shown that optimal broadcasting of quantum coherence is not possible using any kind of cloning transformations. The measure of coherence we have used to show this result is l_1 norm because it is mathematically convenient to compute. The results are briefly discussed here.

6.1 Impossibility of Optimal Broadcasting of Coherence

Let us begin with a situation where we have two distant parties A and B and they share a two qubit mixed state ρ_{12} which can be canonically expressed as,

$$\rho_{12} = \frac{1}{4}\left[I_{2\times 2}\otimes I_{2\times 2} + \sum_{i=1}^{3}(x_i\sigma_i\otimes I) + \sum_{i=1}^{3} (y_i I\otimes\sigma_i) + \sum_{i,j=1}^{3}(t_{ij}\sigma_i\otimes\sigma_j)\right] = \left\{\overrightarrow{X},\ \overrightarrow{Y},\ T_{3\times 3}\right\}, \tag{6.1}$$

where $\overrightarrow{X} = \{x_1,\ x_2,\ x_3\}$ and $\overrightarrow{Y} = \{y_1,\ y_2,\ y_3\}$ are Bloch vectors with $0 \leq \|\overrightarrow{x}\| \leq 1$ and $0 \leq \|\overrightarrow{y}\| \leq 1$. Here, t_{ij}'s ($i,\ j$ = $\{1,2,3\}$) are elements of the correlation matrix ($T = [t_{ij}]_{3\times 3}$), whereas $\sigma_i = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices and I is the identity matrix.

We start with two qubit state ρ_{12} and then apply cloning operations (Local/Nonlocal State Dependent (LSD)(U^{l}_{bhsd})/NLSD (U^{nl}_{bhsd})) Local/Non local State Independent (LSI(U^{l}_{bhsi})/NLSI(U^{nl}_{bhsi})), to produce a composite system $\tilde{\rho}_{1234}$. In short, broadcasting will be possible if we are able to produce more coherent pairs out of initial coherent pair. In principle, in order to broadcast the amount of coherence between the desired pairs $(1,4)/(1,2)$ and $(2,3)/(3,4)$, we need to maximize the amount of coherence between the non-local output pairs, regardless of the states between $(1,3)$ and $(2,4)$. However, for optimal broadcasting of coherence across parties, we need to minimize the amount of coherence within parties, i.e, in the states $(1,3)$ and $(2,4)$.This is because the total amount of coherence (C) produced after cloning operation is equal to the sum of the coherence within parties (C_l) and the coherence across the

parties (C_{nl}), i.e $C = C_l + C_{nl}$. To maximize C_{nl}, we must have $C_l = 0$. In other words, for optimal broadcasting we should have no coherence between the qubits $(1, 3)$ and $(2, 4)$.
Definition: Optimal Broadcasting (Local/Nonlocal) Let A and B share a coherent input state ρ_{12}, with qubits 1 and 2 being with A and B respectively. A coherent input state ρ_{12} is said to be broadcast optimally after the application of local cloning operation $U_1 \otimes U_2$, on the qubits 1 and 2 respectively, if for some values of the input state parameters, the non-local output states, given by

$$\tilde{\rho}_{14} = Tr_{23}\left[U_1 \otimes U_2\left(\rho_{12}\right)\right],$$
$$\tilde{\rho}_{23} = Tr_{14}\left[U_1 \otimes U_2\left(\rho_{12}\right)\right], \tag{6.2}$$

of A and B are coherent i.e $C(\tilde{\rho}_{14}) \neq 0, C(\tilde{\rho}_{23}) \neq 0$, while the local output states, given by

$$\tilde{\rho}_{13} = Tr_{24}\left[U_1 \otimes U_2\left(\rho_{12}\right)\right],$$
$$\tilde{\rho}_{24} = Tr_{13}\left[U_1 \otimes U_2\left(\rho_{12}\right)\right], \tag{6.3}$$

of A and B are incoherent i.e $C(\tilde{\rho}_{13}) = 0, C(\tilde{\rho}_{24}) = 0$.
Similarly, a coherent input state ρ_{12} is said to be broadcast optimally after the application of non local cloning operation U_{12} together on qubits 1 and 2, if for some values of the input state parameters, the non-local output states, given by

$$\tilde{\rho}_{12} = Tr_{34}\left[U_{12}\left(\rho_{12}\right)\right],$$
$$\tilde{\rho}_{34} = Tr_{12}\left[U_{12}\left(\rho_{12}\right)\right], \tag{6.4}$$

of A and B are coherent i.e $C(\tilde{\rho}_{12}) \neq 0, C(\tilde{\rho}_{34}) \neq 0$, while the local output states, given by

$$\tilde{\rho}_{13} = Tr_{24}\left[U_{12}\left(\rho_{12}\right)\right],$$
$$\tilde{\rho}_{24} = Tr_{13}\left[U_{12}\left(\rho_{12}\right)\right], \tag{6.5}$$

of A and B are incoherent i.e $C(\tilde{\rho}_{13}) = 0, C(\tilde{\rho}_{24}) = 0$.
Here, in this work, we show that it is impossible to broadcast coherence optimally in both the cases of local and non local cloning. Without any loss of generality, we show this for the computational basis $\{|0\rangle, |1\rangle\}$ starting with the most general mixed state (6.1) as a resource with the help of the following theorems.

Theorem 6.1.1. *Given a two qubit general mixed state ρ_{12} (given by Eq. (6.1)) and B-H local cloning transformations (state independent optimal U^l_{bhsi} or state dependent U^l_{bhsd}), it is impossible to optimally broadcast the coherence within ρ_{12} into two lesser coherent states: [(a)] $\tilde{\rho}_{14} = Tr_{23}\left[U_1 \otimes U_2\left(\rho_{12}\right)\right]$, [(b)] $\tilde{\rho}_{23} = Tr_{14}\left[U_1 \otimes U_2\left(\rho_{12}\right)\right]$ (Here we have $U_1(U_2) \equiv U^l_{bhsi}(U^l_{bhsd})$) in the computational basis $\{|0\rangle, |1\rangle\}$.*

Proof. Here, we consider the input state as the most general two qubit mixed state ρ_{12} (given by Eq. (6.1)). The B-H state dependent cloning transformation U^{l}_{bhsd} is applied locally to clone qubits ($1 \rightarrow 3$ and $2 \rightarrow 4$) of ρ_{12}. We have local output state as, $\tilde{\rho}_{13} = \{\mu x, \mu x, T^{sd}_{l}\}, \tilde{\rho}_{24} = \{\mu y, \mu y, T^{sd}_{l}\}$, where $T^{sd}_{l} = diag(2\lambda, 2\lambda, 1-4\lambda)$ and the non local output states $\tilde{\rho}_{14} = \tilde{\rho}_{23} = \{\mu x, \mu y, \mu^2 T\}$. Here, $\mu = 1 - 2\lambda$ is the machine parameter of the cloning machine determining the optimality of the transformation. Also, x, y represent the Bloch vector and T represents the correlation matrix of the input state. The coherence calculated by using l_1 norm of the local output states in the computational basis $\{|0\rangle, |1\rangle\}$ are given by, $C(\tilde{\rho}_{13}) = 2\sqrt{x_1^2 + x_2^2} + 2\lambda(1 - 2\sqrt{x_1^2 + x_2^2}) > 0$ on A's side and $C(\tilde{\rho}_{24}) = 2\sqrt{y_1^2 + y_2^2} + 2\lambda(1 - 2\sqrt{y_1^2 + y_2^2}) > 0$ on B's side. Each of these quantities are positive as the norm $||x||, ||y||$ are positive quantities lying between 0 and 1. This is true for all values of λ lying between $[0, 1/2]$.

Now, for $\lambda = 1/6$, the above B-H state dependent quantum cloning machine U^{l}_{bhsd} reduces to state independent B-H optimal local cloner U^{l}_{bhsi}. Subsequently the coherence of local output states changes to $C(\tilde{\rho}_{13}) = 1/3 + (4/3)\sqrt{x_1^2 + x_2^2}) > 0$ and $C(\tilde{\rho}_{24}) = 1/3 + (4/3)\sqrt{y_1^2 + y_2^2}) > 0$. □

Theorem 6.1.2. *Given a two qubit general mixed state ρ_{12} (given by Eq. (6.1)) and B-H non-local cloning transformations (state independent optimal U^{nl}_{bhsi} or state dependent U^{nl}_{bhsd}), it is impossible to optimally broadcast the coherence within ρ_{12} into two lesser coherent states: [(a)] $\tilde{\rho}_{12} = Tr_{34}\left[U_{12}\left(\rho_{12}\right)\right]$, [(b)] $\tilde{\rho}_{34} = Tr_{12}\left[U_{12}\left(\rho_{12}\right)\right]$. (Here we have $U_{12} \equiv U^{nl}_{bhsi}(U^{nl}_{bhsd})$) in the computational basis $\{|0\rangle, |1\rangle\}$.*

Proof. Quite similar to the previous proof, here also we start with the most general version of the two qubit mixed state ρ_{12} (given by Eq. (6.1)). When B-H state dependent cloning transformation U^{nl}_{bhsd} is applied non locally to clone qubits ($1 \rightarrow 3$ and $2 \rightarrow 4$) of ρ_{12}, then, we have local output states as, $\tilde{\rho}_{13} = \{\mu x, \mu x, T^{sd}_{nl}\}, \tilde{\rho}_{24} = \{\mu y, \mu y, T^{sd}_{nl}\}$, where $T^{sd}_{l} = diag(2\lambda, 2\lambda, 1-8\lambda)$ and the non local output states $\tilde{\rho}_{12} = \tilde{\rho}_{34} = \{\mu x, \mu y, \mu T\}$. Here, $\mu = 1 - 4\lambda$ represents the machine parameter, x, y represent the Bloch vector and T represents the correlation matrix of the input state. The coherence is calculated by using l_1 norm measure in the computational basis $\{|0\rangle, |1\rangle\}$. Hence, for the local output states, we have, $C(\tilde{\rho}_{13}) = 2\sqrt{x_1^2 + x_2^2} + 2\lambda(1 - 4\sqrt{x_1^2 + x_2^2}) > 0$ on A's side and $C(\tilde{\rho}_{24}) = 2\sqrt{y_1^2 + y_2^2} + 2\lambda(1 - 4\sqrt{y_1^2 + y_2^2}) > 0$ on B's side. Each of these quantities are positive as the norm $||x||, ||y||$ are positive quantities lying between 0 and 1. This is true for all values of λ lying between $[0, 1/4]$.

For $\lambda = 1/10$, the above B-H state dependent quantum cloning machine U^{nl}_{bhsd} reduces to state independent B-H optimal non-local cloner U^{nl}_{bhsi}. Subsequently the coherence of local output states changes to, $C(\tilde{\rho}_{13}) = 1/5 + (6/5)\sqrt{x_1^2 + x_2^2}) > 0$ and $C(\tilde{\rho}_{24}) = 1/5 + (6/5)\sqrt{y_1^2 + y_2^2}) > 0$.

□

6.2 Non Optimal Broadcasting of Coherence

At this point it is interesting to note that though it is impossible to broadcast optimally QCh between the desired qubits. However, it does not rule out the entire possibility of broadcasting. If we relax the condition $C(\tilde{\rho}_{13}) = 0, C(\tilde{\rho}_{24}) = 0$, then we show that subsequently it will be possible to broadcast QCh. We show this for the most general two qubit mixed state as a resource.

Definition : Non Optimal Broadcasting (Local/Non local) A coherent state ρ_{12} is said to be broadcast non optimally after the application of local cloning operation $U_1 \otimes U_2$, on the qubits 1 and 2 respectively ((or by non local cloning operation U_{12} together on qubits 1 and 2)), if for some values of the input state parameters, the coherence of non-local output states are greater than coherence of local output states i.e $C(\tilde{\rho}_{14}) \geq \{C(\tilde{\rho}_{13}, C(\tilde{\rho}_{24}))\}$ and $C(\tilde{\rho}_{23}) \geq \{C(\tilde{\rho}_{13}, C(\tilde{\rho}_{24}))\}$.

Under this relaxation, the quantum coherence of the cloned state $\tilde{\rho}_{14}, \tilde{\rho}_{23}$ can increase, remains same and also can decrease from the initial resource state ρ_{12} i.e

- $C\left(\tilde{\rho}_{12}\right), C\left(\tilde{\rho}_{34}\right) < C\left(\rho_{12}\right)$
- $C\left(\tilde{\rho}_{12}\right), C\left(\tilde{\rho}_{34}\right) = C\left(\rho_{12}\right)$
- $C\left(\tilde{\rho}_{12}\right), C\left(\tilde{\rho}_{34}\right) > C\left(\rho_{12}\right)$

Interestingly, in this work we show that it is indeed impossible to increase the quantum coherence by local and non local cloning operation though in general it is possible to increase the quantum coherence by local operations. We present this result in the form of the following two theorems, where we have considered local and non local cloning transformations both in state dependent and state independent framework.

Theorem 6.2.1. *Given a two qubit general mixed state ρ_{12} (given by Eq. (6.1)) and B-H local cloning transformations (state independent optimal U^l_{bhsi} or state dependent U^l_{bhsd}), we can always create two* *states $\tilde{\rho}_{14}$ and $\tilde{\rho}_{32}$, such that : (a) $C(\tilde{\rho}_{14}) < C(\rho_{12})$ (b) $C(\tilde{\rho}_{32}) < C(\rho_{12})$ in the computational basis* $\{|0\rangle, |1\rangle\}$.

Proof. We start with the most general representation of two qubit mixed state ρ_{12} (given by Eq. (6.1)). When Buzek Hillary state dependent cloning transformation U^l_{bhsd} is applied locally to ρ_{12}, we get the non local output states as $\tilde{\rho}_{14} = \tilde{\rho}_{23} = \{\mu x, \mu y, \mu^2 T\}$. Here, $\mu = 1 - 2\lambda$ is the machine parameter, x, y represent the Bloch vector and T represents the correlation matrix of the input state. Using l_1-norm as the measure of coherence we find out the coherence in the computational basis $\{|0\rangle, |1\rangle\}$,

$$\begin{aligned}
C(\rho_{12}) \quad = \quad & 1/2[\sqrt{(T_{13}+T_{21})^2+(T_{11}-T_{22})^2} \\
& +\sqrt{(T_{12}-T_{21})^2+(T_{11}+T_{22})^2}] \\
& +1/2[\sqrt{(T_{13}-x_1)^2+(T_{23}-x_2)^2} \\
& +\sqrt{(T_{13}+x_1)^2+(T_{23}+x_2)^2}] \\
& +1/2[\sqrt{(T_{31}-y_1)^2+(T_{32}-y_2)^2} \\
& +\sqrt{(T_{31}+y_1)^2+(T_{32}+y_2)^2}] \\
& = 1/2[a_1+a_2+a_3]
\end{aligned}$$

and

$$\begin{aligned}
C(\tilde{\rho}_{14}(\tilde{\rho}_{23})) \quad = \quad & 1/2[\mu^2\sqrt{(T_{13}+T_{21})^2+(T_{11}-T_{22})^2} \\
& +\mu^2\sqrt{(T_{12}-T_{21})^2+(T_{11}+T_{22})^2}] \\
& +1/2[\mu\sqrt{(\mu T_{13}-x_1)^2+(\mu T_{23}-x_2)^2} \\
& +\mu\sqrt{(\mu T_{13}+x_1)^2+(\mu T_{23}+x_2)^2}] \\
& +1/2[\mu\sqrt{(\mu T_{31}-y_1)^2+(\mu T_{32}-y_2)^2} \\
& +\mu\sqrt{(\mu T_{31}+y_1)^2+(\mu T_{32}+y_2)^2}] \\
& = 1/2[b_1+b_2+b_3].
\end{aligned}$$

Here,

$$
\begin{aligned}
a_1 &= \sqrt{(T_{13}+T_{21})^2+(T_{11}-T_{22})^2} \\
&\quad +\sqrt{(T_{12}-T_{21})^2+(T_{11}+T_{22})^2} \\
a_2 &= \sqrt{(T_{13}-x_1)^2+(T_{23}-x_2)^2} \\
&\quad +\sqrt{(T_{13}+x_1)^2+(T_{23}+x_2)^2} \\
a_3 &= \sqrt{(T_{31}-y_1)^2+(T_{32}-y_2)^2} \\
&\quad +\sqrt{(T_{31}+y_1)^2+(T_{32}+y_2)^2} \\
b_1 &= \mu^2\sqrt{(T_{13}+T_{21})^2+(T_{11}-T_{22})^2} \\
&\quad +\mu^2\sqrt{(T_{12}-T_{21})^2+(T_{11}+T_{22})^2} \\
b_2 &= \mu\sqrt{(\mu T_{13}-x_1)^2+(\mu T_{23}-x_2)^2} \\
&\quad +\mu\sqrt{(\mu T_{13}+x_1)^2+(\mu T_{23}+x_2)^2} \\
b_3 &= \mu\sqrt{(\mu T_{31}-y_1)^2+(\mu T_{32}-y_2)^2} \\
&\quad +\mu\sqrt{(\mu T_{31}+y_1)^2+(\mu T_{32}+y_2)^2}.
\end{aligned}
\tag{6.6}
$$

If we can show that $a_1 > b_1$, $a_2 > b_2$ and $a_3 > b_3$ hold at the same time, then we can always conclude that $C(\tilde{\rho}_{12}) > C(\tilde{\rho}_{14}(\tilde{\rho}_{23}))$.

At first, if we observe the terms a_1 and b_1 carefully, we find that $b_1 = \mu^2 a_1 < a_1$ because $\mu = 1-2\lambda < 1$. Now, we need to compare a_2 and b_2.If we observe the terms in a_2 and b_2, and ignoring the constant $1/2$, they can be thought of as distance between two points in a Cartesian Plane. Let us denote the distance between two points A and B in a 2-dimensional space (like XY Cartesian Plane) by $\|AB\|$.Let us consider $\tilde{b_2} = \sqrt{(\mu T_{13}-x_1)^2+(\mu T_{23}-x_2)^2}+\sqrt{(\mu T_{13}+x_1)^2+(\mu T_{23}+x_2)^2}$ and as $\mu < 1$, so, we have, $b_2 < \tilde{b_2}$. Now, assume 4 points in Cartesian Plane, namely, $A(T_{13}, T_{23})$, $B(x_1, x_2)$,$C(-x_1, -x_2)$ and $D(\mu T_{13}, \mu T_{23})$ without O as the origin. Now, as $\mu < 1$, hence, it is always true that $\|OD\| = \sqrt{(\mu T_{13})^2+(\mu T_{23})^2} = \mu\sqrt{(T_{13})^2+(T_{23})^2} = \mu\|OA\|$; that is, $\|OD\| < \|OA\|$. So, D always lies on $\overline{OA}$, inside ΔABC. Now, from the Figure 6.2, we have, $\tilde{b_2} = \|DC\|+\|DB\|$ and $a_2 = \|AC\|+\|AB\|$. Now, we can prove $\tilde{b_2} < a_2$ by plane geometry.

□

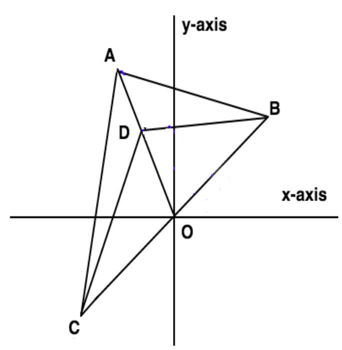

Figure 6.1 ΔABC in XY-Plane, with point D in its interior

Chapter 7

Conclusion and Future Work

In a nutshell, in this work, we re-conceptualize the notion of state dependence in quantum cloning. Based on that, we introduce new state dependent cloning machine which, when used locally, outperforms the state independent cloning machines and when used non locally, performs better for some values of input parameters. Further, we have defined dynamic state dependent cloning which adjusts the optimal machine parameter according to the input state. It is shown to be better than state independent cloning in the context of broadcasting of entanglement by taking various states like non maximally entangled (NME), Werner like (WLS), Bell diagonal (BDS) as examples. This new type of state dependent cloning machine opens new possibilities for generating more number of input copies from a given input state. In the future, it would be interesting to study broadcasting using asymmetric state dependent cloner[31], both locally and non locally. Also, it would be of interest to study broadcasting of bell violations, coherence and other correlations using these state dependent cloners. We have also talked about GGM in chapter 3 as a measure of genuine multiparty entanglement. It would also be interesting to study the broadcasting of genuine multiparty entanglement using higher dimensional cloning transformations.

In Chapter 6, we have shown that optimal broadcasting of coherence is not possible using cloning transformations. In future, it would also be interesting to study how coherence can be broadcast in non optimal fashion, if we use state dependent cloners. Not all entangled states violate bell violations so if we are able to broadcast entanglement, it does not mean we can also broadcast bell violations. The states which would violate bell violations would make a better resource for information processing task. So, a study of broadcasting of bell violations using cloning operations has a potential to produce some very important results.

Related Publications

1. State Dependent and Independent cloner: Cloning and Broadcasting, MK Shukla, I Chakrabarty, S Chatterjee, arXiv:1511.05796 (2015).

2. Broadcasting Quantum Coherence via Cloning, I Chakrabarty, UK Sharma, MK Shukla, arXiv:1605.02458 (2016).

Other publications

Implementation of Kirchhoff-Helmholtz transform on GPU for use in digital in-line holographic microscopy, G Singh, M Shukla, P Bhimalapuram, K. Kothapalli, ACM Proceedings of the 7th ACM India Computing Conference, (2014).

Bibliography

[1] S. Adhikari, I. Chakrabarty, and P. Agrawal. Probabilistic secret sharing through noisy quantum channels. *arXiv preprint arXiv:1012.5570*, 2010.

[2] S. Adhikari, B. Choudhury, and I. Chakrabarty. Broadcasting of inseparability. *Journal of Physics A: Mathematical and General*, 39(26):8439, 2006.

[3] S. Bandyopadhyay and G. Kar. Broadcasting of entanglement and universal quantum cloners. *Physical Review A*, 60(4):3296, 1999.

[4] B. Bellomo, R. L. Franco, and G. Compagno. Entanglement dynamics of two independent qubits in environments with and without memory. *Physical Review A*, 77(3):032342, 2008.

[5] C. H. Bennett. Quantum cryptography: Public key distribution and coin tossing. In *International Conference on Computer System and Signal Processing, IEEE, 1984*, pages 175–179, 1984.

[6] C. H. Bennett. Notes on landauer's principle, reversible computation, and maxwell's demon. *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics*, 34(3):501–510, 2003.

[7] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher. Concentrating partial entanglement by local operations. *Physical Review A*, 53(4):2046, 1996.

[8] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters. Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels. *Physical review letters*, 70(13):1895, 1993.

[9] C. H. Bennett and S. J. Wiesner. Communication via one-and two-particle operators on einstein-podolsky-rosen states. *Physical review letters*, 69(20):2881, 1992.

[10] S. Bose, V. Vedral, and P. L. Knight. Multiparticle generalization of entanglement swapping. *Physical Review A*, 57(2):822, 1998.

[11] D. Bruß, D. P. DiVincenzo, A. Ekert, C. A. Fuchs, C. Macchiavello, and J. A. Smolin. Optimal universal and state-dependent quantum cloning. *Physical Review A*, 57(4):2368, 1998.

[12] V. Bužek and M. Hillery. Quantum copying: Beyond the no-cloning theorem. *Physical Review A*, 54(3):1844, 1996.

[13] V. Bužek and M. Hillery. Universal optimal cloning of arbitrary quantum states: from qubits to quantum registers. *Physical review letters*, 81(22):5003, 1998.

[14] V. Buzek, V. Vedral, M. Plenio, P. Knight, and M. Hillery. Broadcasting of entanglement via local copying. *Physical Review A*, 55(5):3327, 1997.

[15] N. J. Cerf. Asymmetric quantum cloning machines. *J. Mod. Opt.*, 47(quant-ph/9805024):187, 2000.

[16] N. J. Cerf. Pauli cloning of a quantum bit. *Physical review letters*, 84(19):4497, 2000.

[17] I. Chakrabarty. Impossibility of partial swapping of quantum information. *International Journal of Quantum Information*, 5(04):605–609, 2007.

[18] I. Chakrabarty and B. Choudhury. Secretly broadcasting five qubit entangled state among three parties from w-type states. *arXiv preprint arXiv:0804.2568*, 2008.

[19] S. Chatterjee, S. Sazim, and I. Chakrabarty. Broadcasting of quantum correlations: Possibilities and impossibilities. *Physical Review A*, 93(4):042309, 2016.

[20] I. Chattopadhyay, S. K. Choudhary, G. Kar, S. Kunkri, and D. Sarkar. No-flipping as a consequence of no-signalling and non-increase of entanglement under locc. *Physics Letters A*, 351(6):384–387, 2006.

[21] T. M. Cover and J. A. Thomas. *Elements of information theory*. John Wiley & Sons, 2006.

[22] T. Das, S. S. Roy, S. Bagchi, A. Misra, A. S. De, and U. Sen. Generalized geometric measure of entanglement for multiparty mixed states. *arXiv preprint arXiv:1509.02085*, 2015.

[23] A. S. De and U. Sen. Bound genuine multisite entanglement: Detector of gapless-gapped quantum transitions in frustrated systems. *arXiv preprint arXiv:1002.1253*, 2010.

[24] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera. Quantum privacy amplification and the security of quantum cryptography over noisy channels. *Physical review letters*, 77(13):2818, 1996.

[25] L.-M. Duan and G.-C. Guo. Probabilistic cloning and identification of linearly independent quantum states. *Physical review letters*, 80(22):4999, 1998.

[26] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical review*, 47(10):777, 1935.

[27] A. K. Ekert. Quantum cryptography based on bells theorem. *Physical review letters*, 67(6):661, 1991.

[28] W. Gerlach and O. Stern. Das magnetische moment des silberatoms. *Zeitschrift für Physik A Hadrons and Nuclei*, 9(1):353–355, 1922.

[29] W. Gerlach and O. Stern. Der experimentelle nachweis der richtungsquantelung im magnetfeld. *Zeitschrift für Physik A Hadrons and Nuclei*, 9(1):349–352, 1922.

[30] W. Gerlach and O. Stern. Der experimentelle nachweis des magnetischen moments des silberatoms. *Zeitschrift für Physik A Hadrons and Nuclei*, 8(1):110–111, 1922.

[31] I. Ghiu. Asymmetric quantum telecloning of d-level systems and broadcasting of entanglement to different locations using the many-to-many communication protocol. *Physical Review A*, 67(1):012323, 2003.

[32] N. Gisin. Quantum cloning without signaling. *arXiv preprint quant-ph/9801005*, 1998.

[33] N. Gisin and S. Massar. Optimal quantum cloning machines. *Physical review letters*, 79(11):2153, 1997.

[34] L. Hardy and D. D. Song. No signalling and probabilistic quantum cloning. *Physics Letters A*, 259(5):331–333, 1999.

[35] M. Hillery, V. Bužek, and A. Berthiaume. Quantum secret sharing. *Physical Review A*, 59(3):1829, 1999.

[36] R. Horodecki, M. Horodecki, and P. Horodecki. Teleportation, bell's inequalities and inseparability. *Physics Letters A*, 222(1):21–25, 1996.

[37] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki. Quantum entanglement. *Reviews of modern physics*, 81(2):865, 2009.

[38] M. Q. Mechanics. Jj sakurai. *Jim Napolitano*, 1985.

[39] M. Murao, M. Plenio, S. Popescu, V. Vedral, and P. Knight. Multiparticle entanglement purification protocols. *Physical Review A*, 57(6):R4075, 1998.

[40] R. Nepal, R. Prabhu, A. Sen, U. Sen, et al. Maximally-dense-coding-capable quantum states. *Physical Review A*, 87(3):032336, 2013.

[41] M. A. Nielsen and I. L. Chuang. *Quantum computation and quantum information*. Cambridge university press, 2010.

[42] M. A. Nielsen and I. L. Chuang. *Quantum computation and quantum information*. Cambridge university press, 2010.

[43] M. A. Nielsen and I. L. Chuang. *Quantum computation and quantum information*. Cambridge university press, 2010.

[44] H. Ollivier and W. H. Zurek. Quantum discord: a measure of the quantumness of correlations. *Physical review letters*, 88(1):017901, 2001.

[45] M. Oszmaniec, A. Grudka, M. Horodecki, and A. Wójcik. Creating a superposition of unknown quantum states. *Physical review letters*, 116(11):110403, 2016.

[46] A. K. Pati and S. L. Braunstein. Impossibility of deleting an unknown quantum state. *Nature*, 404(6774):164–165, 2000.

[47] A. K. Pati and B. C. Sanders. No-partial erasure of quantum information. *Physics Letters A*, 359(1):31–36, 2006.

[48] J. Preskill. Lecture notes for physics 229: Quantum information and computation. *California Institute of Technology*, 12:chapter 3, 1–5, 1998.

[49] S. Sazim and I. Chakrabarty. A study of teleportation and super dense coding capacity in remote entanglement distribution. *The European Physical Journal D*, 67(8):1–8, 2013.

[50] S. Sazim, I. Chakrabarty, A. Datta, and A. K. Pati. Complementarity of quantum correlations in cloning and deleting of quantum states. *Physical Review A*, 91(6):062311, 2015.

[51] S. Sazim, V. Chiranjeevi, I. Chakrabarty, and K. Srinathan. Retrieving and routing quantum information in a quantum network. *Quantum Information Processing*, 14(12):4651–4664, 2015.

[52] V. Scarani, S. Iblisdir, N. Gisin, and A. Acin. Quantum cloning. *Reviews of Modern Physics*, 77(4):1225, 2005.

[53] B. Schumacher. Quantum coding. *Physical Review A*, 51(4):2738, 1995.

[54] P. W. Shor and J. Preskill. Simple proof of security of the bb84 quantum key distribution protocol. *Physical review letters*, 85(2):441, 2000.

[55] G. Vidal and R. F. Werner. Computable measure of entanglement. *Physical Review A*, 65(3):032314, 2002.

[56] R. F. Werner. Quantum states with einstein-podolsky-rosen correlations admitting a hidden-variable model. *Physical Review A*, 40(8):4277, 1989.

[57] A. G. White, D. F. James, P. H. Eberhard, and P. G. Kwiat. Nonmaximally entangled states: Production, characterization, and utilization. *Physical review letters*, 83(16):3103, 1999.

[58] S. Wiesner. Conjugate coding. *ACM Sigact News*, 15(1):78–88, 1983.

[59] A. Winter and D. Yang. Operational resource theory of coherence. *Physical review letters*, 116(12):120404, 2016.

[60] W. K. Wootters. Entanglement of formation of an arbitrary state of two qubits. *Physical Review Letters*, 80(10):2245, 1998.

[61] W. K. Wootters. Entanglement of formation and concurrence. *Quantum Information & Computation*, 1(1):27–44, 2001.

[62] W. K. Wootters and W. H. Zurek. A single quantum cannot be cloned. *Nature*, 299(5886):802–803, 1982.

[63] D. Zhou, B. Zeng, and L. You. Quantum information cannot be split into complementary parts. *Physics Letters A*, 352(1):41–44, 2006.

[64] M. Zukowski, A. Zeilinger, M. Horne, and A. Ekert. " event-ready-detectors" bell experiment via entanglement swapping. *Physical Review Letters*, 71(26):4287–4290, 1993.